Analysis of statically indeterminate structures by the Method of slope-deflection.

① Indeterminate beams:-

Slope-deflection eq:-

$$M_{ij} = \frac{2EI}{\ell}\left(2\theta_i + \theta_j - 3\frac{\Delta}{\ell}\right) \mp M^F_{ij}$$

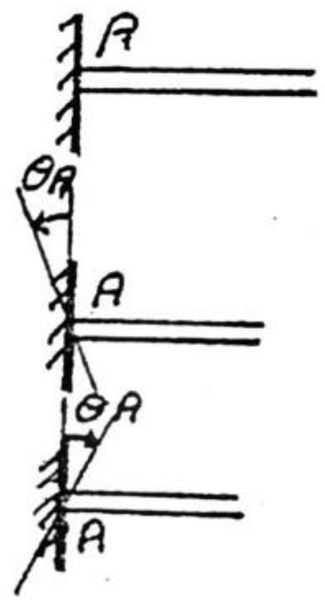
← حفظ

خطوات العمل:

①- نقوم بتحديد المجاهيل

* في هذه الطريقة تكون المجاهيل هي (θ) فقط (pin أو roller) تكون فيه (θ) مجهولة.

* أما المسند الـ (Fixed) فتكون ($\theta=0$) أما لم يذكر او يوضح ان في هذا المسند حصل (rotation) مثلاً :-



$\Rightarrow \theta_A = 0$

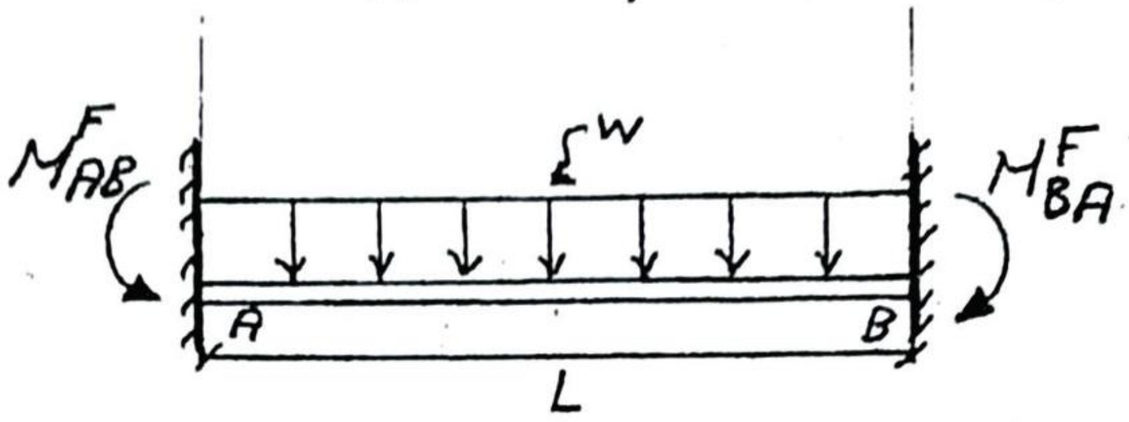
$\Rightarrow \theta_A = \ominus$ ← الدوران عكس عقرب الساعة

$\Rightarrow \theta_A = \oplus$ ← الدوران باتجاه عقرب الساعة

② - أحسب (Fixed-end-Moment) (MF)
وذلك حسب الحالات التالية :

