Chapter five

Integration

5-1- Indefinite integrals :

The set of all anti derivatives of a function is called indefinite integral of the function.

Assume *u* and *v* denote differentiable functions of *x*, and *a*, *n*, and *c* are constants, then the integration formulas are:-

$1)\ \int du = u(x) + c$

$2)\ \int a \cdot u(x)dx = a\int u(x)dx$

$3)\ \int \left(u(x) \mp v(x)\right)dx = \int u(x)\,dx \mp \int v(x)\,dx$

$4)\ \int u^n du = \frac{u^{n+1}}{n+1} + c \quad when \quad n \neq -1 \quad \& \quad \int u^{-1}du = \int \frac{1}{u}du = \ln u + c$

$5)\ \int a^u du = \frac{a^u}{\ln a} + c \qquad \Rightarrow \qquad \int e^u du = e^u + c$

EX-1 **– Evaluate the following integrals:**

$1)\ \int 3x^2\,dx$

$2)\ \int \left(\frac{1}{x^2} + x\right)dx$

$3)\ \int x\sqrt{x^2+1}\,dx$

$4)\ \int \left(2t + t^{-1}\right)^2 dt$

$5)\ \int \sqrt{(z^2 - z^{-2})^2 + 4}\,dz$

$6)\ \int \frac{x+3}{\sqrt{x^2+6x}}\,dx$

$7)\ \int \frac{x+2}{x^2}\,dx$

$8)\ \int \frac{e^x}{1+3e^x}\,dx$

$9)\ \int 3x^3 \cdot e^{-2x^4}\,dx$

$10)\ \int 2^{-4x}\,dx$

Sol. **–**

$1)\ \int 3x^2\,dx = 3\int x^2dx = 3\frac{x^3}{3} + c = x^3 + c$

$$2)\ \left(x^{-2}+x\right)dx=\int x^{-2}\,dx+\int x\,dx=\frac{x^{-1}}{-1}+\frac{x^2}{2}+c=-\frac{1}{x}+\frac{x^2}{2}+c$$

$$3)\ \int x\sqrt{x^2+1}\,dx=\frac{1}{2}\int 2x(x^2+1)^{1/2}dx=\frac{1}{2}\frac{(x^2+1)^{3/2}}{3/2}+c=\frac{1}{3}\sqrt{(x^2+1)3}+c$$

$$4)\ \int\left(2t+t^{-1}\right)^2 dt=\int\left(4t^2+4+t^{-2}\right)dt=4\frac{t^3}{3}+4t+\frac{t^{-1}}{-1}+c=\frac{4}{3}t^3+4t-\frac{1}{t}+c$$

$$5)\ \int\sqrt{(z^2-z^{-2})^2+4}\,dz=\int\sqrt{z^4-2+z^{-4}+4}\,dz=\int\sqrt{z^4+2+z^{-4}}\,dz$$

$$=\int\sqrt{(z^2+z^{-2})^2}\,dz=\int(z^2+z^{-2})\,dz=\frac{z^3}{3}+\frac{z^{-1}}{-1}+c=\frac{1}{3}z^3-\frac{1}{z}+c$$

$$6)\ \int\frac{x+3}{\sqrt{x^2+6x}}dx=\frac{1}{2}\int(2x+6)\cdot(x^2+6x)^{-1/2}dx$$

$$=\frac{1}{2}\cdot\frac{(x^2+6x)^{1/2}}{1/2}+c=\sqrt{x^2+6x}+c$$

$$7)\ \int\frac{x+2}{x^2}dx=\int\left(\frac{x}{x^2}+\frac{2}{x^2}\right)dx=\int\left(x^{-1}+2x^{-2}\right)dx=\ln x+\frac{2x^{-1}}{-1}+c=\ln x-\frac{2}{x}+c$$

$$8)\ \int\frac{e^x}{1+3e^x}dx=\frac{1}{3}\int 3e^x(1+3e^x)^{-1}dx=\frac{1}{3}\ln(1+3e^x)+c$$

$$9)\ \int 3x^3\cdot e^{-2x^4}dx=-\frac{3}{8}\int -8x^3\cdot e^{-2x^4}dx=-\frac{3}{8}\cdot e^{-2x^4}+c$$

$$10)\ \int 2^{-4x}dx=-\frac{1}{4}\int 2^{-4x}\cdot(-4dx)=-\frac{1}{4}\cdot 2^{-4x}\cdot\frac{1}{\ln 2}+c$$

