Calculations of the height of packing (Z)

For dilute mixtures :

Consider mass transfer occur in the column of cross sectional area equal to (S) (m^2), containing packing of specific surface area equal to (a), (m^2 / m^3).

Specific surface area = total surface area / volume of the column

Total surface area = surface area of one pack * number of packing

Volume of column = S * dZ

Total mass transfer area in height dZ (Interfacial area for transfer) = a . S . dZ

The total no. of moles of (A) transfer per unit area per unit time = N_A

Total moles of (A) transfer / time = N_A *a * S * dZ

According to Whitman two-film theory:-

$$N_A = K_{OG} (P_A - P_A^*) \quad ....(1)$$

$$N_A * a * S * dZ = K_{OG} (P_A - P_A^*) * a * S * dZ \quad ....(2)$$

The equation is based on gas phase and is used in calculating the height of the packing (Z) are applied only for dilute or weak solution which leads to the assumption that L_m & G_m are constant through out the solution. Don't forget that the solutions can Be considered dilute if the mole fraction of the solute in the inlet streams (gas & liquid) are less than 0.1 (i.e. $\leq$ 10%). The change in gas composition of (A) through the height (dZ) equal to (dy_A).

Total no. of moles of (A) transfer / time = $N_A * a * S * dZ$

$N_A * a * S * dZ = G_m * dy_A$ ....(3)

$G_m * dy_A = K_{OG} (P_A - P_A^*) * a * S * dZ$ ....(4)

Divide the right hand side of the above equation by (P_T / P_T):-

$G_m * dy_A = K_{OG} (y_A - y_A^*) P_T * a * S * dZ$ ....(5)

$$\int_0^Z dZ = \left(\frac{G_m}{K_{OG} * P_T * a * S}\right) \int_{y_{AT}}^{y_{AB}} \frac{dy_A}{(y_A - y_A^*)}$$

Integration eq.:-

$$Z = \left(\frac{G_m}{K_{OG} * P_T * a * S}\right) * \int_{y_{AT}}^{y_{AB}} \frac{dy_A}{(y_A - y_A^*)} \quad ....(6)$$

$Z = (HTU)_{OG} * (NTU)_{OG}$ [Height of packing for dilute solution for gas phase]

Where:- (HTU) = height of transfer unit.

(NTU) = number of transfer unit.

K_{OG} = is constant depends on the physical properties of gas and hydrodynamic properties of fluid.

G_m = constant for dilute solution $= \left(\frac{G_{top} + G_{Bott.}}{2}\right)$.

P_T = constant, no friction losses.

K_{OG}, K_{OL} = volumetric overall mass transfer coefficient (Kmole/ sec. mole fraction. m^3 packing).

H.W :- Derive the (Z) equation for liquid phases:-

$Z = (HTU)_{OL} * (NTU)_{OL}$ [Height of packing for dilute solution for liquid phase].

Summary:-

For dilute solutions:-

For gases (Overall Mass Transfer):-

$Z = (HTU)_{OG} * (NTU)_{OG}$ **[Height of packing for dilute solution for gas phase]**

$$(HTU)_{OG} = \left(\frac{G_m}{K_{OG} * P_T * a * S}\right)$$

$$(NTU)_{OG} = \int_{y_{AT}}^{y_{AB}} \frac{dy_A}{(y_A - y_A^*)}$$

For liquids (Overall Mass Transfer):-

$Z = (HTU)_{OL} * (NTU)_{OL}$ **[Height of packing for dilute solution for liquid phase]**

$$(HTU)_{OL} = \left(\frac{L_m}{K_{OL} * C_T * a * S}\right)$$

$$(NTU)_{OL} = \int_{X_{AT}}^{X_{AB}} \frac{dX_A}{(X_A^* - X_A)}$$

Where the integration of $(NTU)_{OG} = \int_{y_{AT}}^{y_{AB}} \frac{dy_A}{(y_A - y_A^*)}$ **is :-**

$$(NTU)_{OG} = \left(\frac{y_B - y_T}{(y - y_*)_{Lm}}\right)$$

$$(y - y_*)_{Lm} = \frac{(y_B - y_{B*}) - (y_T - y_{T*})}{\ln\frac{(y_B - y_{B*})}{(y_T - y_{T*})}}$$

For gases (Individual Mass Transfer):-

$Z = (HTU)_G * (NTU)_G$ [Height of packing for dilute solution for gas phase]

$$(HTU)_G = \left(\frac{G_m}{K_G * P_T * a * S}\right)$$

$$(NTU)_G = \int_{y_{AT}}^{y_{AB}} \frac{dy_A}{(y_A - y_{Ai})}$$

For liquids (Individual Mass Transfer):-

$Z = (HTU)_L * (NTU)_L$ [Height of packing for dilute solution for liquid phase]

$$(HTU)_L = \left(\frac{L_m}{K_L * C_T * a * S}\right)$$

$$(NTU)_L = \int_{X_{AT}}^{X_{AB}} \frac{dX_A}{(X_{Ai} - X_A)}$$

The integration of $(NTU)_G = \int_{y_{AT}}^{y_{AB}} \frac{dy_A}{(y_A - y_{Ai})}$ **is :-**

$$(NTU)_G = \left(\frac{y_B - y_T}{(y - y_i)_{Lm}}\right) \text{ where } (y - y_i)_{Lm} = \frac{(y_B - y_{Bi}) - (y_T - y_{Ti})}{\ln\frac{(y_B - y_{Bi})}{(y_T - y_{Ti})}}$$

Notice: 1) $\frac{Lm}{Gm}$ **slope of operating line =** $\frac{(y_B - y_T)}{(x_B - x_T)}$ **from (M. B.).**

2) m **= slope of equilibrium line =** $\frac{(y_B^* - y_T^*)}{(x_B - x_T)}$ **from (equilibrium data).**

Relation between overall and individual mass transfer coefficient

$$\frac{1}{K_{OG}} = \frac{1}{K_G} + \frac{H}{K_L} \quad \text{............(1)} \quad \text{(in gas phase)}$$

$$\frac{1}{K_{OL}} = \frac{1}{K_G} + \frac{1}{H.K_L} \quad \text{............(2)} \quad \text{(in liquid phase)}$$

Multiply equation (1) by (G_m / a . s . P_T):-

$$\frac{Gm}{K_{OG} . a . s . P_T} = \frac{Gm}{K_G . a . s . P_T} + \frac{H . Gm}{K_L . a . s . P_T} \quad \text{...........(3)}$$

$$(HTU)_{OG} = (HTU)_G + \frac{H . Gm}{K_L . a . s . P_T}] * \frac{Lm}{Lm} \frac{C_T}{C_T}$$

$$(HTU)_{OG} = (HTU)_G + (HTU)_L \frac{Gm}{Lm} * \frac{H . C_T}{P_T} \quad \text{..............(4)}$$

$$\therefore P_A = H . C_A \rightarrow \frac{P_A}{P_T} = \frac{H . C_A}{P_T} \rightarrow \therefore y_A = \frac{H . C_T}{P_T} . x_A$$

$\therefore y_A = m . x_A$ **......... straight line equation**

$$(HTU)_{OG} = (HTU)_G + (HTU)_L . m . \frac{Gm}{Lm} \quad \text{..............(5)} \quad \text{(For gas phase)}$$

Same steps can be applied to equation (2) for liquid phase to get :

$$(HTU)_{OL} = (HTU)_L + (HTU)_G . \frac{Lm}{m . Gm} \quad \text{..............(6)} \quad \text{(For liquid phase)}$$

Where $(\frac{Lm}{m . Gm})$ is the absorption factor (A).