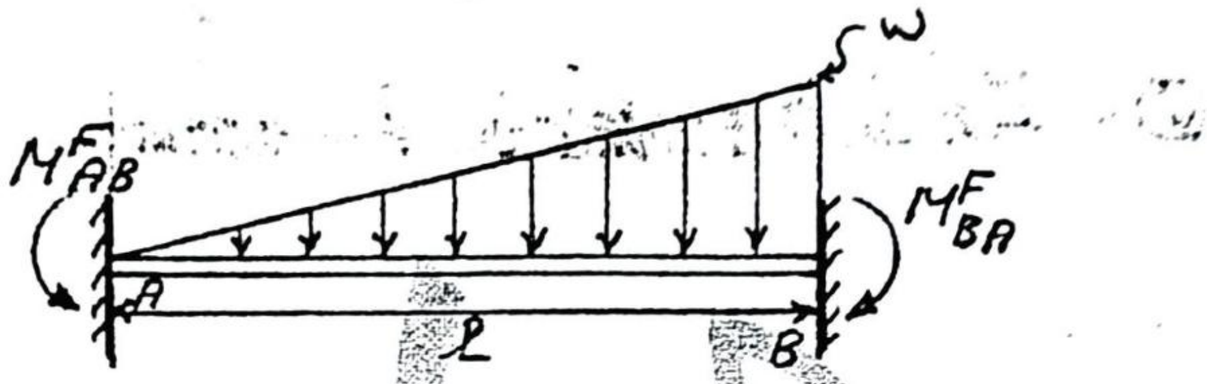
$$M^F_{AB} = -\frac{wl^2}{12}$$

$$M^F_{BA} = +\frac{wl^2}{12}$$



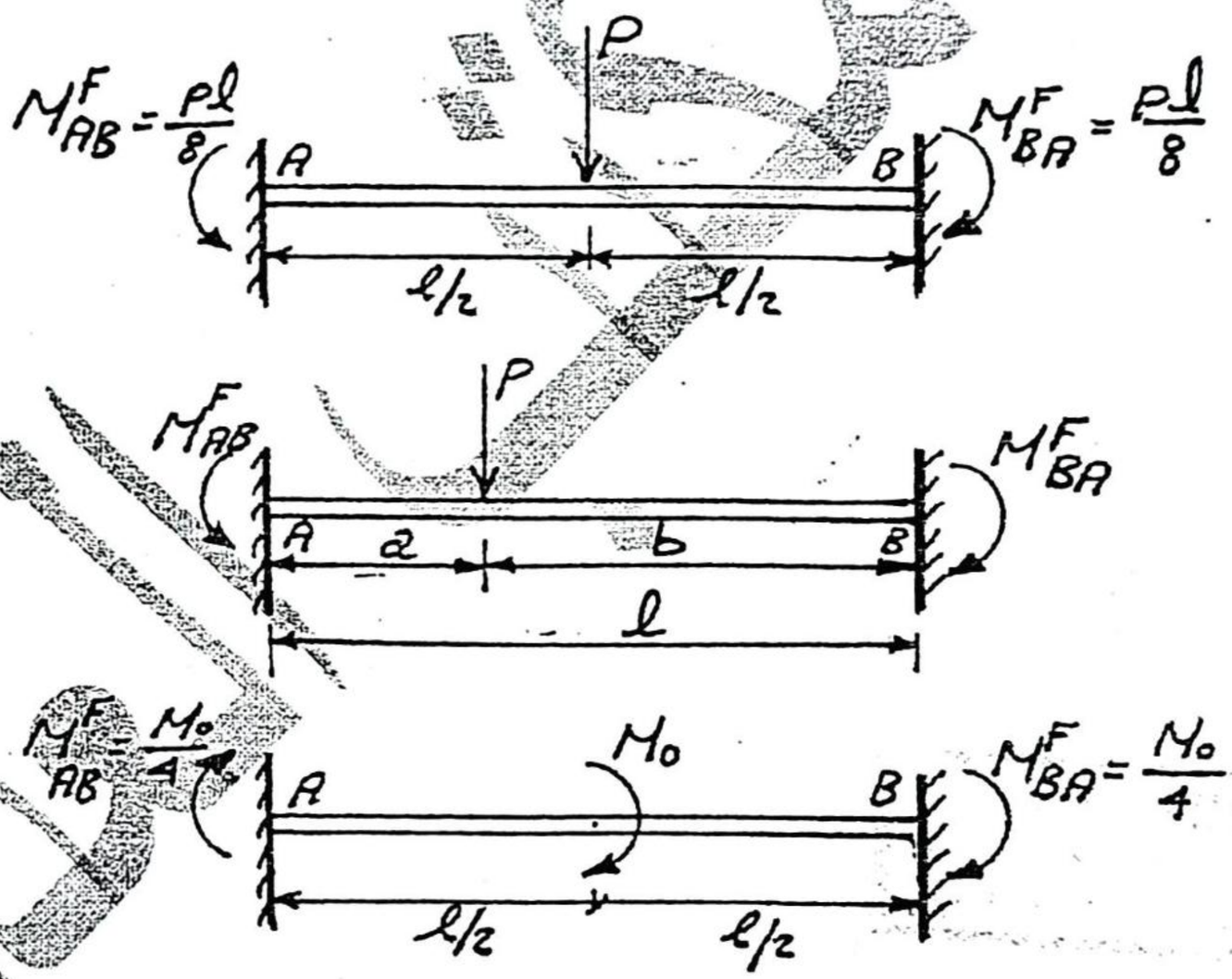
$$M^F_{AB} = -\frac{wl^2}{30}$$

$$M^F_{BA} = +\frac{wl^2}{20}$$



$$M^F_{AB} = -\frac{Pl}{8}$$

$$M^F_{BA} = +\frac{Pl}{8}$$



$$M^F_{AB} = -\frac{P*a*b^2}{l^2}$$

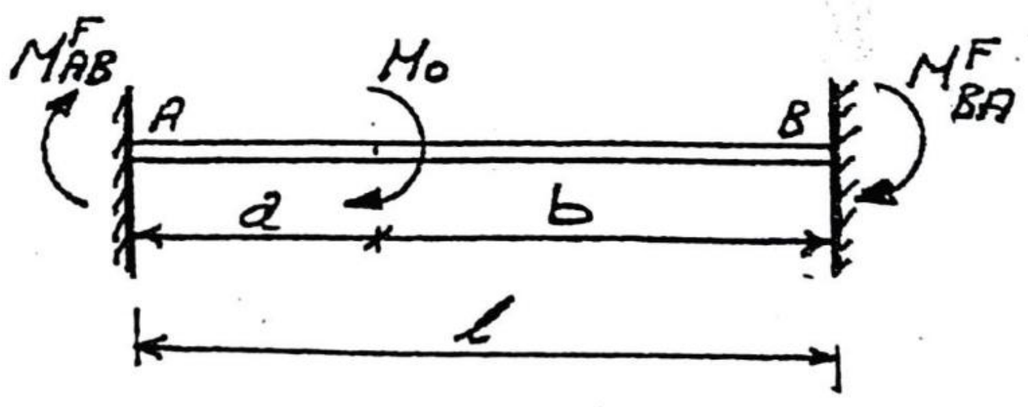
$$M^F_{BA} = +\frac{P*b*a^2}{l^2}$$

$$M^F_{AB} = +\frac{M_o}{4}$$

$$M^F_{BA} = +\frac{M^o}{4}$$

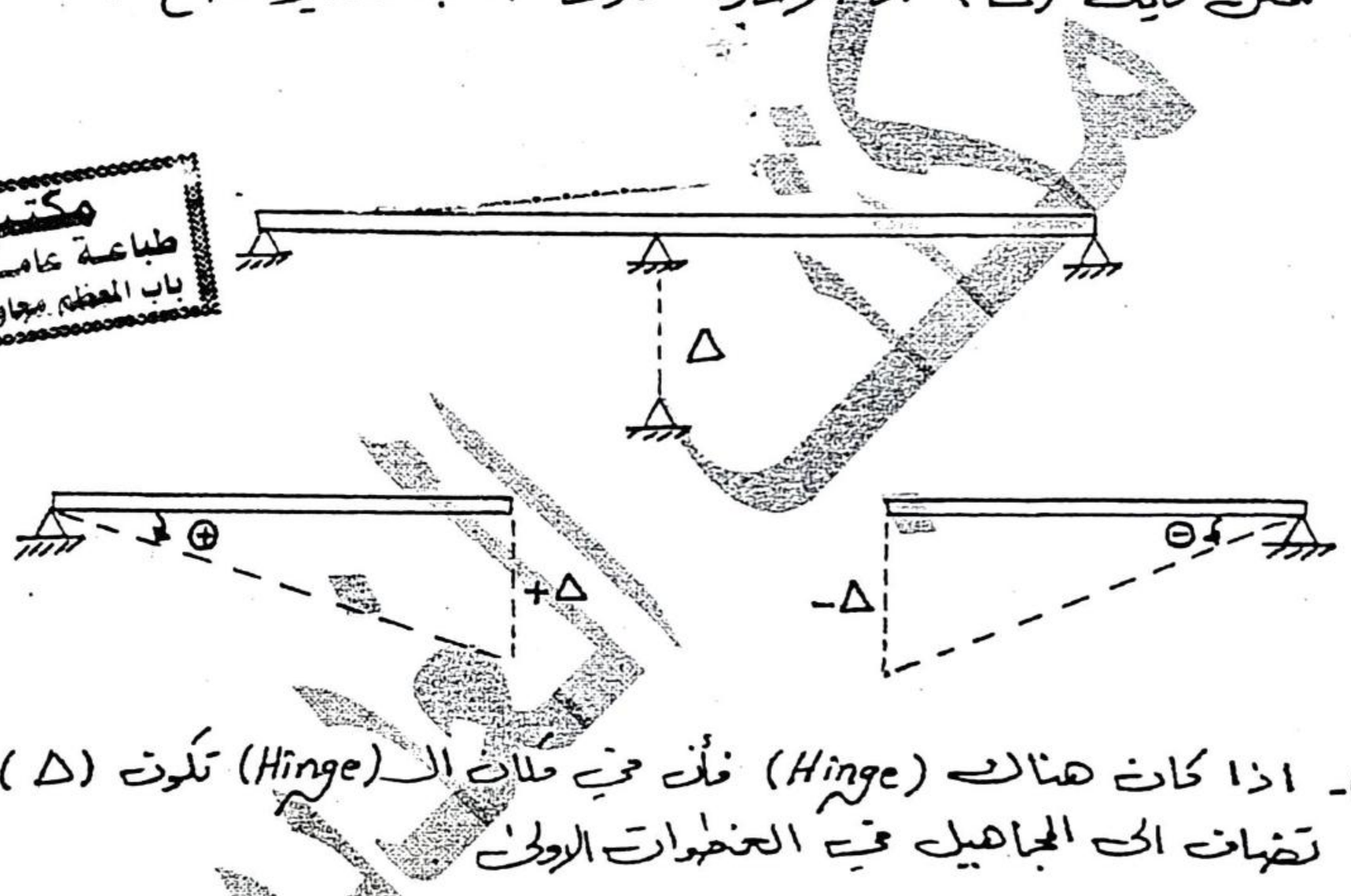
$$M^F_{AB} = +\frac{M_o*a(2b-a)}{l^2}$$

$$M^F_{BA} = +\frac{M_o*b(2a-b)}{l^2}$$



③ . نطبق المعادلات العامة للطريقة وتكون عددها ضعف عدد الفضاءات
* بالنسبة للحد الثالث في هذه المعادلات $\left(-6\frac{\Delta}{l}\right)$

*فأن هذا الحد يعتمد على وجود متبت لـ (Δ) .
① . اذا لم يكن هناك sette. في مسند فأن $(\Delta=0)$.
② . اذا كان هناك مسند احوي على sette. فأن قيمة الـ sette. تمثل قيمة (Δ) أما الاشارة تكون حسب تدوير العنلع .

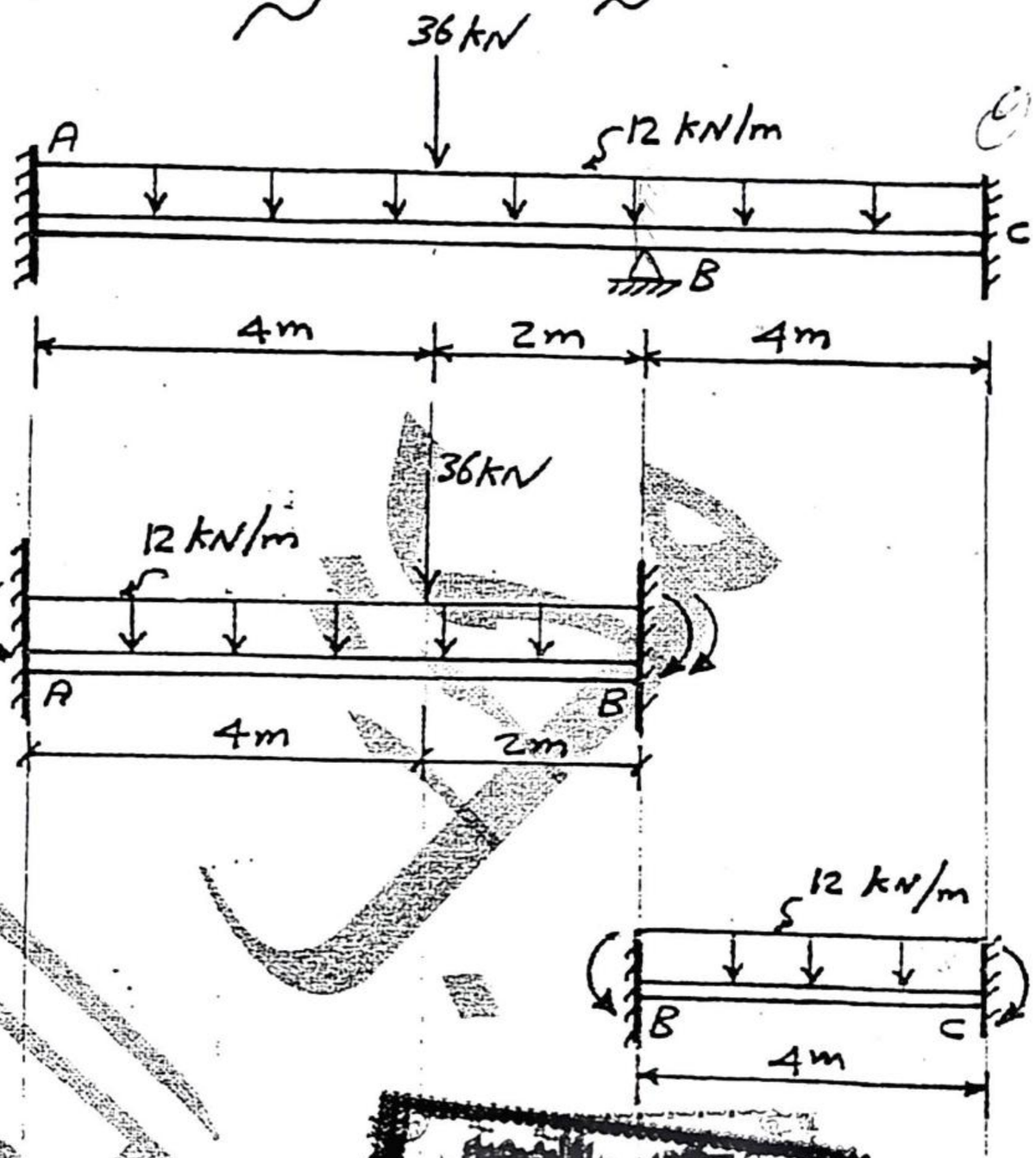


④ . اذا كان هناك (Hinge) فأن في طلاق الـ (Hinge) تكون (Δ) مجهولة تضاف الى المجاهيل في الخطوات الاولى

⑤ . بعد ترتيب المعادلات في الخطوه ③ فأن هذه المعادلات تحوي على مجاهيل محددة في الخطوه ① ولاستخراج هذه المجاهيل نستخدم : (Joint Condition) .