5-2- <u>Integrals of trigonometric functions</u> :

The integration formulas for the trigonometric functions are:

$$6)\ \int \sin u\cdot du=-\cos u+c$$

$$7)\ \int \cos u\cdot du=\sin u+c$$

$$8)\ \int \tan u\cdot du=-\ln|\cos u|+c$$

$$9)\ \int \cot u\cdot du=\ln|\sin u|+c$$

$$10)\ \int \sec u\cdot du=\ln|\sec u+\tan u|+c$$

$$11)\ \int \csc u\cdot du=-\ln|\csc u+\cot u|+c$$

$$12)\ \int \sec^2 u\cdot du=\tan u+c$$

$$13)\ \int \csc^2 u\cdot du=-\cot u+c$$

$$14)\ \int \sec u\cdot\tan u\cdot du=\sec u+c$$

$$15)\ \int \csc u\cdot\cot u\cdot du=-\csc u+c$$

EX-2- **Evaluate the following integrals:**

1) $\int cos(3\theta - 1)d\theta$

2) $\int x \cdot sin(2x^2)dx$

3) $\int cos^2(2y) \cdot sin(2y)dy$

4) $\int sec^3 x \cdot tan x \ dx$

5) $\int \sqrt{2 + sin3t} \cdot cos3t \ dt$

6) $\int \frac{d\theta}{cos^2\theta}$

7) $\int (1 - sin^2 3t) \cdot cos3t \ dt$

8) $\int tan^3(5x) \cdot sec^2(5x) \ dx$

9) $\int sin^4 x \cdot cos^3 x \ dx$

10) $\int \frac{cot^2\sqrt{x}}{\sqrt{x}} dx$

Sol.-

1) $\frac{1}{3}\int 3cos(3\theta - 1)d\theta = \frac{1}{3}sin(3\theta - 1) + c$

2) $\frac{1}{4}\int 4x \cdot sin(2x^2)dx = -\frac{1}{4}cos(2x^2) + c$

3) $-\frac{1}{2}\int (cos 2y)^2 \cdot (-2 sin 2y \ dy) = -\frac{1}{2} \cdot \frac{(cos 2y)^3}{3} + c = -\frac{1}{6}(cos 2y)^3 + c$

4) $\int sec^2 x \cdot (sec x \cdot tan x \cdot dx) = \frac{sec^3 x}{3} + c$

5) $\frac{1}{3}\int (2 + sin 3t)^{1/2}(3cos 3t \ dt) = \frac{1}{3} \cdot \frac{(2 + sin 3t)^{3/2}}{3/2} + c = \frac{2}{9}\sqrt{(2 + sin 3t)^3} + c$

6) $\int \frac{d\theta}{cos^2\theta} = \int sec^2\theta \cdot d\theta = tan\theta + c$

7) $\int (1 - sin^2 3t) \cdot cos 3t \ dt = \frac{1}{3}\int 3cos 3t \ dt - \frac{1}{3}\int (sin3t)^2 \cdot 3cos3t \ dt$

$= \frac{1}{3}sin 3t - \frac{1}{3} \cdot \frac{sin^3 3t}{3} + c = \frac{1}{3} \cdot sin 3t - \frac{1}{9}sin^3 3t + c$

8) $\frac{1}{5}\int tan^3 5x \cdot (5 sec^2 5x \ dx) = \frac{1}{5} \cdot \frac{tan^4 5x}{4} + c = \frac{1}{20}tan^4 5x + c$

9) $\int sin^4 x \cdot cos^3 x \ dx = \int sin^4 x \cdot (1 - sin^2 x) \cdot cos x \ dx$

$= \int sin^4 x \cdot cos x \ dx - \int sin^6 x \cdot cos x \ dx = \frac{sin^5 x}{5} - \frac{sin^7 x}{7} + c$