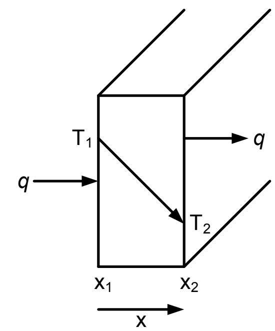
Steady-State Conduction One Dimension

To examine the applications of Fourier's law of heat conduction to calculation of heat flow in some simple one-dimensional systems, we may take the following different cases:

1- The plane wall

A) One material



Using Fourier's law

$$q = -kA\frac{dT}{dx} \quad by \int \quad \Rightarrow$$

$$q = -kA\ \frac{T_2 - T_1}{x_2 - x_1}$$

$$Flow = \frac{potential\ (Driving\ Force)}{Resistance}$$

$$I = \frac{V}{R}$$

$$\therefore \frac{\Delta x}{kA} = Thermal Resistance$$

- When the thermal conductivity is considered constant
-

$$q = -\frac{kA}{\Delta x}(T_2 - T_1)$$

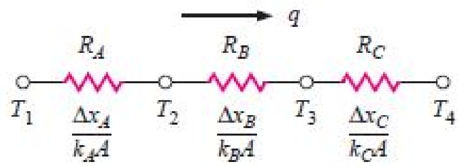
- When the thermal conductivity varies with temperature, the k can be described as

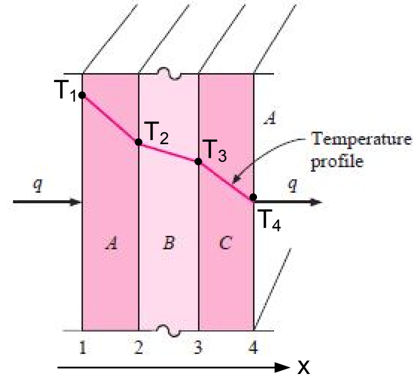
$$k = k_0(1 + \beta T)$$

k_0 and β are constants. The resultant equation for the heat flow is

$$q = -\frac{k_0 A}{\Delta x}\left[(T_2 - T_1) + \frac{\beta}{2}(T_2^2 - T_1^2)\right]$$

B) More than one material (Composite wall)





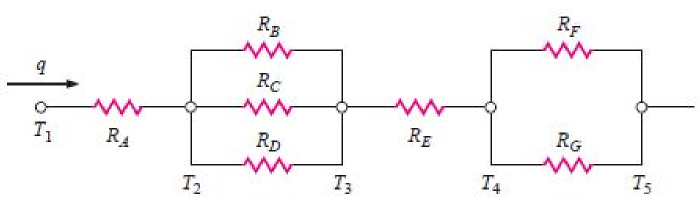
The heat flow must be the same through all sections, therefore,

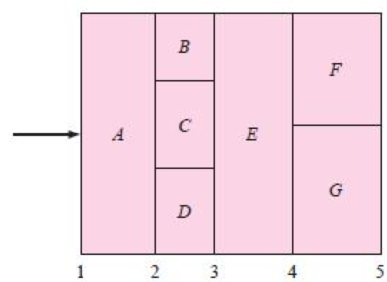
$$q = -k_A A \frac{T_2 - T_1}{\Delta x_A} = -k_B A \frac{T_3 - T_2}{\Delta x_B} = -k_C A \frac{T_4 - T_3}{\Delta x_C}$$

Solving these three equations simultaneously, the heat flow is written:

$$q = \frac{T_1 - T_4}{\Delta x_A / k_A A + \Delta x_B / k_B A + \Delta x_C / k_C A}$$

For series and parallel one-dimensional heat transfer through a composite wall and electrical analog:





$$\frac{1}{R_1} = \frac{1}{R_A}; \quad \frac{1}{R_2} = \frac{1}{R_B} + \frac{1}{R_C} + \frac{1}{R_D}; \quad \frac{1}{R_3} = \frac{1}{R_E}; \quad \frac{1}{R_4} = \frac{1}{R_F} + \frac{1}{R_G}$$

$$\therefore \quad \frac{1}{R_{th}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$

(R_{th} is the thermal resistances)

