

Chapter Two

Pressure and Its Measurement . Abdulkareem Abdulwahab

2.1/ Fluid pressure at a point :

Consider a small area dA in large mass of fluid. If the fluid is static , then the force exerted by fluid on the area dA will always be perpendicular to the surface dA . Let dF is the force acting on the area dA in the normal direction.

Then the ratio of $\frac{dF}{dA}$ is known as the pressure (P). Hence mathematically the pressure at a point in a fluid at rest (static) is :

$$P = \frac{dF}{dA}$$

If the force (F) is uniformly distributed over the area (A) , the pressure at any point is given by :

$$P = \frac{F}{A} \quad (2.1)$$

The unit of pressure are (1) kgf / cm^2 (in MKS) (meter – kilogram – second)
 (2) $\text{Newton} / \text{m}^2$ (N / m^2) (in SI unit) . N / m^2 is known as Pascal (1 bar = 100 kpa = 10^5 Pascal)

2.2/ Pascal Law :

It states that the pressure or intensity of pressure at a point in a static fluid is equal in all directions .This is proved as :

The fluid element is of very small directions , i.e , (dx , dy and ds) .

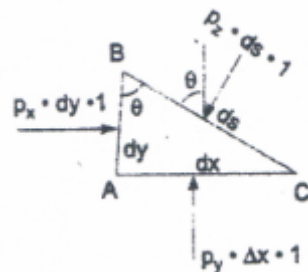


Fig.(2.1) Forces on a fluid element .

Consider an arbitrary fluid element of wedge shape in a fluid mass at rest , as shown in Fig.(2.1) . Let the width of the element perpendicular to the plane of

paper is unity and P_x , P_y and P_z are the pressure acting on the face AB, AC and BC respectively. Let angle ABC is Θ . Then the forces acting on the element are:

1. Pressure force normal to the surfaces.
2. Weight of the element in the vertical direction.

Force on the face AB = $P_x \times \text{area of face AB}$

$$= P_x \times dy \times 1$$

Force on the face AC = $P_y \times dx \times 1$

Force on the face BC = $P_z \times ds \times 1$

Weight of element = mass of element $\times g$

$$= (\text{volume} \times \rho) \times g = \left(\frac{AB \times AC}{2} \times 1\right) \times \rho \times g$$

$$\sum F_x = 0$$

$$P_x \times dy \times 1 - P_z (ds \times 1) \sin(90 - \Theta) = 0$$

$$P_x \times dy - P_z \times ds \times \cos \Theta = 0$$

But, from Fig.(2.1), $ds \cos \Theta = AB = dy$

$$P_x \times dy - P_z \times dy = 0$$

$$P_x = P_z$$

Similarly, $\sum F_y = 0$

$$P_y \times dx \times 1 - P_z \times ds \times 1 \times \cos(90 - \Theta) - \frac{dx \times dy}{2} \times 1 \times \rho \times g = 0$$

$$P_y \times dx - P_z ds \sin \Theta - \frac{dx dy}{2} \times \rho \times g = 0$$

But, $ds \sin \Theta = dx$, and the element has very small, therefore the weight is negligible (third term), therefore,

$$P_y = P_z$$

$$\text{Therefore, } P_x = P_y = P_z \quad (2.2)$$

This equation shows that the pressure at any point in x, y and z direction is equal.

2.3 / Pressure variation in a fluid at rest (fluid static) :

The pressure at any point in a fluid at rest is obtained by the hydrostatic law which states that the rate of increase of pressure in a vertically downward direction must be equal to the specific weight (weight density) of the fluid at the point. This is proved as :

Consider a small fluid element as shown in Fig.(2.2) .

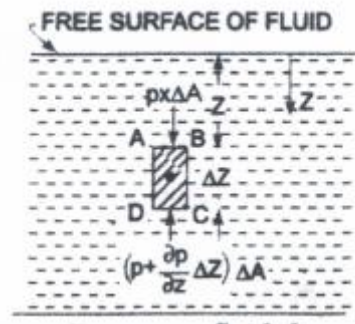


Fig.(2.2) Forces on a fluid element

Let , ΔA - cross – section area of element .

ΔZ - Height of fluid element .

P - pressure on face AB .

Z – distance of fluid element from free surface.

The forces acting on the fluid element are :

1. Pressure force on AB = $p \times \Delta A$ (acting perpendicular to face AB in the downward direction).
2. Pressure force on CD = $(p + \frac{\partial p}{\partial z} \Delta Z) \times \Delta A$ (acting perpendicular to face CD vertically upward direction).
3. Weight of fluid element = $\gamma \times \text{volume} = \rho g (\Delta A \times \Delta Z)$.
4. Pressure forces on surface BC and AD are equal and opposite.

