

Theory of structure

Stability and determinacy of structures

Beams

- ❖ Total equation of equilibrium of beam

$$\sum F_X = 0$$

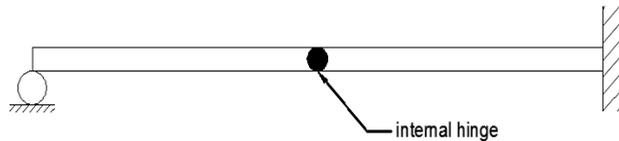
$$\sum F_Y = 0$$

$$\sum M = 0$$

- ❖ Equation of condition

Internal hinge:-

$$\sum M = 0$$



$$C=1$$

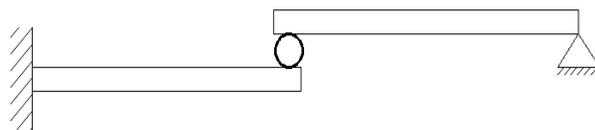
$$C = m - 1$$

Roller:-

$$\sum M = 0$$

$$\sum F_X = 0$$

$$C=2$$



Let r = No. of reaction

1- If $r < c+3$, unstable

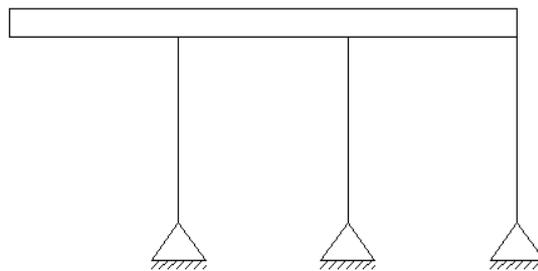
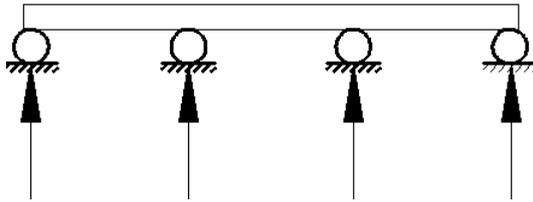
2- $r = c+3$, determine if stable

3- $r > c+3$, indeterminate if stable

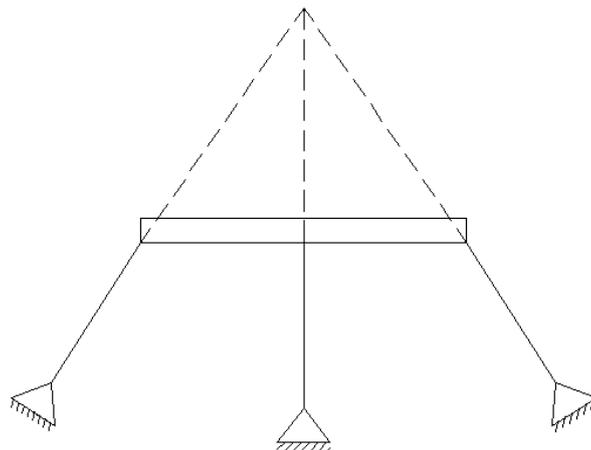
Let (m) degree of indeterminate

$$m = r - (c+3)$$

- ❖ the structure is said to be unstable if one of the following facts counter
- 1- $r < c+3$
 - 2- The reaction element constitutes a parallel force system.



- 3- The reaction element constitutes a concurrent force system.



