Introduction to Amperes' Law

Ampere's law is one of the useful law which relates the net magnetic field along the closed-loop to the electric current which passes through the loop. It was discovered by André-Marie Ampère in the year 1826. This expression for the relation between the magnetic field and the current which produces it is termed as the Ampere's law.

The definition states that the line integral of the magnetic field around some arbitrary path will be proportional to the net electric current enclosed by such path. Further, Maxwell extended this law with the inclusion of magnetic fields which arise without current by other causes. Thus, Ampere-Maxwell's law is one of the four Maxwell equations.

The integral form of the law is as below:

$\oint Bds=\mu 0I$

Where :

μ0 is the permeability of free space and 'I' is current.

Also, in the form of summation:

Sum $B\Delta S=\mu 0I$

Concept of Ampere's law

For Ampère's Law, all currents have to be steady. So, currently does not change with time. Also, only currents crossing the area inside the path must be considered and will have some contribution to the magnetic field. Currents should be taken with their algebraic signs. One may use the right hand's rule to determine its directions and signs. The total magnetic circulation will be zero if the magnetic field is normal to the selected path at

any point. Ampère's Law will also be useful while calculating the magnetic fields of current distributions with a high degree of symmetry.

Ampere's Law

We can find the net electric field due to *any* distribution of charges by first writing the differential electric field $\overrightarrow{dE}$ due to a charge element and then summing the contributions of $\overrightarrow{dE}$ from all the elements. However, if the distribution is complicated, we may have to use a computer. Recall, however, that if the distribution has planar, cylindrical, or spherical symmetry, we can apply Gauss' law to find the net electric field with considerably less effort.

Similarly, we can find the net magnetic field due to *any* distribution of currents by first writing the differential magnetic field $\overrightarrow{dB}$ (Eq. 29-3) due to a current-length element and then summing the contributions of $\overrightarrow{dB}$ from all the elements. Again we may have to use a computer for a complicated distribution. However, if the distribution has some symmetry, we may be able to apply **Ampere's law** to find the magnetic field with considerably less effort. This law, which can be derived from the Biot–Savart law, has traditionally been credited to André-Marie Ampère (1775–1836), for whom the SI unit of current is named. However, the law actually was advanced by English physicist James Clerk Maxwell.

Ampere's law is

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{i\,d\vec{s}\times\hat{r}}{r^2} \qquad \text{(Biot–Savart law).} \qquad (29\text{-}3)$$

$$\oint \vec{B}\cdot d\vec{s} = \mu_0 i_{enc} \qquad \text{(Ampere's law).} \qquad (29\text{-}14)$$

The loop on the integral sign means that the scalar (dot) product $\vec{B}.\overrightarrow{ds}$ is to be integrated around a *closed* loop, called an *Amperian loop.* The current *i*enc is the *net* current encircled by that closed loop.

To see the meaning of the scalar product $\vec{B}.\overrightarrow{ds}$ and its integral, let us first apply Ampere's law to the general situation of Fig. 29-11.The figure shows cross sections of three long straight wires that carry currents i_1, i_2, and i_3 either directly into or directly out of the page. An arbitrary Amperian loop lying in the plane of the page encircles two of the currents but not the third. The counterclockwise direction marked on the loop indicates the arbitrarily chosen direction of integration for Eq. 29-14.

To apply Ampere's law, we mentally divide the loop into differential vector elements $\overrightarrow{ds}$ that are everywhere directed along the tangent to the loop in the direction of integration. Assume that at the location of the element $\overrightarrow{ds}$ shown in Fig. 29-11, the net magnetic field due to the three currents is . Because the wires are perpendicular to the page, we know that the magnetic field at $\overrightarrow{ds}$ due to each current is in the plane of Fig. 29-11, thus, their net magnetic field at must also be in that plane. However, we do not know the orientation of within the plane. In Fig. 29-11, $\vec{B}$ is arbitrarily drawn at an angle θ to the direction of $\overrightarrow{ds}$.

The scalar product $\vec{B}.\overrightarrow{ds}$ on the left side of Eq. 29-14 is equal to $B \cos\theta\, ds$.

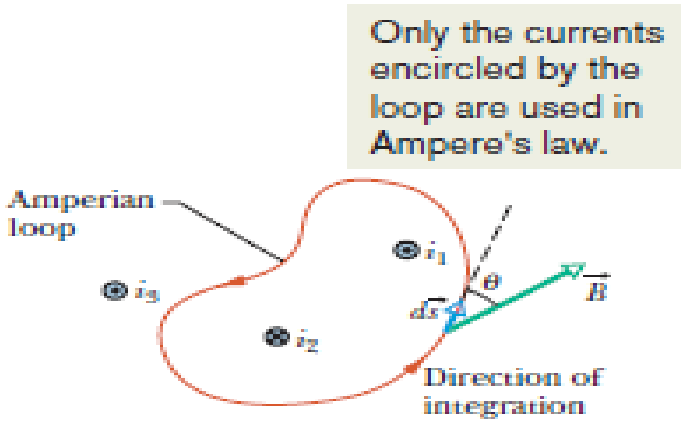


Fig. 29-11 Ampere's law applied to an arbitrary Amperian loop that encircles two long straight wires but excludes a third wire. Note the directions of the currents.