كل ⊿ او ⊿ يعتبر (Condition) ويعوض العزوم دائماً موجب
اما العزم الخارجي فنعوض عكس الاشارة دائماً .
بالتالي نحصل على معادلات تحل آنياً ونجد المجاهيل ونعوض هذه المجاهيل في معادلات العزوم.

Ex:- Using Slope Deflection Method to Find reaction and draw shear force and bending moment Diagram.
$EI = Const.$



Sol.

① - unknowns:

θ_B only.

② - F.E.M

$$M^F_{AB} = -\frac{12(6)^2}{12} - \frac{36*4(2)^2}{(6)^2} = -52$$

$$M^F_{BA} = +\frac{12(6)^2}{12} + \frac{36*2(4)^2}{(6)^2} = +68$$

$$M^F_{Bc} = -\frac{12(4)^2}{12} = -16$$

$$M^F_{CB} = +\frac{12(4)^2}{12} = +16$$

③ - S.D.E

$$M_{ij} = \frac{2EI}{\ell}(2\theta_i + \theta_j - 3\frac{\Delta}{\ell}) + M^F_{ij}$$

$$M_{AB} = \frac{2EI}{6}(2\theta_A + \theta_B - 0) - 52 = \frac{1}{3}EI\theta_B - 52 \quad \text{——①}$$

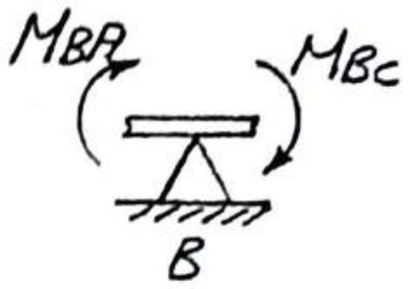
$$M_{BA} = \frac{2EI}{6}(2\theta_B + \theta_A - 0) + 68 = \frac{2}{3}EI\theta_B + 68 \quad \text{——②}$$

$$M_{Bc} = \frac{2EI}{4}(2\theta_B + \theta_c - 0) - 16 = EI\theta_B - 16 \quad \text{——③}$$

$$M_{CB} = \frac{2EI}{4}(2\theta_c + \theta_B - 0) + 16 = \frac{1}{2}EI\theta_B + 16 \quad \text{——④}$$

④ - Additional eq.

Use joint B:



$\Sigma M_B = 0$ ⟲

$M_{BA} + M_{BC} = 0$

$\frac{2}{3} EI\theta_B + 68 + EI\theta_B - 16 = 0$

$\frac{2}{3} EI\theta_B + EI\theta_B = -52 \Rightarrow \frac{5}{3} EI\theta_B = -52$

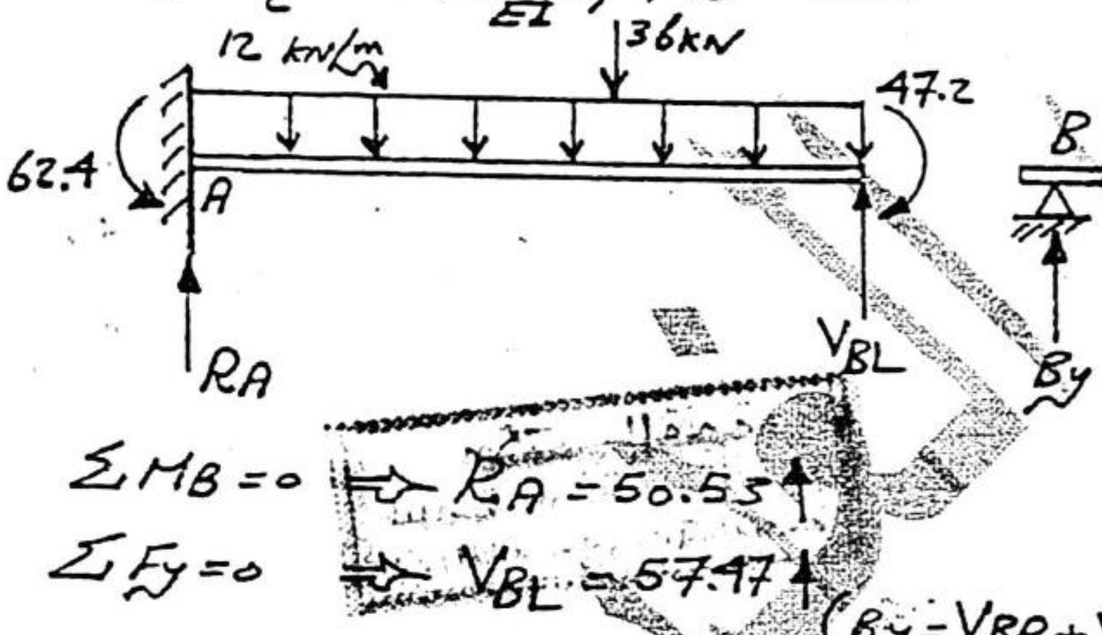
$\theta_B = \frac{-31.2}{EI}$

$M_{AB} = \frac{1}{3} EI \left(\frac{-31.2}{EI}\right) - 52 = -62.4 \text{ kN·m}$

$M_{BA} = \frac{2}{3} EI \left(\frac{-31.2}{EI}\right) + 68 = 47.2 \text{ kN·m}$

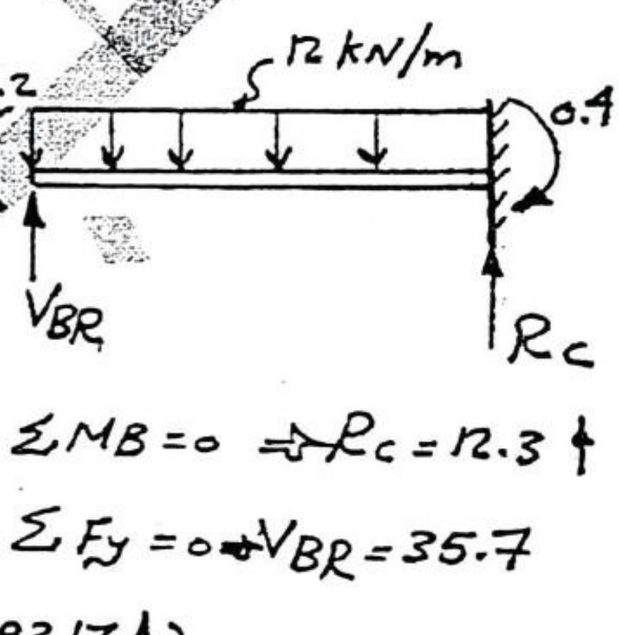
$M_{BC} = EI \left(\frac{-31.2}{EI}\right) - 16 = -47.2 \text{ kN·m}$