4- Internal geometric instability:-

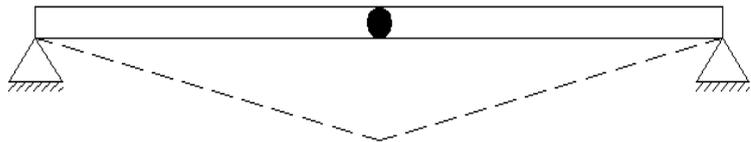
Example

$r = 4$

$c = 1$

$r = c + 3$

$4 = 4$



The beam is unstable because the Internal geometric instability

Example

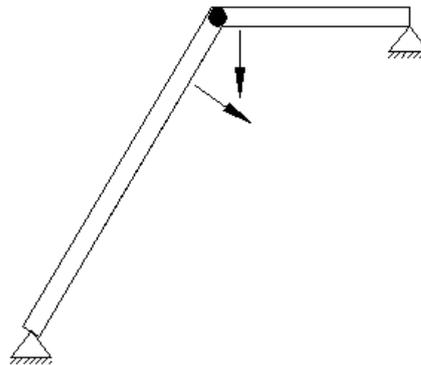
$r = 4$

$c = m - 1, c = 1$

$r = c + 3$

$4 = 4$

The beam is determinate if stable



Example

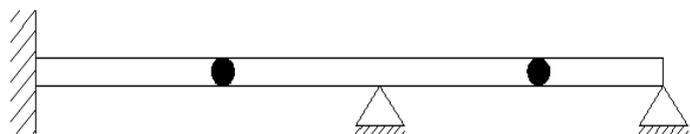
$r = 7$

$c = 2$

$r > c + 3$

$7 > 5$

The beam is indeterminate 2nd degree if stable



Example

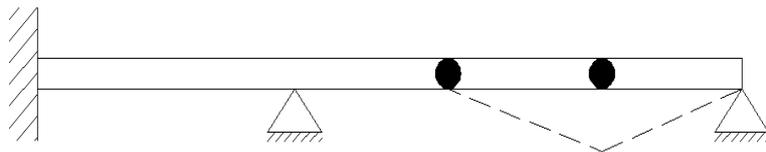
$r = 7$

$c = 2$

$r > c + 3$

$7 > 5$

The beam is unstable

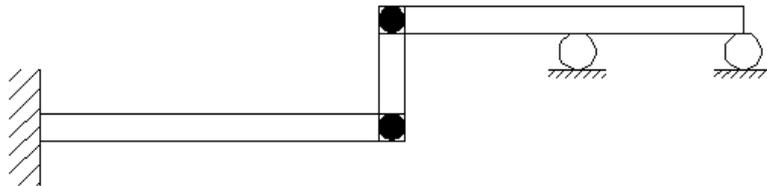


Examples:-

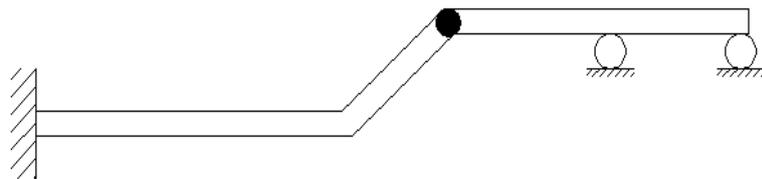
Beam	r	c	c+3	state	Stability & determinate.
	3	0	3	$r = c + 3$	Stable & deter.
	4	0	3	$r > c + 3$	Stable & indeter. First degree
	6	1	4	$r > c + 3$	Stable & indeter. Second degree
	6	2	5	$r > c + 3$	unstable
	3	0	3	$r = c + 3$	unstable

Home Works

H.W1: Find the stability and determinacy of beam.



H.W2: Find the stability and determinacy of beam.



Stability and Determinacy of Trusses

$b + r = \text{unknown}$

$j = \text{equations}$

- 1- $b + r < 2j$, the truss is unstable
- 2- $b + r = 2j$, the truss is determinate if stable
- 3- $b + r > 2j$, the truss is indeterminate if stable

Let (m) equal to the degree of indeterminate

$$m = (b + r) - 2j$$

$b = \text{No. of bars}$

$r = \text{No. of reactions}$

$j = \text{No. of joints}$

Examples: - Find the stability and determinacy of trusses below.