Forequilibrium of fluid element , we have

$$p\Delta A - (p + \frac{\partial p}{\partial z} \Delta Z) \Delta A + \rho g (\Delta A \times \Delta Z) = 0$$

$$p\Delta A - p\Delta A - \frac{\partial p}{\partial z} \Delta Z \Delta A + \rho g \times \Delta A \times \Delta Z = 0$$

$$- \frac{\partial p}{\partial z} \Delta Z \Delta A + \rho g \times \Delta A \Delta Z = 0$$

$$\frac{\partial p}{\partial z} \Delta Z \Delta A = \rho g \times \Delta A \Delta Z$$

$$\frac{\partial P}{\partial Z} = \gamma$$

$$\frac{dP}{dZ} = \gamma \quad , \quad d p = \gamma dz \quad , \quad \int dp = \gamma \int dz$$

$$P = \gamma Z \quad (2.3)$$

Equation (2.3) states that the rate of increase of pressure in vertical direction is equal to weight density (γ) of the fluid at that point. This is Hydrostatic Law . (Z is called pressure head) .

2.4 / Absolute , Gauge , Atmospheric , And Vacuum Pressures

The pressure on the fluid is measured in two difference systems. In one system , it is measured above the absolute zero or complete vacuum and it is called the absolute pressure and in other system , pressure is measured above the atmospheric pressure and is called gauge pressure .

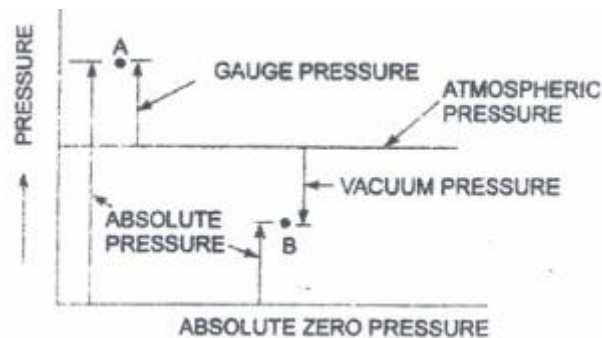


Fig.(2.3) Relationship between pressure.

The relationship between the absolute pressure , gauge pressure and vacuum pressure are shown in Fig.(2.3) .

Mathematically :

$$P_{abs.} = P_{atm.} \pm P_{gauge} \quad (2.4)$$

$$P_{A(abs)} = P_{atm.} + P_{gauge}$$

$$P_{B(abs)} = P_{atm.} - P_{gauge(vacuum)}$$

The values of atmospheric pressure at sea level at 15^oc :

$$P_{atm.} = 101.3 \text{ KN/m}^2(\text{ kpa}) \quad , \quad P_{atm.} = 10^5 \text{ N/m}^2(\text{ Pascal})$$

$P_{\text{atm.}} = 76 \text{ cm Hg.}$, $P_{\text{atm.}} = 10 \text{ m(water)}$, $P_{\text{atm.}} = 14.7 \text{ psi}$

$P_{\text{atm.}} = 14.7 \text{ psi}$. $P_{\text{atm.}} = 1\text{bar}$.

2.5/ Measurement of pressure :

The pressure of a fluid is measured by the following devices :

1. Manometers .
2. Mechanical Gauges .

2.5.1/ Manometers :

Manometers are defined as the devices used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same or another column of the fluid. They are classified as :

(1) simple manometers , (2) Differential manometers

2.5.2 / Simple Manometers :

A simple manometer consists of a glass tube having one of its ends connected to a point where pressure is to be measured and other end remains open to atmosphere. Common types of simple manometers are :

1. Piezometer. ,
2. U – tube Manometer. ,
3. Single Column Manometer.

1. Piezometer :

It is simple form of manometer , used for measuring gauge pressures , as shown in Fig.(2.4)

$$P_A = \rho g h = \gamma h \quad \text{N/m}^2 \text{ (Pascal)} \quad (2. 5)$$

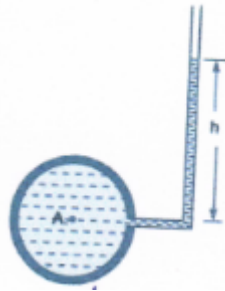


Fig.(2.4) Piezometer.

2. U– tube Manometer :

It consists of glass tube bent in U- shape , one end of which is connected to a point at which pressure is to be measured and other end remains open to

the atmosphere as shown in Fig.(2.5) . In this manometer , we can measure positive pressure (gauge pressure) and negative pressure (vacuum) .