$M_{CB} = \frac{1}{2} EI \left(\frac{-31.2}{EI}\right) + 16 = 0.4 \text{ kN·m}$



$\Sigma M_B = 0 \Rightarrow R_A = 50.55 \uparrow$

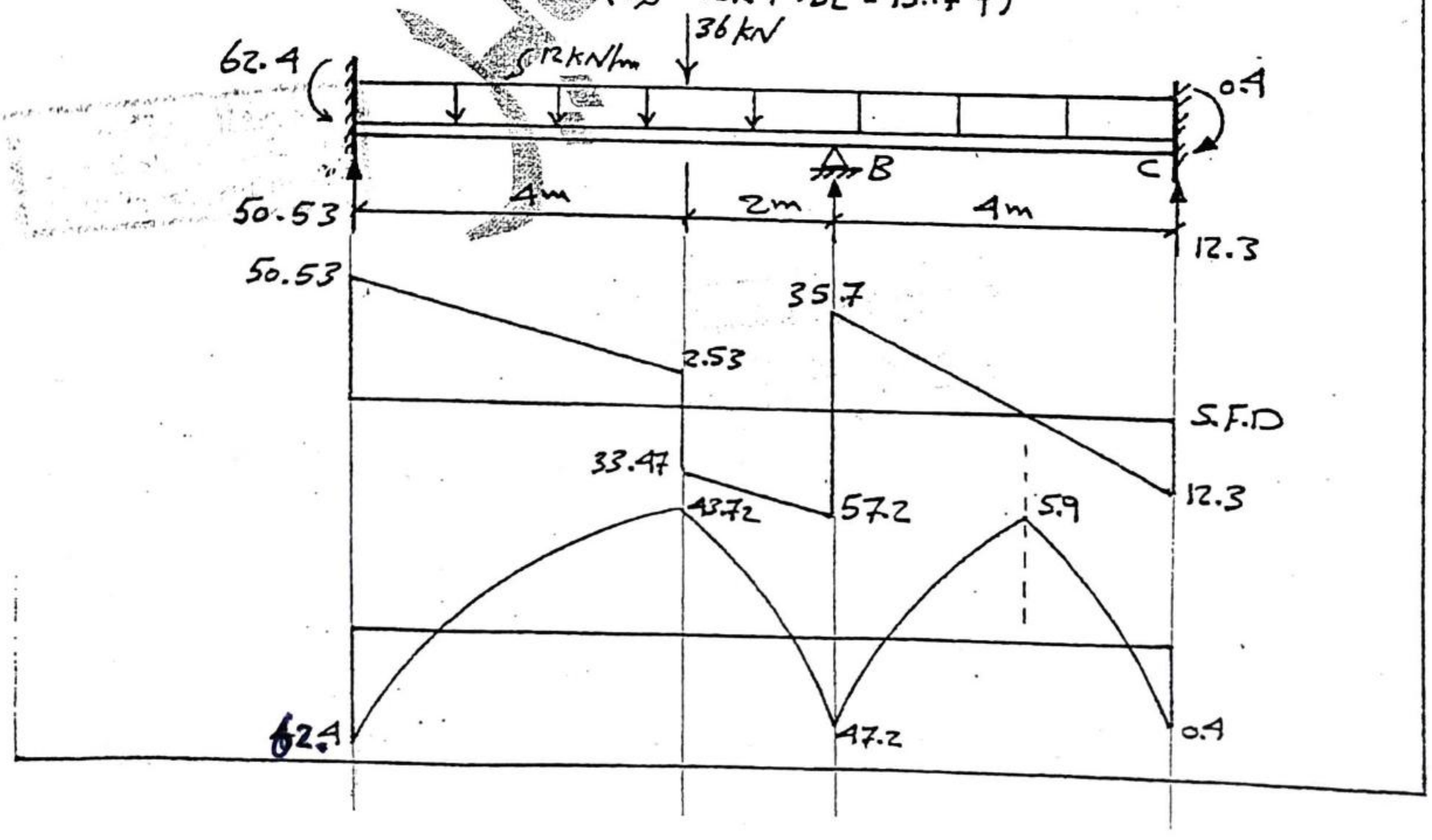
$\Sigma F_y = 0 \Rightarrow V_{BL} = 57.47 \uparrow$



$\Sigma M_B = 0 \Rightarrow R_C = 12.3 \uparrow$

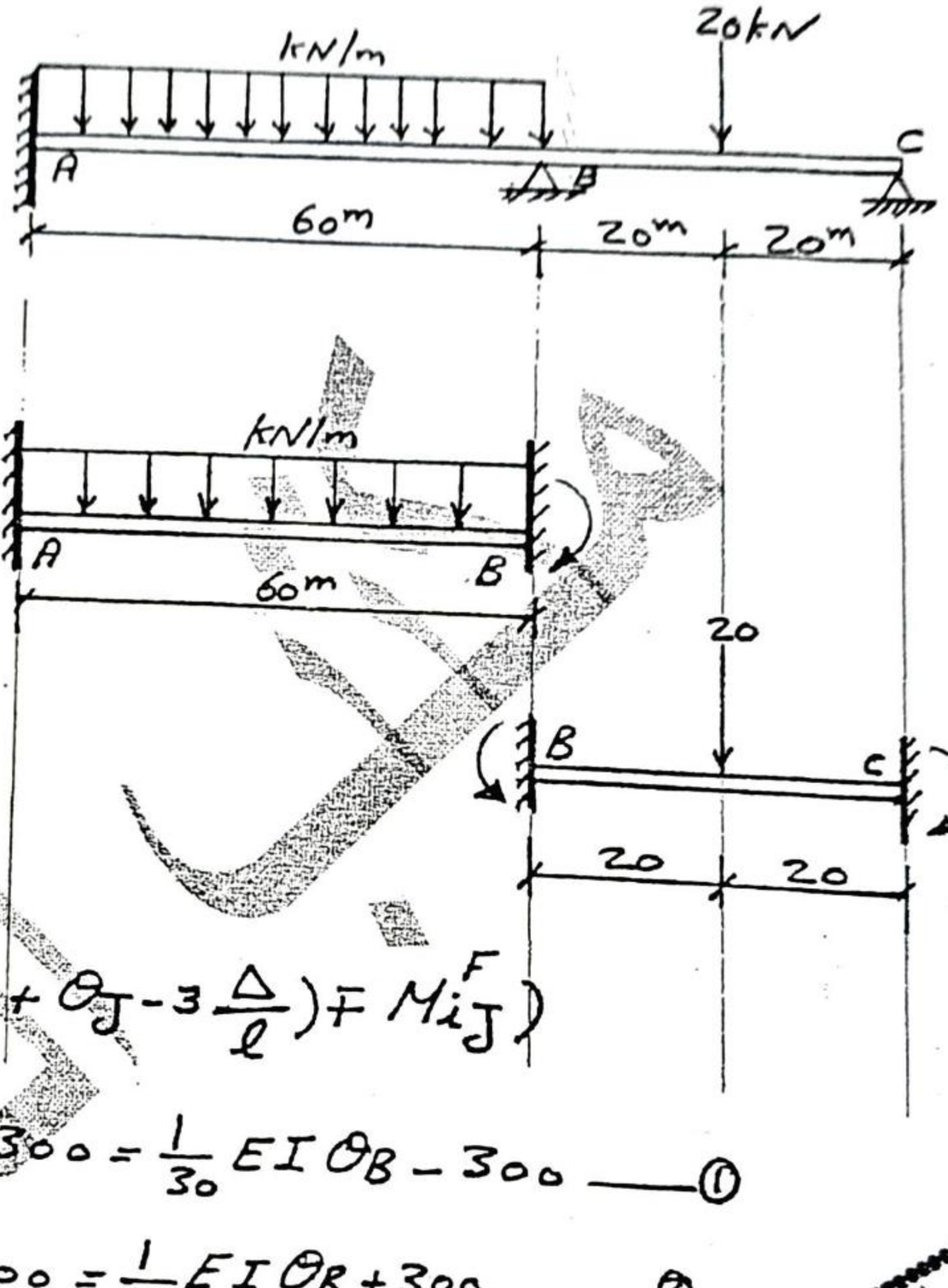
$\Sigma F_y = 0 \Rightarrow V_{BR} = 35.7$

$(B_y = V_{BR} + V_{BL} = 93.17 \uparrow)$



Ex:- Using slope deflection method to find reaction and draw shear and bending moment diagram.

EI = Constant.



Sol.

① - Unknowns:

θ_B , θ_C .

② - F.E.M.

$$M^F_{AB} = -\frac{1(60)^2}{12} = -300$$

$$M^F_{BA} = +\frac{1(60)^2}{12} = 300$$

$$M^F_{BC} = -\frac{20(40)}{8} = -100$$

$$M^F_{CB} = \frac{+20(40)}{8} = 100$$

③ - S.D.E $\left(M_{iJ} = \frac{2EI}{\ell}\left(2\theta_i + \theta_J - 3\frac{\Delta}{\ell}\right) \mp M^F_{iJ}\right)$

$$M_{AB} = \frac{2EI}{60}(2\overset{0}{\theta_A} + \theta_B - 0) - 300 = \frac{1}{30}EI\theta_B - 300 \quad \text{——①}$$

$$M_{BA} = \frac{2EI}{60}(2\theta_B + \overset{0}{\theta_A} - 0) + 300 = \frac{1}{15}EI\theta_B + 300 \quad \text{——②}$$

$$M_{BC} = \frac{2EI}{40}(2\theta_B + \theta_C - 0) - 100 = \frac{1}{10}EI\theta_B + \frac{1}{20}EI\theta_C - 100 \quad \text{——③}$$

$$M_{CB} = \frac{2EI}{40}(2\theta_C + \theta_B - 0) + 100 = \frac{1}{10}EI\theta_C + \frac{1}{20}EI\theta_B + 100 \quad \text{——④}$$

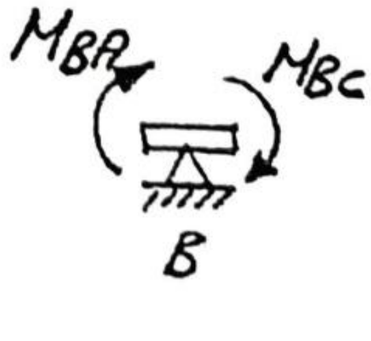
④ - Additional eq.