Ex1

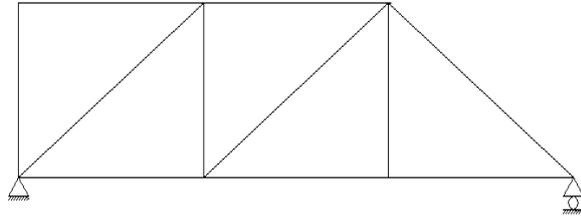
$r = 3, b = 11, j = 7$

$b+r = 14$

$2j = 14$

$b+r = 2j$

The truss is stable & determinate



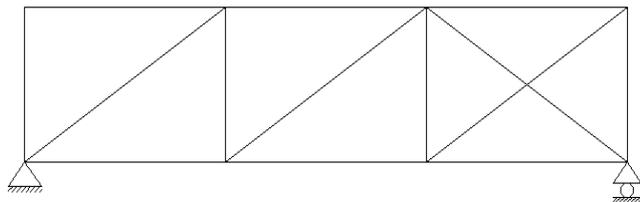
Ex2

$r = 3, b = 14, j = 8$

$b+r = 17$

$2j = 16$

$b+r > 2j$, the truss is stable & indeterminate 1st degree



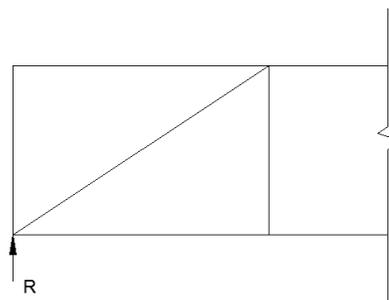
Ex3

$r = 3$

$b = 13$

$2j = 16$

$b+r = 2j$, the truss is unstable because of $\sum F_y \neq 0$, in this section



Ex4

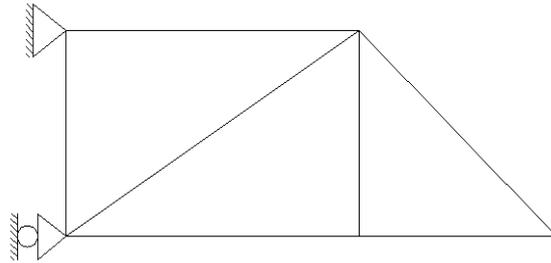
$$r = 3, b = 7, j = 5$$

$$b + r = 10$$

$$2j = 10$$

$$b + r = 2j$$

The truss is stable & determinate



Ex5

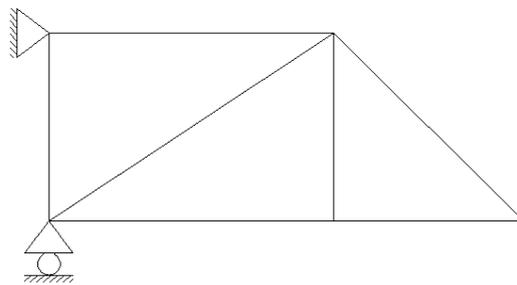
$$r = 3, b = 7, j = 5$$

$$b + r = 10$$

$$2j = 10$$

$$b + r = 2j$$

The truss is unstable



Ex6

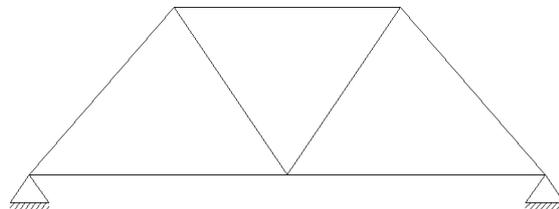
$$r = 4, b = 7, j = 5$$

$$b + r = 11$$

$$2j = 10$$

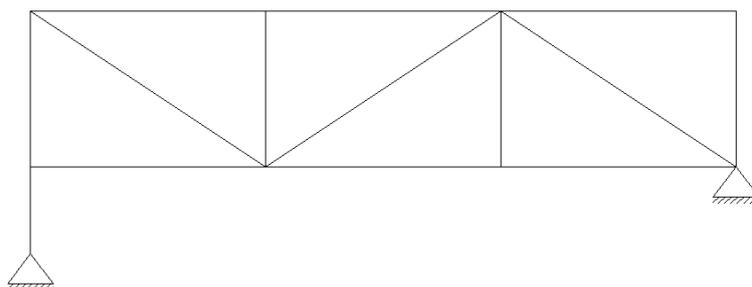
$$b + r > 2j$$

