المادة: الرياضيات

المرحلة: الاولى

2022-2023

Lecture (3)

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Composition of Functions: (تركيب الدوال)

DEFINITION: If f and g are functions, the composite (f ∘ g) ((f composed with g)) or g ∘ f ((g composed with f)) are defined by:
(f o g)(x) = f (g(x)) and (*g* ∘ *f*)(*x*) =*g* (*f*(*x*)) respectively

Examples 1: Find the formula for (f o g)(x) and (*g* ∘ *f*)(*x*) if g(x) =x^2 and f(x) = x - 7,
then find the value of f(g(2)) and g(f(2)).

SOL:

A: (f o g)(x) = f (g(x)) = f (x^2) = x^2 – 7.
f(g(2))= 2^2 – 7= - 3.

B: (g o f)(x) = g (f(x)) = g(x - 7) = $(x - 7)^2$.
g(f(2))= $(2 - 7)^2$= $(-5)^2$.

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Examples 2: Find the formula for (f o g)(x) and (*g* ∘ *f*)(*x*) if f(x) = $x^2 + 1$ and g(x) =$\sqrt{x}$,
then find the value of f(g(3)) and g(f(3)).

SOL:

A: (f o g)(x) = f (g(x)) = f ($\sqrt{x}$) = $(\sqrt{x})^2$+ 1= $x + 1$.
f(g(3))= $3 + 1$ =4.

B: (g o f)(x) = g (f(x)) = g($x^2 + 1$) = $\sqrt{x^2 + 1}$.
g(f(3))= $\sqrt{3^2 + 1}$ = $\sqrt{10}$.

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H.W: Finding formulas for composites If f (x) = x and g(x) = x +1,
Find:

(a) (f o g)(x) **(b) (g o f)(x)** **(c) (f o f)(x)** **(d) (g o g)(x)**

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Even Function, Odd Function:(الدالة الفردية والدالة الزوجية)

A function y = f(x) is an

even function of x if $f(-x) = f(x)$

odd function of x if $f(-x) = -f(x)$

for every x in the function's domain.

Examples: Recognizing Even and Odd functions

1) $f(x) = x^2$
   $f(-x) = (-x)^2 = x^2$
   $-f(x) = -(x)^2$
   Even function

2) $f(x) = x^2 + 1$
   $f(-x) = (-x)^2 + 1 = x^2 + 1$
   $-f(x) = -(x)^2 - 1$
   Even function

3) $f(x) = x$
   $f(-x) = -x$
   $-f(x) = -x$
   odd function

4) $f(x) = x + 1$
   $f(-x) = -x + 1$
   $-f(x) = -x - 1$
   Not Even function , Not odd function

Lecture (3)

5) $f(x) = x - 1$

$f(-x) = -x - 1$

$-f(x) = -x + 1$

Not Even function , Not odd function

6) $f(x) = x^3$

$f(-x) = (-x)^3 = -(x)^3$

$-f(x) = -(x)^3$

Odd function

H.W:

1) $f(x) = x^3 - 3.$
2) $f(x) = x^3 + x^2 - 3$
3) $f(x) = x^2 - x$
4) $f(x) = \frac{1}{x}$

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symmetry of the function (تماثل الدالة)

If f(x,y) = 0 is any function then:

1. Symmetry about x-axis: If f(x,-y) = f(x,y)
2. Symmetry about y-axis: If f(-x,y) = f(x,y) It is called an **even function.**
3. Symmetry about the origin: If f(-x,-y) = f(x,y) It is called an **odd function**

Examples : Check the symmetry of the following curves:

1) $\mathbf{y = x^2}$

Sol \ $f(x,y) = x^2 - y = 0$

$f(x,-y) = x^2 - (-y) = x^2 + y \Rightarrow f(x,-y) \neq f(x,y)$ **NOT OK**

$f(-x,y) = (-x)^2 - (y) = x^2 - y \Rightarrow f(-x,y) = f(x,y)$ **OK**

$f(-x,-y) = (-x)^2 - (-y) = x^2 + y \Rightarrow f(-x,-y) \neq f(x,y)$ **NOT OK**

So the function has symmetry only about y-axis. It is called an even function.

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2) $y = x^3$

Sol \ $f(x, y) = x^3 - y = 0$

$f(x, -y) = x^3 - (-y) = x^3 + y$ ⟹ $f(x, -y) \neq f(x, y)$ NOT OK

$f(-x, y) = (-x)^3 - (y) = -x^3 - y$ ⟹ $f(-x, y) \neq f(x, y)$ NOT OK

$f(-x, -y) = (-x)^3 - (-y) = -x^3 + y = x^3 - y$ ⟹

$f(-x, -y) = f(x, y)$ OK

So the function has symmetry only about origin. It is called an odd function.

3) $x^2 = y^2 + 4$

Sol \ $f(x, y) = y^2 - x^2 + 4 = 0$

$f(x, -y) = (-y)^2 - x^2 + 4 = y^2 - x^2 + 4$ ⟹ $f(x, -y) = f(x, y)$ OK

$f(-x, y) = y^2 - (-x)^2 + 4 = y^2 - x^2 + 4$ ⟹ $f(-x, y) = f(x, y)$ OK

$f(-x, -y) = (-y)^2 - (-x)^2 + 4 = y^2 - x^2 + 4$ ⟹ $f(-x, y) = f(x, y)$ OK

So the function has symmetry about x-axis, y-axis and the origin.

H.W:

1) $y = 3x^2 + 2.$

2) $x^2 + y^2 = 1$