① - Use Joint B.

$$M_{BA} + M_{BC} = 0$$

$$\frac{1}{15}EI\theta_B + 300 + \frac{1}{10}EI\theta_B + \frac{1}{20}EI\theta_C - 100 = 0$$

$$0.16667\,EI\theta_B + 0.05\,EI\theta_C = -200 \quad \text{——①}$$



③. use Joint C:

$\Sigma M_{CB} = 0$

$M_{CB} = 0 \Rightarrow \frac{1}{10} EI\theta_C + \frac{1}{20} EI\theta_B + 100 = 0$

$0.1 EI\theta_C + 0.05 EI\theta_B = -100$ ——— ②

من حل ① و ② احصل على :

$\theta_B = -\frac{1058.8}{EI}$

$\theta_C = -\frac{470.6}{EI}$

$M_{AB} = \frac{1}{30} EI\left(-\frac{1058.8}{EI}\right) - 300 = 335.3 \text{ kN.m}$

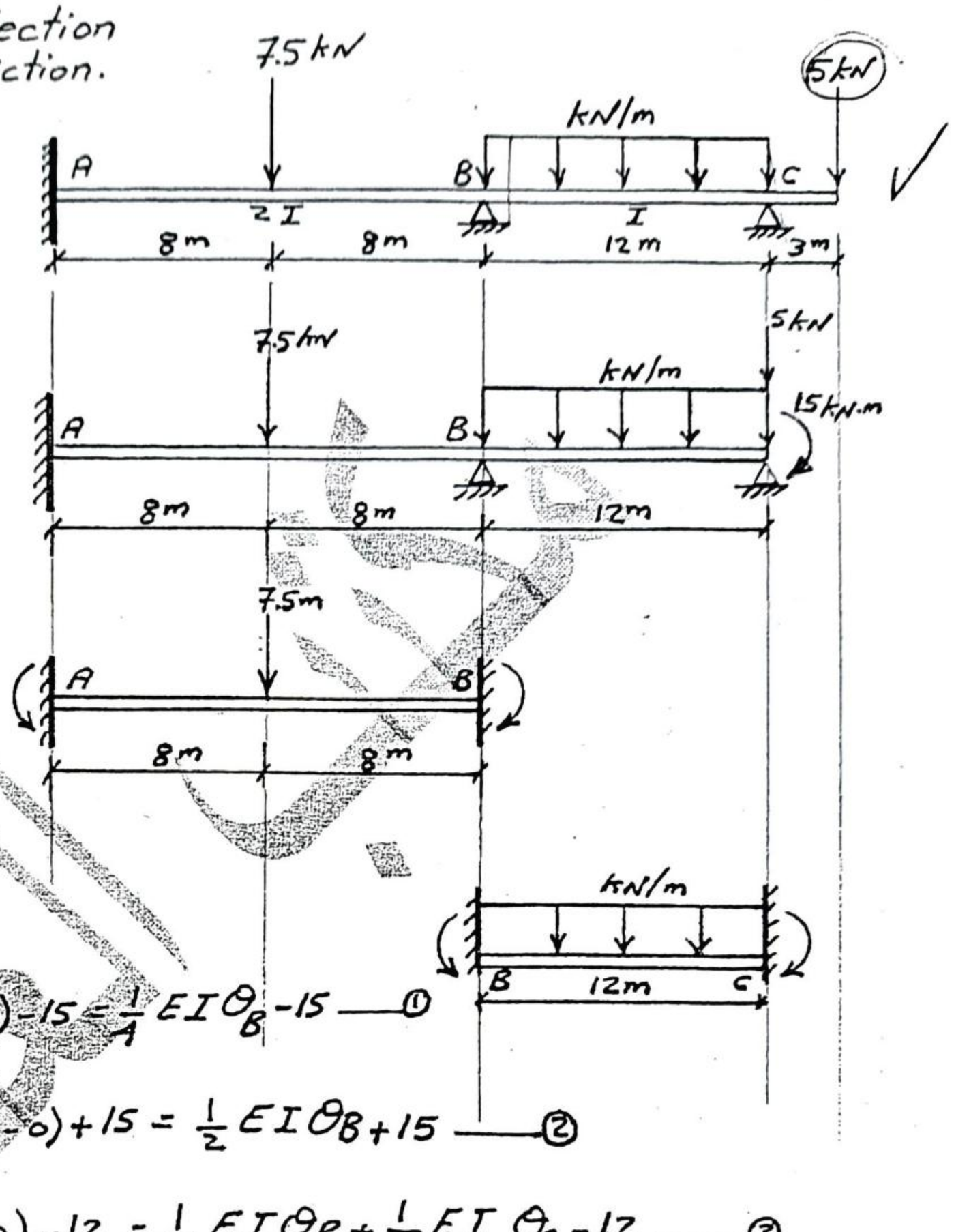
$M_{BA} = \frac{1}{15} EI\left(-\frac{1058.8}{EI}\right) + 300 = 229.5 \text{ kN.m}$

$M_{BC} = \frac{1}{10} EI\left(-\frac{1058.8}{EI}\right) + \frac{1}{20} EI\left(\frac{-470.6}{EI}\right) - 100 = -229.5 \text{ kN.m}$

$M_{CB} = \frac{1}{10} EI\left(\frac{-470.6}{EI}\right) + \frac{1}{20} EI\left(\frac{-1058.8}{EI}\right) + 100 = 0$

رسم قوة القص والعزم (H.W) :

Ex:- use slope deflection method find all reaction.



Sol.

① - Unknowns:

θ_B , θ_C

② - F.E.M

$$M^F_{AB} = -\frac{75(16)}{8} = -15\ kN.m$$

$$M^F_{BA} = +15\ kN.m$$

$$M^F_{BC} = -\frac{1\times(12)^2}{12} = -12\ kN.m$$

$$M^F_{CB} = 12\ kN.m$$

③ - S.D.E:

$$M_{AB} = \frac{2(2EI)}{16}(2\theta_A + \theta_B - 0) - 15 = \frac{1}{4}EI\theta_B - 15 \quad \text{---} ①$$

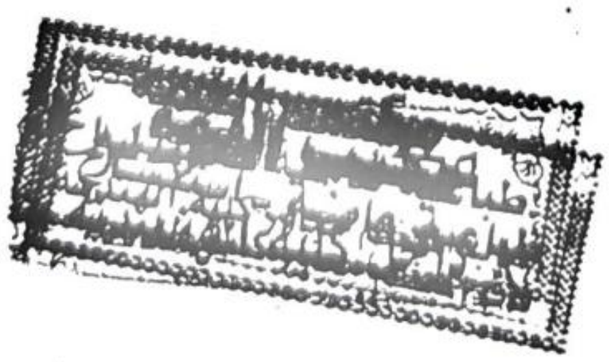
$$M_{BA} = \frac{2(2EI)}{16}(2\theta_B + \theta_A - 0) + 15 = \frac{1}{2}EI\theta_B + 15 \quad \text{---} ②$$

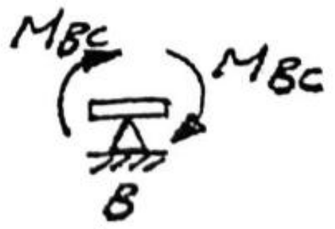
$$M_{BC} = \frac{2EI}{12}(2\theta_B + \theta_C - 0) - 12 = \frac{1}{3}EI\theta_B + \frac{1}{6}EI\theta_C - 12 \quad \text{---} ③$$

$$M_{CB} = \frac{2EI}{12}(2\theta_C + \theta_B - 0) + 12 = \frac{1}{3}EI\theta_C + \frac{1}{6}EI\theta_B + 12 \quad \text{---} ④$$

④ - Add. eq.

① - Use Joint B:





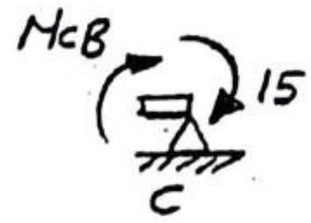
$$\Sigma M_B = 0$$

$$M_{BA} + M_{BC} = 0$$

$$\frac{1}{2}EI\theta_B + 15 + \frac{1}{3}EI\theta_B + \frac{1}{6}EI\theta_C - 12 = 0$$

$$0.8333EI\theta_B + 0.1666EI\theta_C = -3 \quad \text{---} ⑤$$

② - Use joint c:



$\Sigma M_c = 0$

$M_{CB} - 15 = 0$

$\frac{1}{3} EI\theta_C + \frac{1}{6} EI\theta_B + 12 - 15 = 0$

$0.333\, EI\theta_C + 0.1666\, EI\,\theta_B = 3$ ——— ②

من معادلة ① و ② نحصل على:-

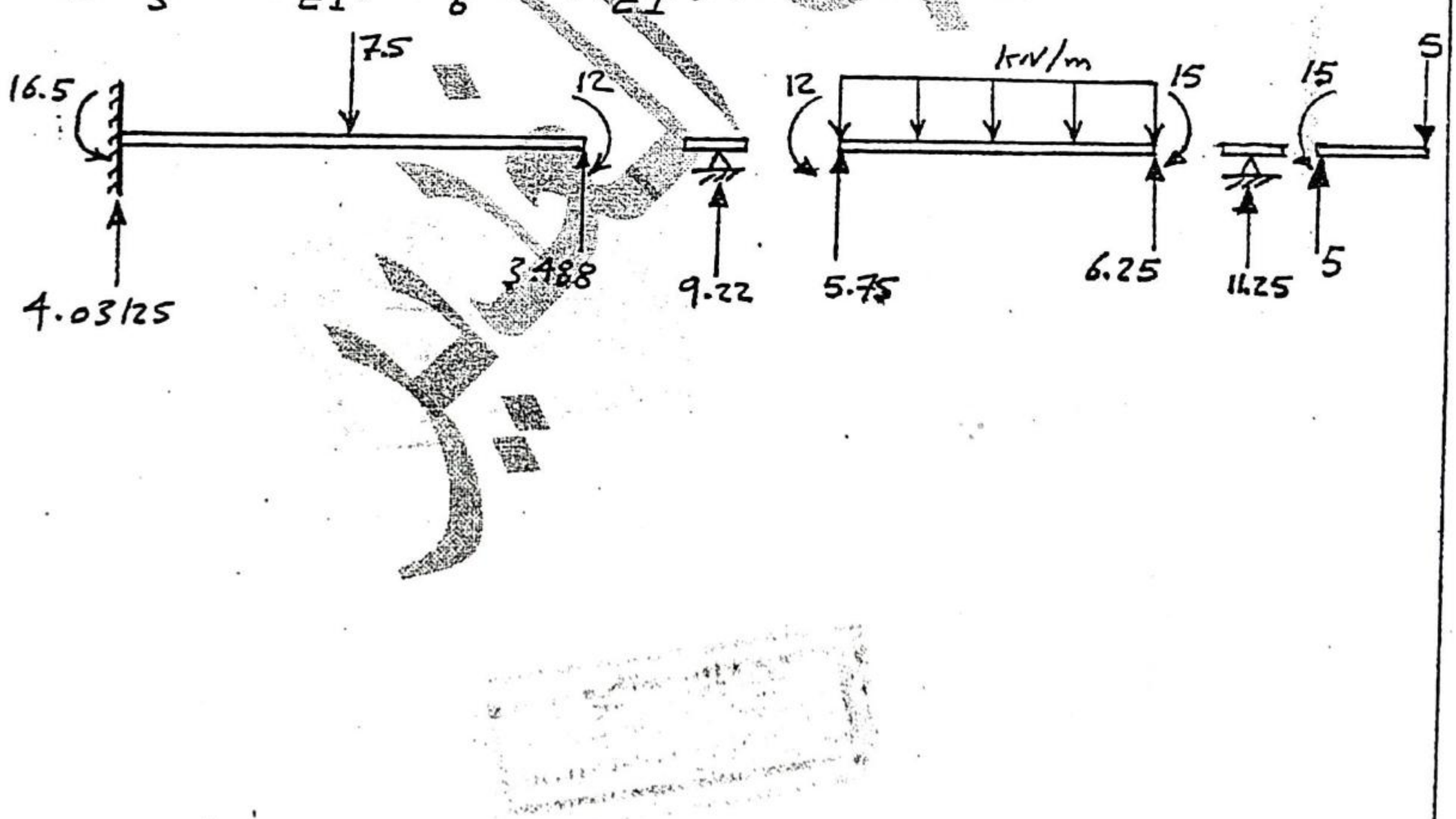
$\theta_B = -\frac{6}{EI}$, $\theta_C = \frac{12}{EI}$

$M_{AB} = \frac{1}{4} EI\left(\frac{-6}{EI}\right) - 15 = -16.5\ kN.m$

$M_{BA} = \frac{1}{2} EI\left(\frac{-6}{EI}\right) + 15 = +12\ kN.m$

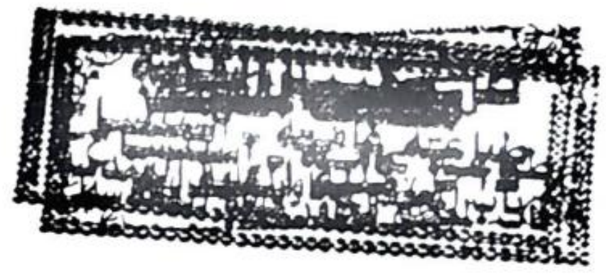
$M_{BC} = \frac{1}{3} EI\left(\frac{-6}{EI}\right) + \frac{1}{6} EI\left(\frac{12}{EI}\right) - 12 = -12\ kN.m$

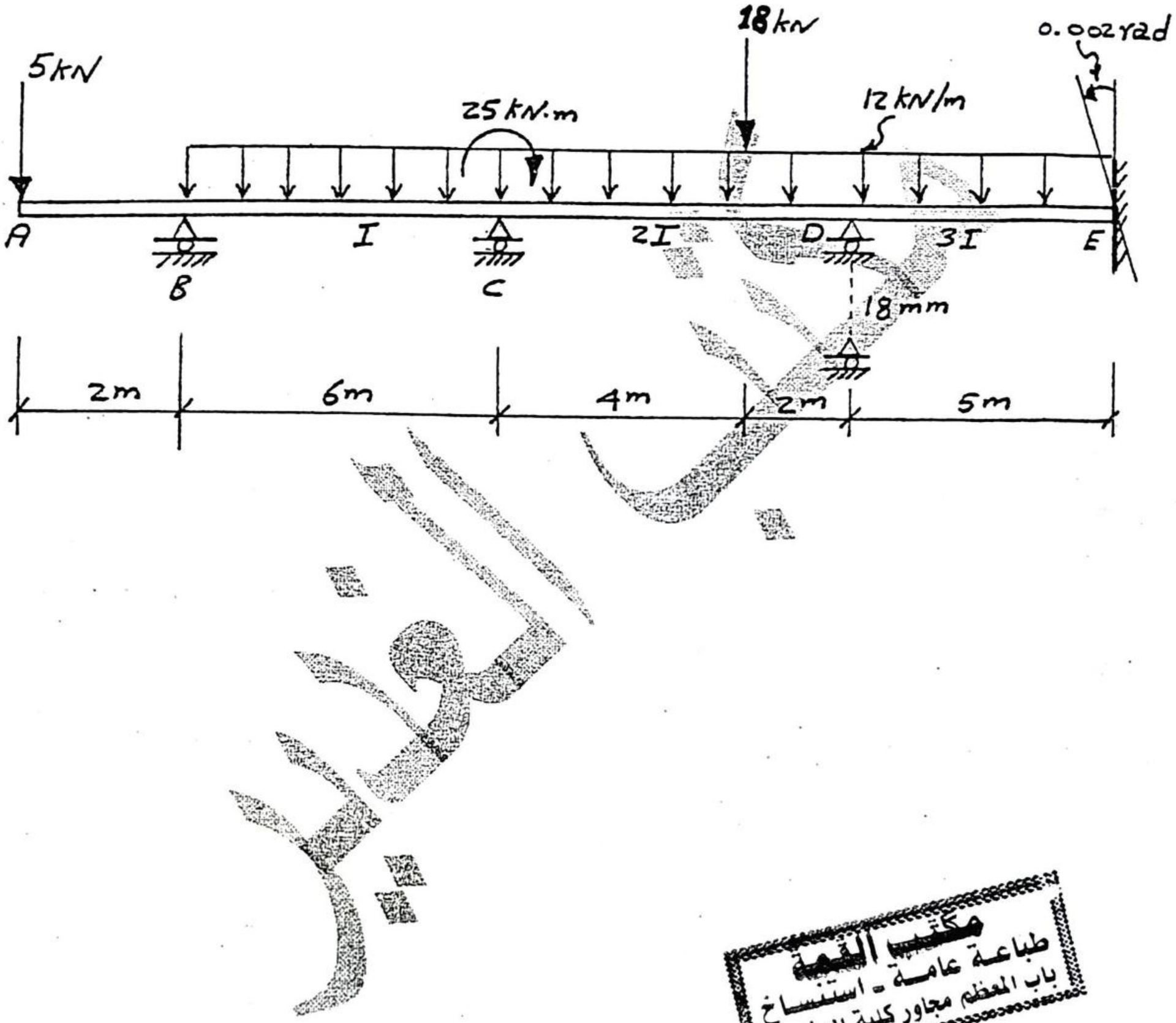
$M_{CB} = \frac{1}{3} EI\left(\frac{12}{EI}\right) + \frac{1}{6} EI\left(\frac{-6}{EI}\right) + 12 = 15\ kN.m$



H.W:- Analyze the beam shown.

