المادة: الرياضيات
المرحلة: الاولى
2023-2022

Lecture (1)

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PREREQUISITES FOR CALCULUS (المتطلبات الاساسية للتفاضل و التكامل)

Sets and Intervals (المجموعات و الفترات)

DEFINITIONS:

Set: is a collection of things under certain conditions.

Example 1:

A={1,3,5,8,10};

A is a set, 1,3,5,8,10 are elements.

Real Numbers (R): is a set of all rational and irrational numbers. R= {-∞, +∞},

-∞ ←——————— 0 ———————→ +∞

Integer Numbers (I): a set of all irrational numbers.

I = {-∞,----,-3,-2,-1,0,1,2,3,----,+∞} negative and positive numbers only.

Natural Numbers (N): consist of zero and positive integer numbers only.

N ={0,1,2,3,----,+∞}

Intervals: is a set of all real numbers between two points on the real number line. (it is a subset of real numbers)

1. **Open interval:** is a set of all real numbers between A&B excluded (A&B are not elements in the set). $\{x: A < x < B\}$ or (A, B).

   -∞ ←——— A(——— X ———)B ———→ +∞

2. **Closed interval:** is a set of all real numbers between A&B included (A&B are elements in the set). $\{x: A \leq x \leq B\}$ or [A, B].

   -∞ ←——— A[——— X ———]B ———→ +∞

3. **Half-Open interval (Half-Close):** is a set of all real numbers between A & B with one of the end-points as an element in the set.

   a) (A, B]= $\{x: A < x \leq B\}$ -∞ ←——— A(——— X ———]B ———→ +∞

   b) [A, B)= $\{x: A \leq x < B\}$ -∞ ←——— A[——— X ———)B ———→ +∞

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TABLE 1.1 Types of intervals

	Notation	Set description	Type	Picture
Finite:	(a, b)	$\{x \mid a < x < b\}$	Open	
	$[a, b]$	$\{x \mid a \leq x \leq b\}$	Closed	
	$[a, b)$	$\{x \mid a \leq x < b\}$	Half-open	
	$(a, b]$	$\{x \mid a < x \leq b\}$	Half-open	
Infinite:	(a, ∞)	$\{x \mid x > a\}$	Open	
	$[a, \infty)$	$\{x \mid x \geq a\}$	Closed	
	$(-\infty, b)$	$\{x \mid x < b\}$	Open	
	$(-\infty, b]$	$\{x \mid x \leq b\}$	Closed	
	$(-\infty, \infty)$	$\mathbb{R}$ (set of all real numbers)	Both open and closed	

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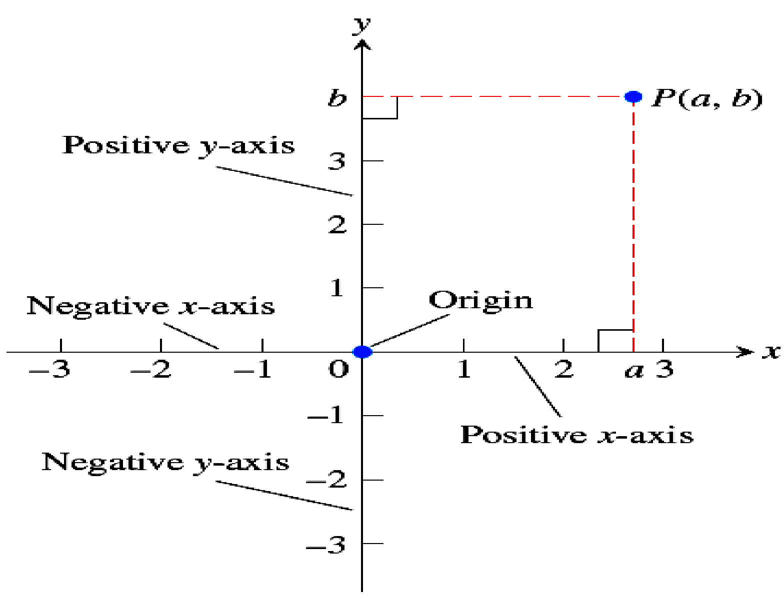
Coordinate in the Plane (الاحداثيات في الفراغ او المستوى)

Each point in the plane can be represented with a pair of real numbers (a,b), the number a is the horizontal distance from the origin to point P, while b is the vertical distance from the origin to point P. The origin divides the x-axis into positive x axis to the right and the negative x-axis to the left, also, the origin divides the y-axis into positive y-axis upward and the negative x-axis downward. The axes divide the plane into four regions called quadrants.



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Distance between Points and (Mid-Point Formula):

Distance between points in the plane is calculated with a formula that comes from Pythagorean Theorem:

❖ Distance Formula for Points in the Plane

The distance between $P(x_1, y_1)$ and $Q(x_2, y_2)$

$$\mathbf{d = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

and the mid-point formula:

$$\mathbf{x_0 = \frac{x_1 + x_2}{2} \quad , y_0 = \frac{y_1 + y_2}{2}}$$

Example 2: find the distance between **P(-1,2)** and **Q(3,4)** and find the mid-point:

Sol.:

$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(3 - (-1))^2 + (4 - 2)^2} = \sqrt{20} = 2\sqrt{5}$$

$$x_0 = \frac{x_1+x_2}{2} \quad , x_0 = \frac{-1+3}{2} = 1 \text{ and } \quad y_0 = \frac{y_1+y_2}{2} \quad , y_0 = \frac{2+4}{2} = 3.$$