The truss is stable & indeterminate 1st degree

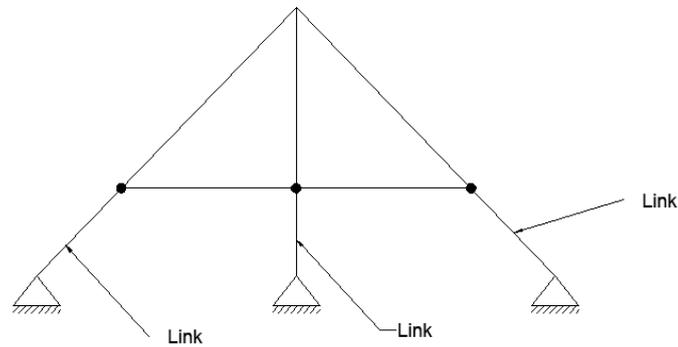


Home works

H.W1



H.W2



Stability and Determinacy of Frames

1- Open frames

$r < C+3$, unstable

$r = C+3$, determinate if stable

$r > C+3$, indeterminate if stable

Ex1:- Find the stability and determinacy of frame below

$$C_1 = m-1, C_1 = 2-1 = 1$$

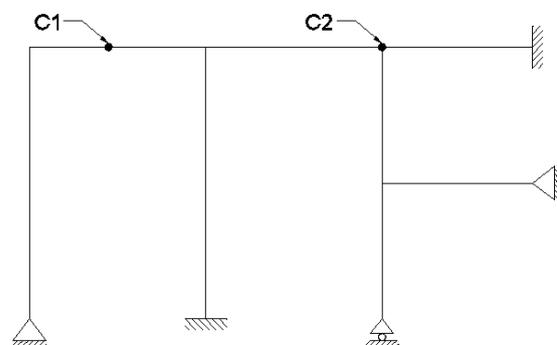
$$C_2 = m-1, C_2 = 3-1 = 2$$

$$C = C_1 + C_2, C = 3$$

$$r = 11$$

$$C+3 = 6$$

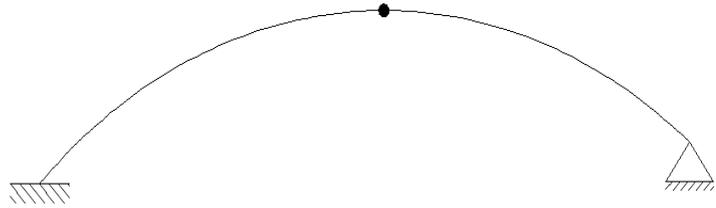
$r > C+3$, the frame is stable & indeterminate 5th degree.



Ex2:-

$r = 5$

$C = 2 - 1 = 1$



$r > C + 3$, the frame is stable & indeterminate 1st degree.

Ex3:-

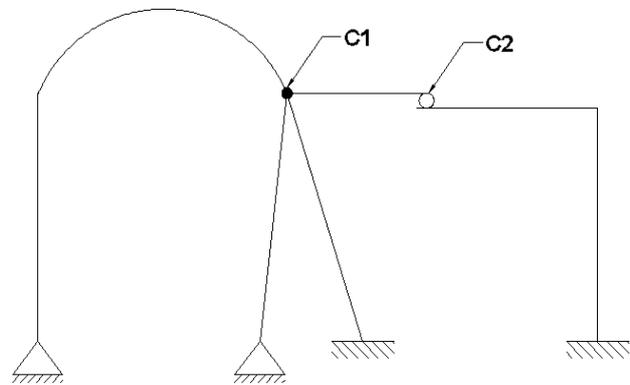
$C_1 = m - 1, C_1 = 4 - 1 = 3$

$C_2 = 2$

$C = C_1 + C_2, C = 5$

$r = 10$

$C + 3 = 8$



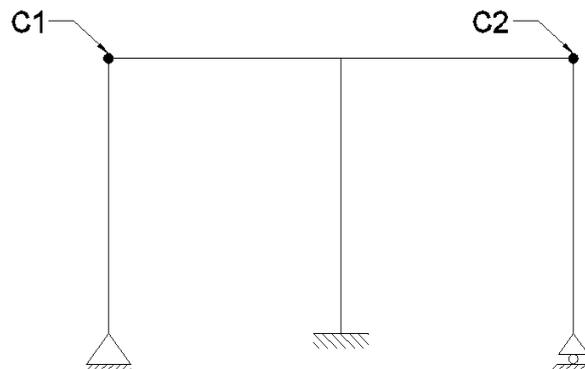
$r > C + 3$, the frame is stable & indeterminate 2nd degree.