$(EI = 3 * 10^4\ kN.m^2)$

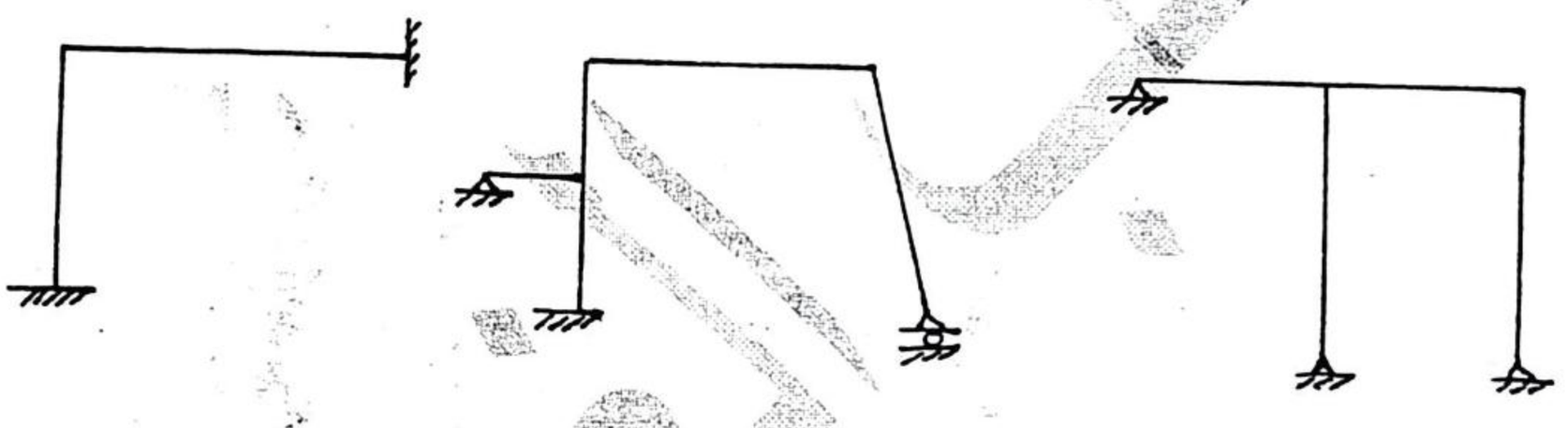




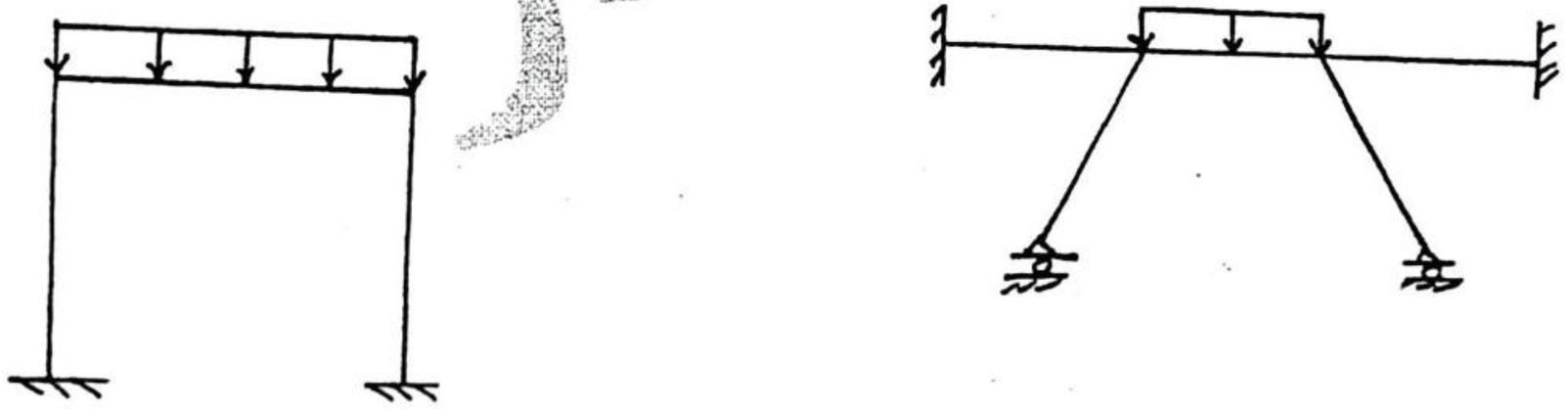
Rigid Frame without Joint Translation
(without side sway)

Frame that have no side sway are either restrained or are symmetrical with respect to both loading and geometry; in this type of structures (Δ) value will equal Zero in the slope-deflection eq.

Examples for these types of frame.

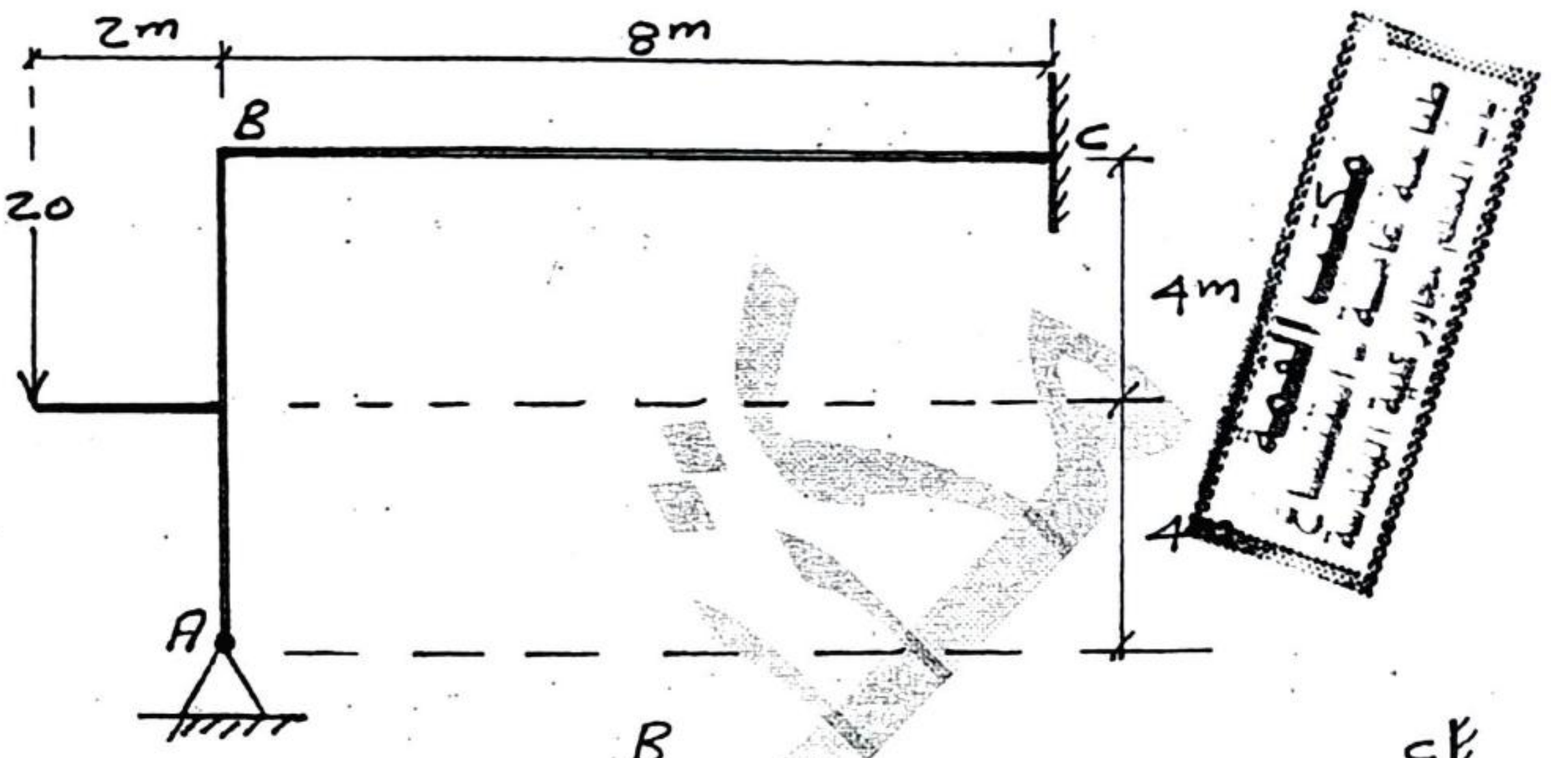


(restrained frames)



(symmetrical frames)

Ex:- Draw the bending moment diagram for member (Bc) only. Use slope-deflection method, support (c) settl. (1cm) down, EI = Constant.



Sol.

① - No-Side sway ($\Delta = 0$)

② - unknowns: θ_A, θ_B

③ - $M^F_{Bc} = M^F_{cB} = 0$

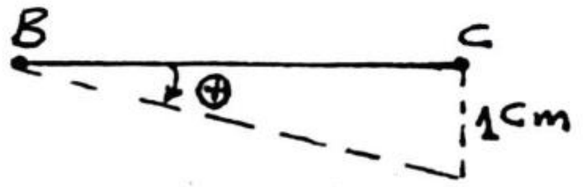
$$M^F_{AB} = M^F_{BA} = -\frac{40}{4} = -10$$

④ - $M_{iJ} = \frac{2EI}{\ell}(2\theta_i + \theta_J - 3\frac{\Delta}{\ell}) \mp M^F_{iJ}$

$$M_{AB} = \frac{2EI}{8}(2\theta_A + \theta_B - 0) - 10 = \frac{1}{2}EI\theta_A + \frac{1}{4}\theta_B - 10$$

$$M_{BA} = \frac{2EI}{8}(2\theta_B + \theta_A - 0) - 10 = \frac{1}{2}EI\theta_B + \frac{1}{4}EI\theta_A - 10$$

$$M_{Bc} = \frac{2EI}{8}(2\theta_B + \cancel{\theta_c}^{0} - 3*\frac{+0.01}{8})$$



$$M_{Bc} = \frac{1}{2}EI\theta_B - 9.375*10^{-4}EI$$

$$M_{cB} = \frac{2EI}{8}(2\cancel{\theta_c}^{0} + \theta_B - 3\frac{0.01}{8})$$

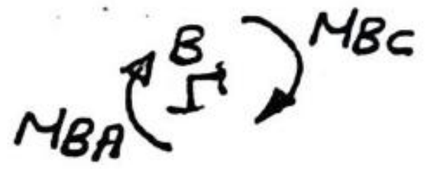
$$M_{cB} = \frac{1}{4}EI\theta_B - 9.375*10^{-4}EI$$

⑤- Joint Condition:

① use joint A.

$$\Sigma M_A = 0 \Rightarrow M_{AB} = 0$$

$$\frac{1}{2} EI\theta_A + \frac{1}{4} EI\theta_B - 10 = 0$$



②- Use Joint B.

$$\Sigma M_B = 0 \Rightarrow M_{BA} + M_{BC} = 0$$

$$\frac{1}{2} EI\theta_B + \frac{1}{4} EI\theta_A - 10 + \frac{1}{2} EI\theta_B - 9.375 * 10^{-4} EI = 0$$

$$EI\theta_B + \frac{1}{4} EI\theta_A - 9.375 * 10^{-4} EI - 10 = 0 \quad \text{---------- ②}$$

