Lecturer: Dr. Hussam Jawad Kadhim AL_Janabi
Email: hussam.jawad@uomus.edu.iq

Experiment No. 3

Boolean Algebra and Logic Simplification

1. Introduction

Boolean algebra is a branch of algebra that deals with binary variables and logical operations. It is based on a mathematical structure that follows specific rules for logical operations, such as AND, OR, and NOT. Boolean algebra is fundamental in digital logic design, computer science, and set theory

1.1 Objective

To design a logical circuit and simplify it using Boolean algebra rules.

1.2 Work Environment

The CircuitMaker software is used to design and simulate logical circuits.

1.3 Theory

Consider the Boolean expression $A \cdot \overline{B} + \overline{A} \cdot B + A \cdot B = Y$, a logic diagram for which is in Figure 1. Note that six gates must be used to implement this logic circuit, which performs the logic detailed in the truth table Figure 2. From examination of the truth table, it is determined that a *single 2-input OR gate* will perform the function. It is found that the OR gate shown in Figure 3 will be the simplest method of performing this logic. The logic circuits in Figures 1 and 3 perform the same logic function. A designer would choose the simplest, least expensive circuit, as shown in Figure 3. It has been demonstrated that the unsimplified Boolean expression $A \cdot \overline{B} + \overline{A} \cdot B + A \cdot B = Y$ could be simplified to A+B = *Y*. The simplification was done by a simple examination of the truth table and recognizing the OR pattern. Many Boolean expressions can be greatly simplified.

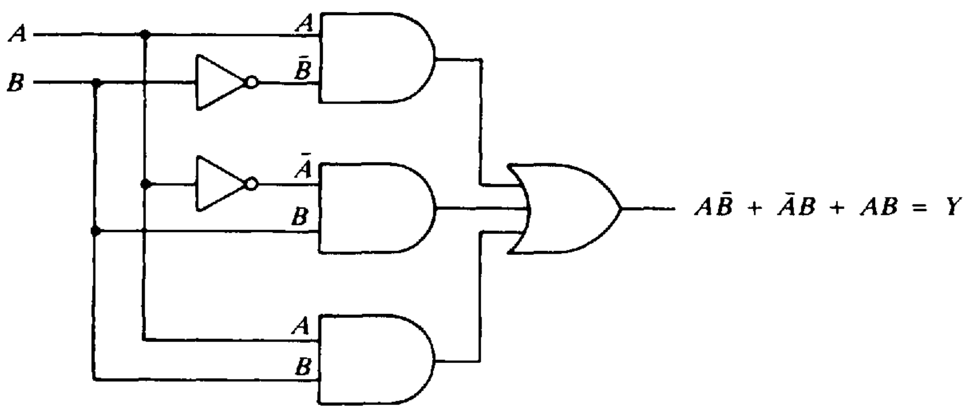


Figure 1

Inputs		Output
B	A	Y
0	0	0
0	1	1
1	0	1
1	1	1

Figure 2

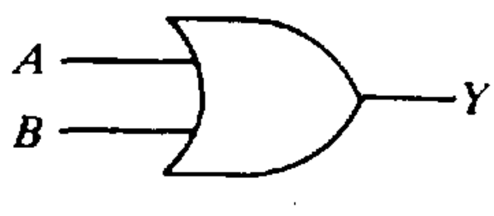


Figure 3

Lecturer: Dr. Hussam Jawad Kadhim AL_Janabi
Email: hussam.jawad@uomus.edu.iq

Rules of Boolean Algebra

TABLE 4–1

Basic rules of Boolean algebra.

1. $A + 0 = A$	7. $A \cdot A = A$
2. $A + 1 = 1$	8. $A \cdot \overline{A} = 0$
3. $A \cdot 0 = 0$	9. $\overline{\overline{A}} = A$
4. $A \cdot 1 = A$	10. $A + AB = A$
5. $A + A = A$	11. $A + \overline{A}B = A + B$
6. $A + \overline{A} = 1$	12. $(A + B)(A + C) = A + BC$

A, *B*, or *C* can represent a single variable or a combination of variables.

Figure 4

Experiment

Using CircuitMaker, complete the following steps:

1- Construct the logical circuit for the given Boolean expression:

AB + A(B + C) + B(B + C).

2- Create the truth table for the circuit.

3- Simplify the Boolean expression using Boolean algebra rules.

4- Draw the simplified logic circuit.

5- Generate the truth table after simplification.

Input			output
C	B	A	Y
0	0	1	
0	0	0	
0	1	1	
0	1	0	
1	0	1	
1	0	0	
1	1	1	
1	1	0	

Solution

The following is not necessarily the only approach.

Step 1: Apply the distributive law to the second and third terms in the expression, as follows:

$$AB + AB + AC + BB + BC$$

Step 2: Apply rule 7 ($BB = B$) to the fourth term.

$$AB + AB + AC + B + BC$$

Step 3: Apply rule 5 ($AB + AB = AB$) to the first two terms.

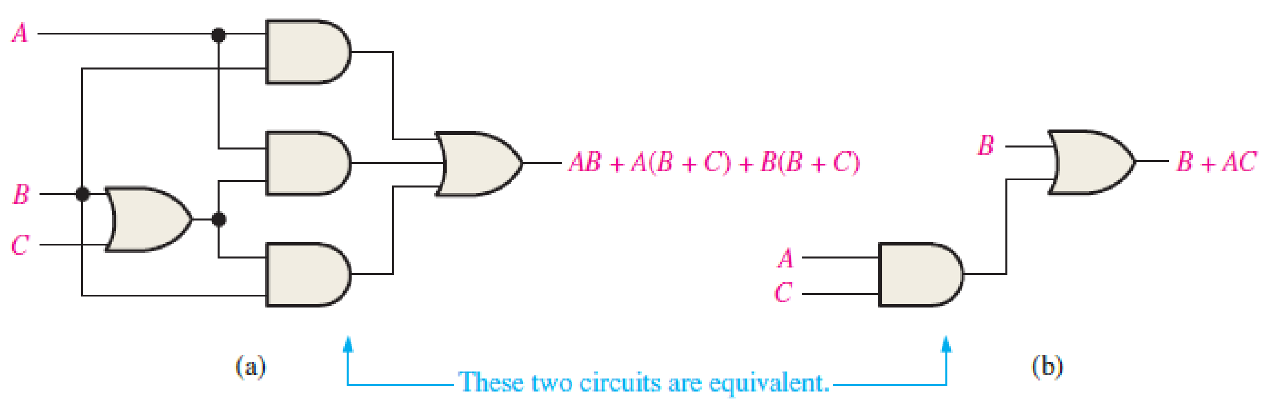
$$AB + AC + B + BC$$

Step 4: Apply rule 10 ($B + BC = B$) to the last two terms.

$$AB + AC + B$$

Step 5: Apply rule 10 ($AB + B = B$) to the first and third terms.

$$B + AC$$



Lecturer: Dr. Hussam Jawad Kadhim AL_Janabi
Email: hussam.jawad@uomus.edu.iq

Calculate the truth table after the simplification

Input			Output
C	B	A	Y
0	0	1	
0	0	0	
0	1	1	
0	1	0	
1	0	1	
1	0	0	
1	1	1	
1	1	0	

Discussion:

1- Why are Boolean algebra rules important in logic circuit design?

2- Explain the rules used in this experiment.

3- Illustrate the logic circuits before and after simplification

4- Compare the truth tables before and after simplification.