Ex4:-

$r = 6$

$C = 2$

$r > C + 3$



The frame is unstable because of internal geometric instability

2- Closed Frames:-

$$3b+r < 3j+c, \text{ unstable}$$

$$3b+r = 3j+c, \text{ determinate if stable}$$

$$3b+r > 3j+c, \text{ indeterminate if stable}$$

Where,

$$3b+r = \text{unknown}$$

$$3j+c = \text{equations}$$

$$b = \text{No. of members}$$

$$r = \text{No. of reactions}$$

$$j = \text{No. of joints}$$

Ex1:-

$$b = 10$$

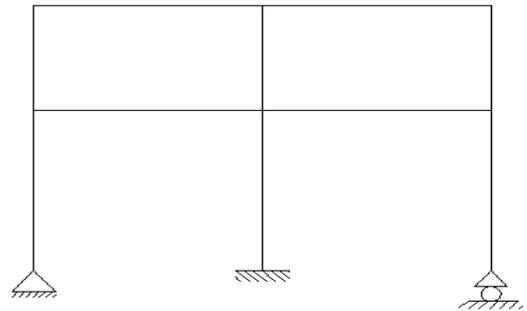
$$r = 6$$

$$j = 9$$

$$3b+r = 36$$

$$3j+c = 27$$

$$3b+r > 3j+c, \text{ stable \& indeterminate } 9^{\text{th}} \text{ degree}$$



Ex2:-

$$b = 10$$

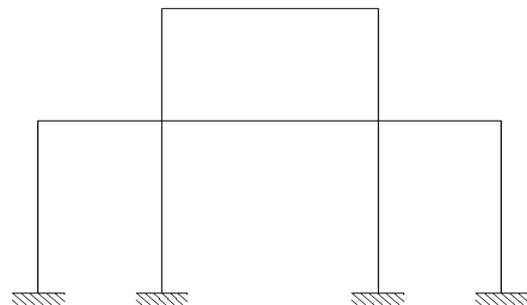
$$r = 12$$

$$j = 10$$

$$3b+r = 42$$

$$3j+c = 30$$

$$3b+r > 3j+c, \text{ stable \& indeterminate } 12^{\text{th}} \text{ degree}$$



Ex3:-

$$b = 4$$

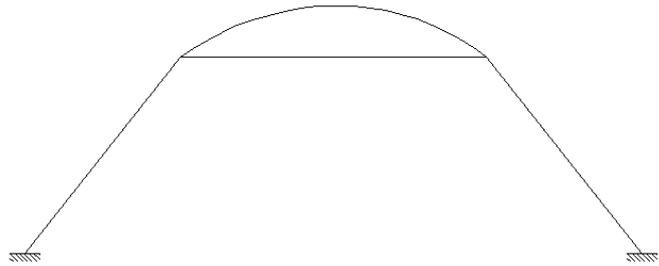
$$r = 6$$

$$j = 4$$

$$c = 0$$

$$3b+r = 18$$

$$3j+c = 12$$

 $3b+r > 3j+c$, stable & indeterminate 6th degree


Ex4:-

$$b = 9$$

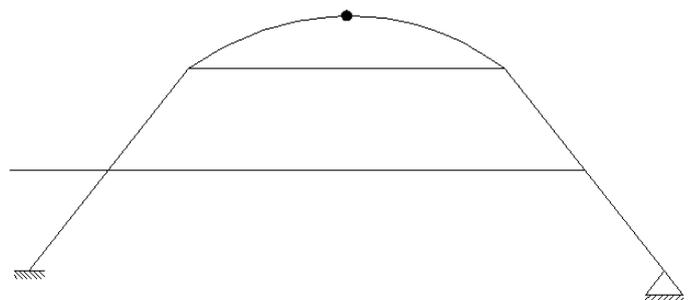
$$r = 5$$

$$j = 7$$