Let B is the point at which pressure is to be measured (p) . The datum line is A – A .

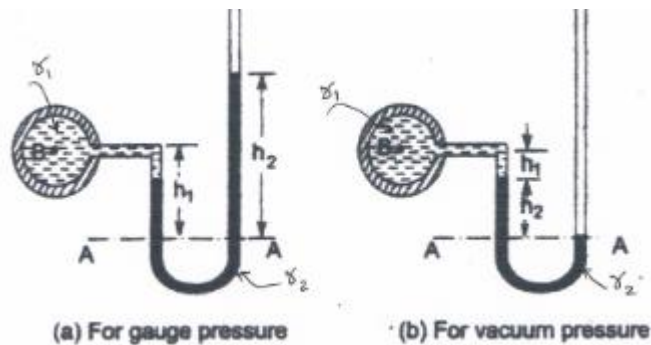


Fig.(2.5) U- tube Manometer.

If we want to measure the pressure (p) at point B .

There are two legs in the manometer , if there is equilibrium between two legs (right and left legs) over the datum (A – A) , i . e the pressure at each leg over the datum are equal .

Mathematically ,

(a) For gauge pressure :

Pressure at left leg = Pressure at right leg

$$P + \gamma_1 h_1 = \gamma_2 h_2$$

$$P = \gamma_2 h_2 - \gamma_1 h_1 \quad \text{N/m}^2 \quad (2.6)$$

(b) For vacuum (negative) pressure :

Pressure at left leg = pressure at right leg

$$P + \gamma_1 h_1 + \gamma_2 h_2 = 0$$

$$P = - \gamma_1 h_1 - \gamma_2 h_2 \quad \text{N/m}^2 \quad (2.7)$$

2.6/ Differential Manometers :

Differential manometer are the devices used for measuring the difference of pressures between between two points in a pipe or in two different pipes . A differential manometer consists of a U – tube , containing a heavy liquid (liquid manometer) , frequently is mercury (Hg). Most commonly types of differential manometers are :

1.U-tube differential manometer. , 2 – Inverted U-tube differential manometer . Fig.(2.6) shows the differential manometer of U-tube type.

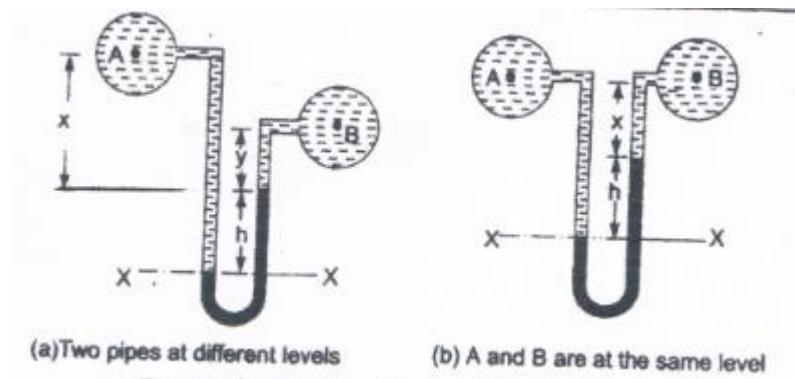


Fig.(2.6) U-tube differential manometer .

In Fig.(2.6) (a) , let the two points A & B are at different level and also contains liquids at different specific gravity (S) (sp. gr.).

Level X - X , level of equilibrium , the pressures in the left leg equal the pressures in right leg :

$$p_A + \gamma_A (x + h) = p_B + \gamma_B y + \gamma_m h$$

$$p_A - p_B = \gamma_B y + \gamma_m h - \gamma_A (x + h) \quad (2.8)$$

In Fig.(2.6) (b) ,

$$p_A + \gamma_A (x + h) = p_B + \gamma_B x + \gamma_m h$$

$$p_A - p_B = \gamma_B x + \gamma_m h - \gamma_A (x + h) \quad (2.9)$$

2. Inverted U-tube differential manometer :

It consists of an inverted U-tube . The two ends of the tube are connected to the points whose difference of pressure is to be measured. It is used for measured difference of low pressure . Fig.(2.7) shows an inverted U- tube differential manometer connected to the two points A & B. Let the pressure at A is more than the pressure at B.