Example 3: find the distance between **R(2,-3)** and **S(6,1)** and find the mid-point:

Sol.:

$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{(6 - 2)^2 + (1 - (-3))^2} = \sqrt{16 + 16} = \sqrt{32} = 2\sqrt{8}$$

$$x_0 = \frac{x_1+x_2}{2} \quad , x_0 = \frac{2+6}{2} = 4 \text{ and } \quad y_0 = \frac{y_1+y_2}{2} \quad , y_0 = \frac{-3+1}{2} = -1.$$

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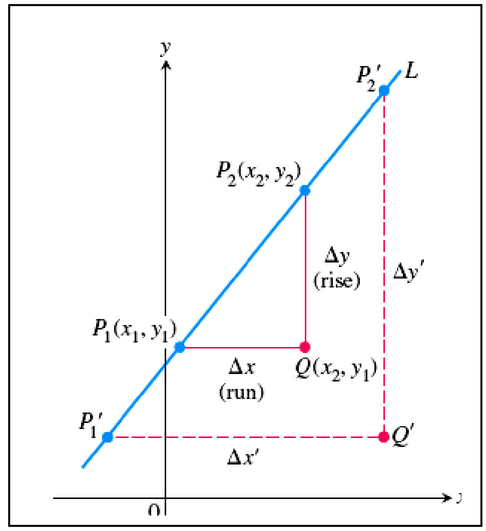
Slope and Equation of Line

- **Slope (الميل):** The constant

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$



is the slope of non-vertical line $P_1 P_2$.

Note1: Horizontal line have (**m=0**) (Δy=0), and the vertical line has no slope or the slope of vertical line is undefined (Δx=0).

Note2: Parallel lines have the same slope
In the the lines are parallel then **(m1= m2).**

Note3: If two non-vertical lines L1and L2 are perpendicular, their slopes **m1** and **m2** satisfy

m1*m2 = -1,

so each slope is the negative reciprocal of the other.

$$m_1 = \frac{1}{m_2} \text{ and } m_2 = \frac{1}{m_1}$$

Example 4: Find the slope of the straight line through the two points **P(3,2)** and **Q(4,4)** :

Sol.:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4-2}{4-3} = 2.$$

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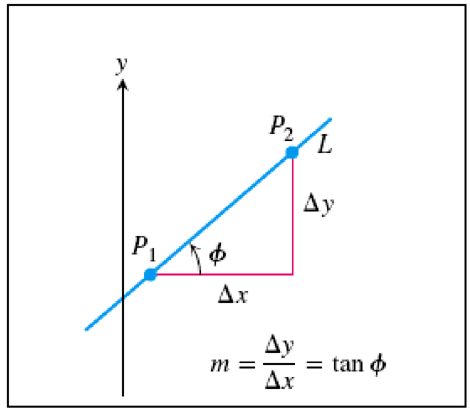
❖ Point-Slope Equation:



We can write an equation for a non-vertical straight line L if we know its slope m and the coordinate of one point $P_1(x_1, y_1)$ on it. If $P(x, y)$ is any other point on L, then we can use two points P_1 and P to compute the slope,

$$\mathbf{m = \frac{y - y_1}{x - x_1}}$$

so that $y - y_1 = m(x - x_1)$

or $y = y_1 + m(x - x_1)$

The equation $\mathbf{y = y_1 + m(x - x_1)}$

is the point-slope equation of the line that passes through the point $\mathbf{P_1(x_1, y_1)}$ and has slope m.

Example 5: write an equation for the line pass through the point **(2,3)** with slope **(-3/2).**

Sol.: we substitute $x_1 = 2$, $y_1 = 3$, and $m = -3/2$ into the point-slope equation and obtain

$$\mathbf{y = y_1 + m(x - x_1)}$$

$$\mathbf{y = 3 + \frac{-3}{2}(x - 2)}$$

$$\mathbf{y = -\frac{3}{2}x + 6.}$$

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Example 6: A line pass through two points: write an equation for the line through
(-2,-1) and **(3,4)**

Sol.: The line's slope is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-4}{-2-3} = \frac{-5}{-5} = 1.$$

We can use this slope with either of the two given points in the point-slope equation;

With (x1,y1) = (–2, –1)	**With (x2,y2) = (3, 4)**
y = –1 + 1 . (x–(–2))	**y = 4 + 1 . (x–3)**
y = –1 + x + 2	**y = 4 + x –3**
y = x + 1	**y = x + 1**

Note: The equation:

y = mx + b

is called the **slope-intercept equation** of the line with slope m and y-intercept b

Note: **The general form of straight line equation** is

Ax + By + C= 0

Example 7: finding the slope and y-Intercept for the line 8x + 4y = 20.

Sol.: solve the equation for y to put it in slope-intercept form :

8x + 4y = 20
4y = –8x + 20
y =-8/4 x +4.
y =-2 x +4.

The slope **m = – 2** the y-intercept is **b = 4**.

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H.W:

1. finding the slope and y-Intercept for the line $4x + 2y = 4$.
2. write an equation for the line pass through **(-1,-1)** and **(1,2).**
3. write an equation for the line pass through the point **(1,-1)** with slope **(4).**
4. Find the **slope** of the straight line through the two points **P(3,-2)** and **Q(3,6)**.
5. write an equation for the horizontal line pass through the point **(2,-2)** .