$$c = m-1 \Rightarrow c = 1$$

$$3b+r = 32$$

$$3j+c = 22$$

 $3b+r > 3j+c$, stable & indeterminate 10th degree


Ex5:-

$$b = 10$$

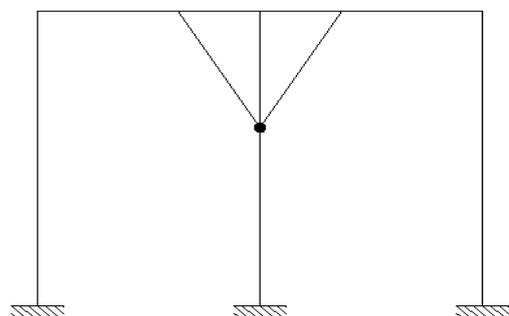
$$r = 9$$

$$j = 9$$

$$c = m-1 \Rightarrow c = 4-1 \Rightarrow c = 3$$

$$3b+r = 39$$

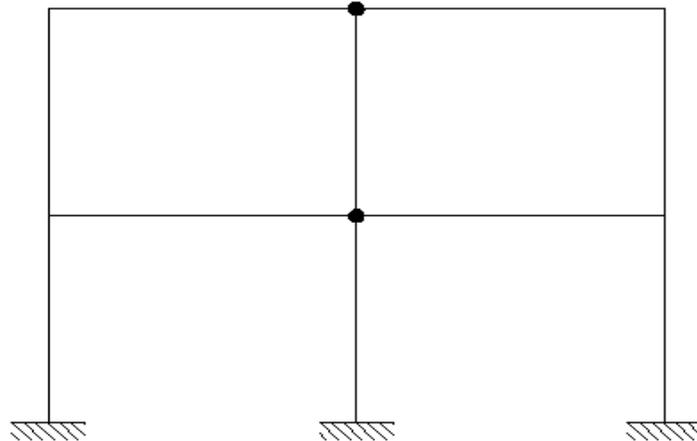
$$3j+c = 30$$

 $3b+r > 3j+c$, stable & indeterminate 9th degree


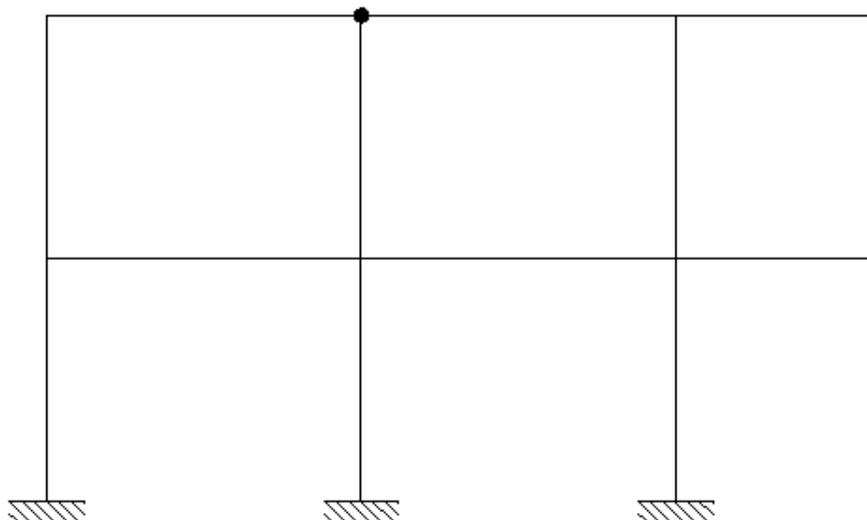
ملاحظة:- اذا جاء ال internal hinge في بداية او نهاية الضلع فيحسب منه (c & j) اما
اذا جاء في داخل الضلع فيحسب منه c فقط

Home work:

H.W1: Find the stability and determinacy of frame below



H.W2: Find the stability and determinacy of frame below



Stability and Determinacy of Composite Structure

Unknowns	Equations
1- Each truss member give one unknown	1- each member carry moment give (3 equations)
2- reactions	2- each joint connect truss members only give (2 equations)
3- each joint connect member carry moment give unknown in these equation (2*(m-1))	

Ex1:- Find the stability and determinacy of composite structure as shown below.