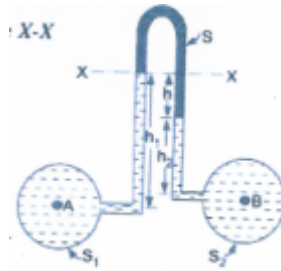


Fig.(2.7)

Taking X - X as datum line , then ,

Pressure at the left leg below the X - X = $p_A - \gamma_1 h_1$

Pressure at the right leg below the X- X = $p_B - \gamma_2 h_2 - \gamma_m h$

Pressure at the left leg = pressure at the right leg

$$P_A - \gamma_1 h_1 = p_B - \gamma_2 h_2 - \gamma_m h$$

$$P_A - p_B = \gamma_1 h_1 - \gamma_2 h_2 - \gamma_m h \quad (2.10)$$

1.7 / Inclined Single column Manometer :

Fig.(2.8) shows the inclined single column manometer. This manometer is more sensitive . Due to inclination the distance moved by the heavy liquid in the right side will be more.

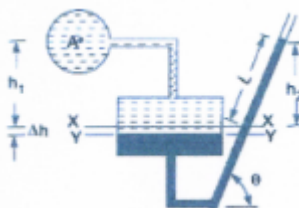


Fig.(2.8) Inclined manometer

Let L – length of heavy liquid moved in right side from X - X

Θ - inclination of right leg with horizontal.

h_2 - vertical rise of heavy liquid in right leg from X - X

$$(L \sin \Theta)$$

$$P_A = \gamma_2 h_2 - \gamma_1 h_1 \quad (\text{but } h_2 = L \sin \Theta)$$

$$P_A = \gamma_2 L \sin \Theta - \gamma_1 h_1 \quad (2.11)$$

2.8/ Micromanometer :

It is used for determine small differences in pressure .With two gage liquids , immiscible in each other and in the fluid to be measured , a large gage difference R , as shown in Fig.(2.9) can be produced for a small pressure difference. The heavier gage liquid fills the lower U-tube up to O - O then the lighter gage liquid is added to both sides , filling the larger reservoir up to 1 - 1 . The gas or liquid in the system fills the space above 1 - 1 . The gas or liquid in the system fills the space above 1 - 1 . When the pressure at C is slightly greater than at D, the menisci move as indicated in Fig.(2.9) . The volume of liquid displaced in each reservoir equals the displacement in the U -tube , thus ,

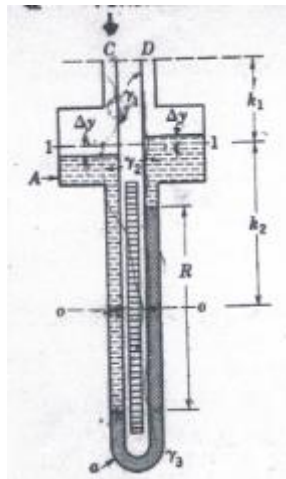


Fig.(2.9) Micromanometer

$$\Delta y \cdot A = \frac{R}{2} \cdot a \quad , \quad \Delta y = \frac{R a}{2A}$$

In which , A is area of reservoir , a is area of U - tube .

The manometer equation may be written starting at C ,

$$P_c + (k_1 + \Delta y)\gamma_1 + (k_2 - \Delta y + \frac{R}{2})\gamma_2 - R\gamma_3 - (k_2 - \frac{R}{2} + \Delta y)\gamma_2 -$$

$$(k_1 - \Delta y) \gamma_1 = p_D$$

In which γ_1 , γ_2 , γ_3 are the specific weights. Simplifying and substituting for Δy gives :

$$p_C - p_D = R \left[\gamma_3 - \gamma_2 \left(1 - \frac{a}{A} \right) - \gamma_1 \frac{a}{A} \right] \quad (2.12)$$

The quantity in bracket is a constant for specified gage and fluids, hence, the pressure difference is directly proportional to R .

1.8 / Bourdon Gage (Mechanical) :

The bourdon pressure gage as shown in Fig.(2.10) is typical of the devices used for measuring gage pressure .

The bourdon gage (shown schematically) in Fig.(2.11) . In the gage , a bent tube (A) of elliptical cross section is held rigidly at (B) and its free end is connected to a pointer (C) by a link (D) . When pressure is

admitted to the tube , its cross section tends to become circular , causing the tube to straighten and move the pointer to the right over the graduated scale .



Fig.(2.10) typical of Bourdon gage

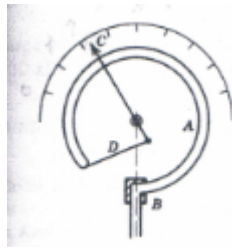


Fig.(2.11) Schematically shown of Bourdon gage

The pointer rests at zero on the scale , when the gauge is disconnected , in this condition the pressure inside and outside of the tube are the same.