From ① $\theta_A = \dfrac{20 - 0.5 EI\theta_B}{EI}$ → تعوض في ②

$$EI\theta_B + \frac{1}{4} EI\left(\frac{20 - 0.5 EI\theta_B}{EI}\right) - 9.375 * 10^{-4} EI - 10 = 0$$

$$EI\theta_B + 5 - 0.125 EI\theta_B - 9.375 * 10^{-4} EI - 10 = 0$$

$$\theta_B = \frac{5 + 9.375 * 10^{-4} EI}{0.875 EI}$$

$$\theta_A = \frac{20 - 0.5EI\left(\dfrac{5 + 9.375 * 10^{-4} EI}{0.875 EI}\right)}{EI}$$

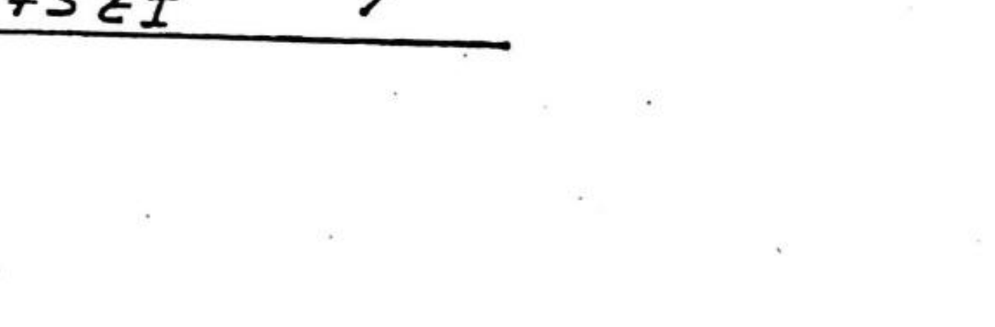
$$\theta_A = \frac{17.143 - 5.357 * 10^{-4} EI}{EI}$$

$$M_{BC} = \frac{1}{2} EI\left(\frac{5 + 9.375 * 10^{-4} EI}{0.875 * EI}\right) - 9.375 * 10^{-4} EI$$

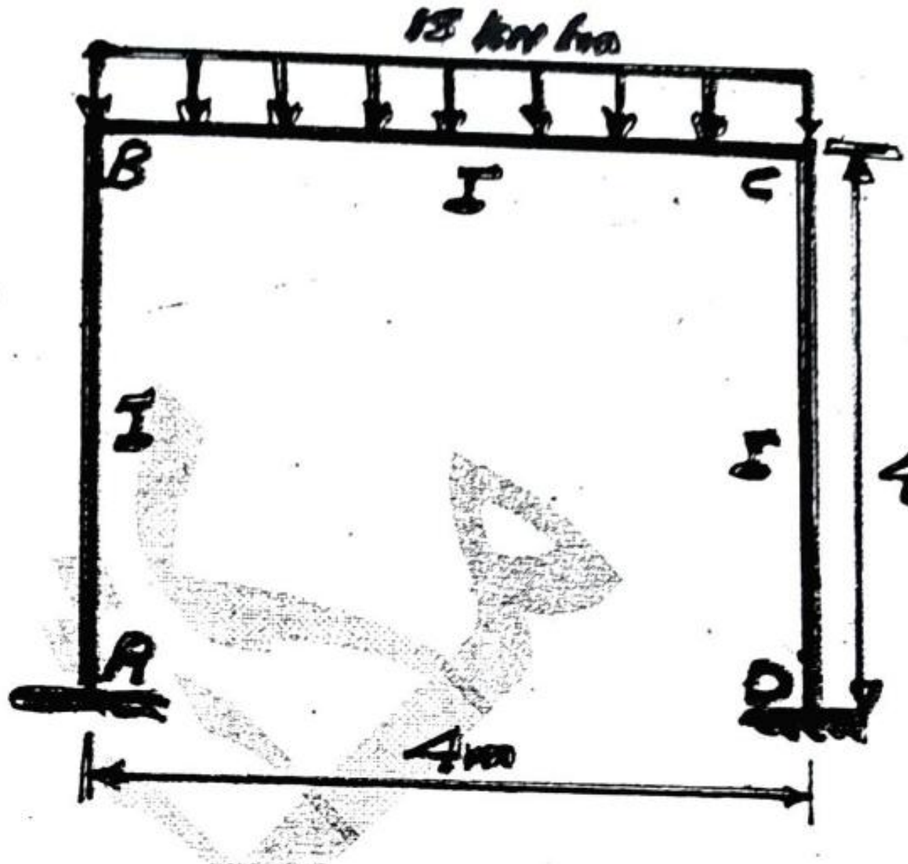
$$M_{BC} = -1.16$$

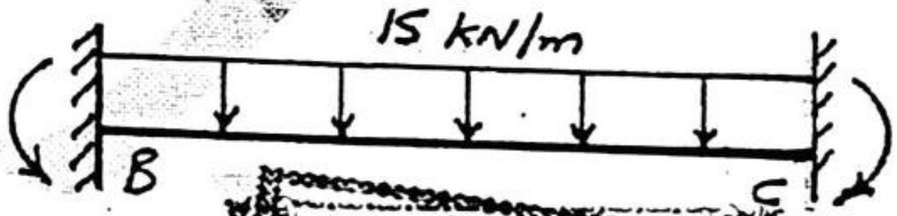
$$M_{CB} = \frac{1}{4} EI\left(\frac{5 + 9.375 * 10^{-4} EI}{0.875 EI}\right) - 9.375 * 10^{-4} EI$$

$$M_{CB} = -5.268$$



Ex:- Analyze the frame shown (Find moments in joints)





Sol.

① - without Side sway (symmetrical $\Delta = 0$)

② - unknowns:

θ_B, θ_C, but $\theta_B = -\theta_C$

ملحوظة: بسبب التناظر

③ - F.E.M

$$M^F_{AB} = M^F_{BA} = M^F_{CD} = M^F_{DC} = 0$$

$$M^F_{BC} = -\frac{15(4)^2}{12} = -20\ kN.m$$

$$M^F_{CB} = +\frac{15(4)^2}{12} = 20\ kN.m$$

④ - S.D.E $\left(M_{ij} = \frac{2EI}{\ell}\left(2\theta_i + \theta_j - 3*\frac{\Delta}{\ell}\right) \mp M^F_{ij}\right)$

$$M_{AB} = \frac{2EI}{4}\left(2\overset{0}{\cancel{\theta_A}} + \theta_B - 0\right) = \frac{1}{2}EI\theta_B \quad \text{——} ①$$

$$M_{BA} = \frac{2EI}{4}\left(2\theta_B + \overset{0}{\cancel{\theta_A}} - 0\right) = EI\theta_B \quad \text{——} ②$$

$$M_{BC} = \frac{2EI}{4}\left(2\theta_B + \overset{-\theta_B}{\cancel{\theta_C}} - 0\right) - 20 = \frac{1}{2}EI\theta_B - 20 \quad \text{——} ③$$

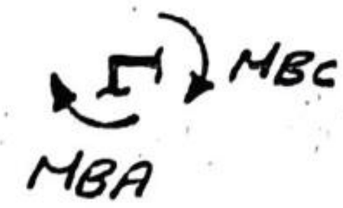
$$M_{CB} = \frac{2EI}{4}\left(2\overset{-\theta_B}{\cancel{\theta_C}} + \theta_B - 0\right) + 20 = -\frac{1}{2}EI\theta_B + 20 \quad \text{——} ④$$

$$M_{CD} = \frac{2EI}{4}\left(2\overset{-\theta_B}{\cancel{\theta_C}} + \overset{0}{\cancel{\theta_D}} - 0\right) = -EI\theta_B \quad \text{——} ⑤$$

$$M_{DC} = \frac{2EI}{4}\left(2\overset{0}{\cancel{\theta_D}} + \overset{-\theta_B}{\cancel{\theta_C}} - 0\right) = -\frac{1}{2}EI\theta_B \quad \text{——} ⑥$$

⑤- Joint Conditions.

Use Joint B



$$\Sigma M_B = 0$$

$$M_{BA} + M_{BC} = 0$$

$$EI\theta_B + \frac{1}{2}EI\theta_B - 20 = 0$$

$$1.5\,EI\,\theta_B = 20$$

$\theta_B = \frac{40}{3EI}$ ← تعوض في معادلات العزوم

$$M_{AB} = \frac{1}{2}EI\left(\frac{40}{3EI}\right) = \frac{20}{3}\ kN.m$$

$$M_{BA} = EI\left(\frac{40}{3EI}\right) = \frac{40}{3}\ kN.m$$

$$M_{BC} = \frac{1}{2}EI\left(\frac{40}{3EI}\right) - 20 = -\frac{40}{3}\ kN.m$$

$$M_{CB} = -\frac{1}{2}EI\left(\frac{40}{3EI}\right) + 20 = \frac{40}{3}\ kN.m$$

$$M_{CD} = -EI\left(\frac{40}{3EI}\right) = -\frac{40}{3}\ kN.m$$

$$M_{DC} = -\frac{1}{2}EI\left(\frac{40}{3EI}\right) = -\frac{20}{3}\ kN.m$$