Solution:

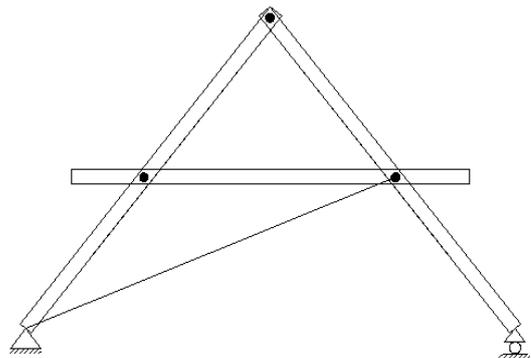
Equations

$$(3*3) + 0 = 9$$

Unknowns

$$1+3+ (3*(2(2-1))) = 10$$

Unknowns > Equations, Stable & indeterminate 1st degree



Ex2:- Find the stability and determinacy of composite structure as shown below.

Solution:

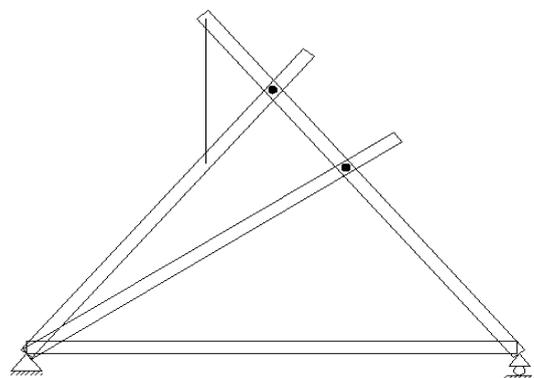
Equations

$$(4*3) + 0 = 12$$

Unknowns

$$1+3+ (3*(2(2-1))) + (2(3-1)) = 14$$

Unknowns > Equations, Stable & indeterminate 2nd degree



Ex3:- Find the stability and determinacy of composite structure as shown below.

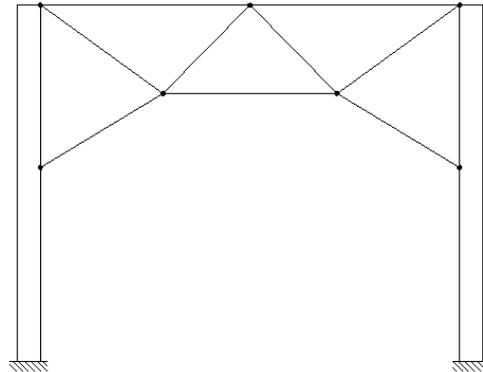
Solution:

Equations

$$(2 \times 3) + (3 \times 2) = 12$$

Unknowns

$$9 + 6 + 0 = 15$$



Unknowns > Equations, Stable & indeterminate 2nd degree

Ex4:- Find the stability and determinacy of composite structure as shown below.

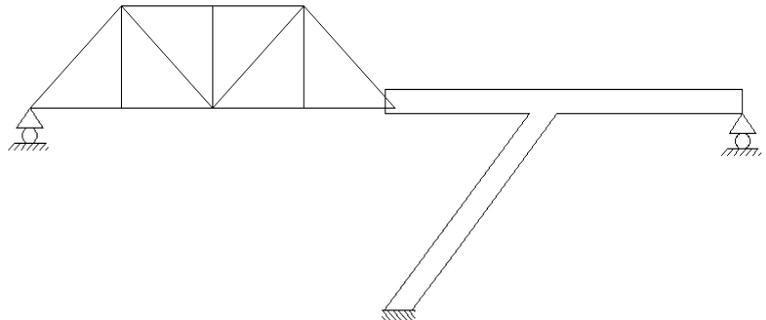
Solution:

Equations

$$(1 \times 3) + (7 \times 2) = 17$$

Unknowns

$$13 + 5 + 0 = 18$$



Unknowns > Equations, Stable & indeterminate 1st degree

H.w: Find the stability and determinacy of composite structure as shown below.