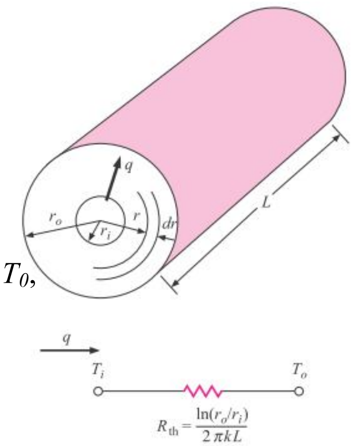
Generally, one-dimensional heat-flow equation for this type of problem may be written

$$q = \frac{\Delta T_{\text{overall}}}{\sum R_{\text{th}}}$$

2- Radial systems

A) Cylindrical

i- One material



Consider a long cylinder of inside radius r_i, outside radius r_o, and length L. The inner side temperature is T_i, The outer side is T_0, when the heat flows only in a radial direction.
The area for heat flow in the cylindrical system is

$$A_r = 2\pi rL$$

So that Fourier's law is written

$$q_r = -kA_r \frac{dT}{dr}$$

or

$$q_r = -2\pi\ krL \frac{dT}{dr}$$

$$\frac{q}{2\pi\ kL} \int_{r_i}^{r_0} \frac{dr}{r} = -\int_{T_i}^{T_0} dT$$

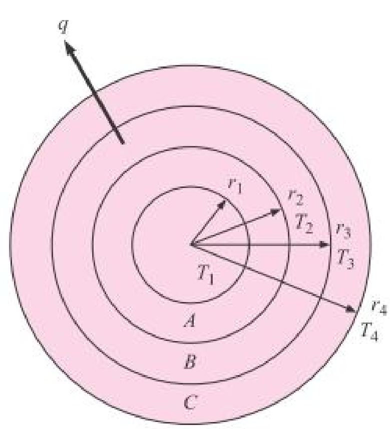
The solution is

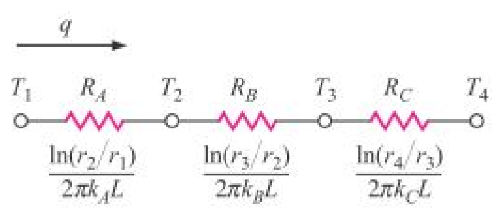
$$q = \frac{2\pi kL\ (T_i - T_o)}{\ln\ (r_o/r_i)}$$

and the thermal resistance in this case is

$$R_{th} = \frac{\ln\ (r_o/r_i)}{2\pi kL}$$

ii- Multi-Layer cylindrical wall

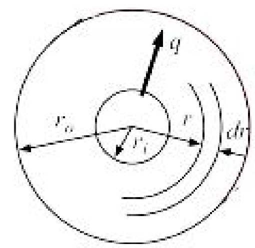




For the system shown, the solution is:

$$q = \frac{2\pi L\,(T_1 - T_4)}{\ln(r_2/r_1)/k_A + \ln(r_3/r_2)/k_B + \ln(r_4/r_3)/k_C}$$

B) Spherical



$Ar = 4\pi\, r^2$

Spherical systems may also be treated as one-dimensional when the temperature is a function of radius only. The heat flow is then

$$q_r = -kA_r \frac{dT}{dr}$$

or

$$q_r = -4k\pi\, r^2 \frac{dT}{dr}$$

$$\frac{d}{4\pi\, k}\int_{r_i}^{r_0} \frac{dr}{r^2} = -\int_{T_i}^{T_0} dT$$

$$q = \frac{4\pi k\,(T_i - T_o)}{1/r_i - 1/r_o}$$

The thermal resistance in spherical system is:

$$R_{th} = \frac{1}{4\pi k}\left(\frac{1}{r_i} - \frac{1}{r_0}\right)$$

Example) **An outside wall of a building consists of 0.1m layer of common brick [k=0.69 W/m.K] and 25mm layer of fiber glass [k=o.o5 W/m.K]. Calculate the heat flow with through the wall for a 45°C temperature differences.**

Solution

$$q = \frac{\Delta T}{\sum R_{th}} = \frac{\Delta T_{overall}}{\frac{\Delta x_b}{k_b A} + \frac{\Delta x_f}{k_f A}}$$

$$\Rightarrow \quad q = \frac{45}{\frac{0.1}{0.69} + \frac{0.025}{0.05}} = 69.78\, W/m^2$$