Chapter Six

Momentum Equation Dr.Abdulkareem A.Wahab

6.1 / Introduction :

In the based on the law of conservation of momentum or on the momentum principle , which states that the net force acting on a fluid mass is equal to the change in momentum of flow per unit time in the direction.

The force acting on a fluid mass (m) is given by the Newton's second law of motion :

$$F = m\,a$$

Where (a) is the acceleration acting in the same direction F .

But $a = \frac{dv}{dt}$

$$F = m\,\frac{dv}{dt} \qquad (6.1)$$

$$F\,dt = m\,dv \qquad (6.2)$$

The equation (6.2) is known as the (impulse – momentum) equation , and states that the impulse of a force (F) acting on a fluid of mass (m) in a short intervalof time (dt) is equal to the change of momentum d (mv) in the direction of force .

6.2 / Force exerted by a flowing fluid on a pipe bend :

The impulse – momentum equation (6.2) is used to determine the resultant force exerted by a flowing fluid on a pipe bend .

Consider two sections (1) & (2) , as shown in Fig.(6.1)

V_1 , P_1 , A_1 are velocity , pressure and area at section (1)

V_2 , P_2 , A_2 are velocity , pressure and area at section (2) .

Let F_x , F_y be the components of force exerted by the flowing fluid on the bend in x – direction and y – direction respectively .

The other external forces acting on the fluid are p_1A_1 and p_2A_2 on the sections (1) and (2) respectively. Then momentum equation in x – direction is given by:

$$\sum F_x = \dot{m}\,\Delta v_x \qquad (6.3)$$

And in y – direction $\sum F_y = \dot{m}\, \Delta v_y$(6.4)

There are two cases for the bend , first case when the angle of bend is $\theta \leq 90^o$ and the second case , when the angle of bend is $90^o \leq \theta \geq 180^o$.

First case ($\theta \leq 90^o$) :

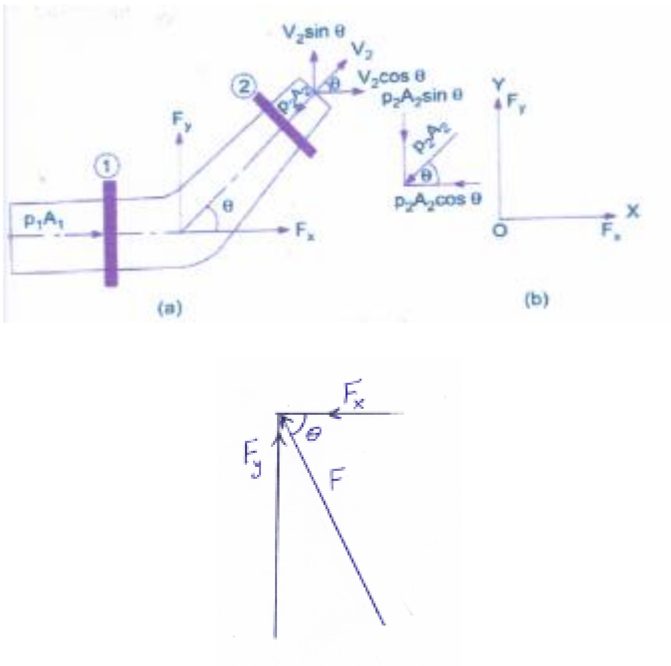

Fig.(6.1) Forces on bend

Applying eq.(6.3) , in x – direction :

$$p_1 A_1 - p_2 A_2 \cos\theta - F_x = \rho Q (V_2 \cos\theta - V_1)$$

$$F_x = - \rho Q (V_2 \cos\theta – V_1) + p_1 A_1 - p_2 A_2 \cos\theta$$

$$F_x = \rho Q (V_1 – V_2 \cos\theta) + p_1 A_1 – p_2 A_2 \cos\theta \qquad (6.5)$$

Similarly the momentum equation (6 .4) in y – direction gives (the weight of bend is acting in y – direction only , and neglected) :

$$F_y - p_2 A_2 \sin\theta = \rho Q (v_2 \sin\theta - 0)$$

$$F_y = \rho Q v_2 \sin\theta + p_2 A_2 \sin\theta \qquad (6.6)$$

The resultant force (F_R) acting on the bend :

$$F_R = \sqrt{F_x^2 + F_y^2} \qquad (6.7)$$

And the angle made by the resultant force (F_R) with horizontal direction is given by :

$$\tan\theta = \frac{F_y}{F_x} \qquad (6.8)$$

<u>Second case (180 ≥ θ ≥ 90 </u>:

When the angle θ (angle of bend) with horizontal direction less than 180^o and more than 90^o , as shown in Fig. (6 .2).

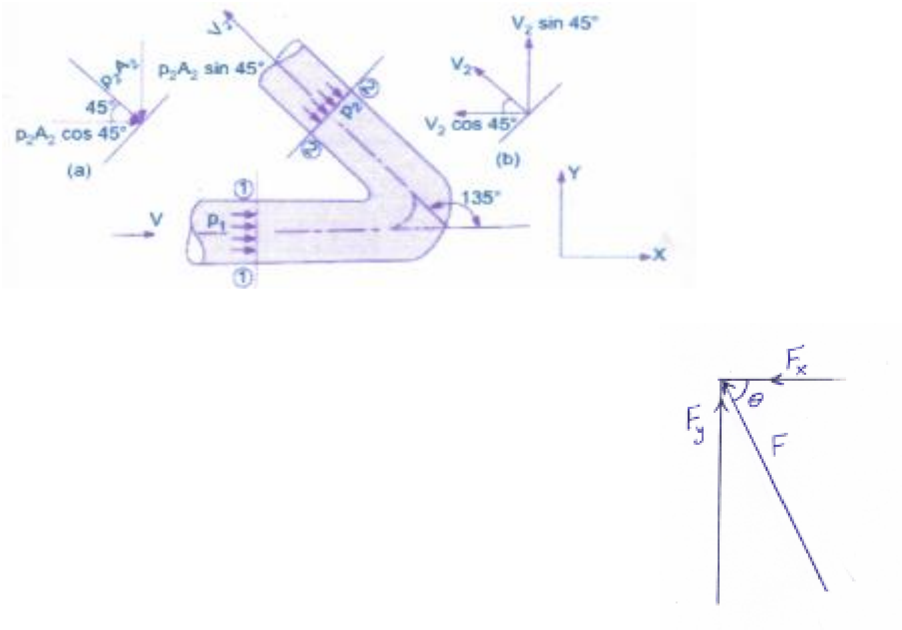

Fig.(6.2)

Applying eq.(6.3) in x – direction :

$$\sum F_x = \dot{m}\,\Delta v_x \qquad (6.3)$$

$$p_1A_1 + p_2A_2\cos\theta - F_x = \rho Q\,(V_2\cos\theta - V_1)$$

$$F_x = p_1A_1 + p_2A_2\cos\theta - \rho Q\,(V_2\cos\theta - V_1) \qquad (6.9)$$

Applying eq.(6.4) in y – direction :

$$\sum F_y = \dot{m}\,\Delta v_y \qquad (6.4)$$

$$F_y - p_2A_2\sin\theta = \rho Q\,(V_2\sin\theta - 0)$$

$$F_y = \rho Q\,(V_2\sin\theta) + p_2A_2\sin\theta \qquad (6.10)$$

The resultant of forces (F_R) :

$$F_R = \sqrt{F_x^2 + F_y^2} \qquad (6.11)$$

$$\tan\theta = \frac{F_y}{F_x} \qquad (6.12)$$