Thus, Ampere's law can be written as

$$\oint \vec{B} \cdot d\vec{s} = \oint B \cos\theta \, ds = \mu_0 i_{enc} \qquad (29\text{-}15)$$

We can now interpret the scalar product $\vec{B}.\overrightarrow{ds}$ as being the product of a length *ds* of the Amperian loop and the field component $B \cos\theta$ tangent to the loop.Then we can interpret the integration as being the summation of all such products around the entire loop.

When we can actually perform this integration, we do not need to know the direction of $\vec{B}$ before integrating. Instead, we arbitrarily assume$\vec{B}$ to be generally in the direction of integration (as in Fig. 29-11). Then we use the following curled–straight right-hand rule to assign a plus sign or a minus sign to each of the currents that make up the net encircled current *i*enc:

Finally, we solve Eq. 29-15 for the magnitude of . If *B* turns out positive, then the direction we assumed for $\vec{B}$ is correct. If it turns out negative, we neglect the minus sign and redraw $\overrightarrow{B}$in the opposite direction.

$$i_{enc} = i_1 - i_2.$$

(Current i_3 is not encircled by the loop.) We can then rewrite Eq. 29-15 as

$$\oint B \cos\theta \, ds = \mu_0(i_1 - i_2). \qquad (29\text{-}16)$$

You might wonder why, since current *i*3 contributes to the magnetic-field magnitude *B* on the left side of Eq. 29-16, it is not needed on the right side. The answer is that the contributions of current *i*3 to the magnetic field cancel out because the integration in Eq. 29-16 is made around the full loop. In contrast, the contributions of an encircled current to the magnetic field do not cancel out.

We cannot solve Eq. 29-16 for the magnitude *B* of the magnetic field because for the situation of Fig. 29-11 we do not have enough information to simplify and solve the integral. However, we do know the outcome of the integration; it must be equal to $\mu_0 (i_1 - i_2)$, the value of which is set by the net current passing through the loop.

We shall now apply Ampere's law to two situations in which symmetry does allow us to simplify and solve the integral, hence to find the magnetic field.

Ampère's law with Maxwell's addition

Ampère's law with Maxwell's addition states that magnetic fields can be generated in two ways: by electric current (this was the original "Ampère's law") and by changing electric fields (this was "Maxwell's addition", which he called displacement current). In integral form, the magnetic field induced around any closed loop is proportional to the electric current plus displacement current (proportional to the rate of change of electric flux) through the enclosed surface.

Maxwell's addition to Ampère's law is particularly important: it makes the set of equations mathematically consistent for non static fields, without changing the laws of Ampere and Gauss for static fields. However, as a consequence, it predicts that a changing magnetic field induces an electric field and vice versa. Therefore, these equations allow self-sustaining "electromagnetic waves" to travel through empty space (see electromagnetic wave equation).

The speed calculated for electromagnetic waves, which could be predicted from experiments on charges and currents, matches the speed of light; indeed, light *is* one form of electromagnetic radiation (as are X-rays, radio waves, and others). Maxwell understood the connection between electromagnetic waves and light in 1861, thereby unifying the theories of electromagnetism and optics.

Formulation in SI units convention

Name	Integral equations	Differential equations
Gauss's law	$\oiint_{\partial\Omega} \mathbf{E}\cdot\mathrm{d}\mathbf{S} = \frac{1}{\varepsilon_0}\iiint_{\Omega}\rho\,\mathrm{d}V$	$\nabla\cdot\mathbf{E} = \frac{\rho}{\varepsilon_0}$
Gauss's law for magnetism	$\oiint_{\partial\Omega} \mathbf{B}\cdot\mathrm{d}\mathbf{S} = 0$	$\nabla\cdot\mathbf{B} = 0$
Maxwell–Faraday equation (Faraday's law of induction)	$\oint_{\partial\Sigma} \mathbf{E}\cdot\mathrm{d}\boldsymbol{\ell} = -\frac{\mathrm{d}}{\mathrm{d}t}\iint_{\Sigma}\mathbf{B}\cdot\mathrm{d}\mathbf{S}$	$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$
Ampère's circuital law (with Maxwell's addition)	$\oint_{\partial\Sigma} \mathbf{B}\cdot\mathrm{d}\boldsymbol{\ell} = \mu_0\left(\iint_{\Sigma}\mathbf{J}\cdot\mathrm{d}\mathbf{S} + \varepsilon_0\frac{\mathrm{d}}{\mathrm{d}t}\iint_{\Sigma}\mathbf{E}\cdot\mathrm{d}\mathbf{S}\right)$	$\nabla\times\mathbf{B} = \mu_0\left(\mathbf{J} + \varepsilon_0\frac{\partial\mathbf{E}}{\partial t}\right)$

Formulation in Gaussian units convention

Name	Integral equations	Differential equations
Gauss's law	$\oiint_{\partial\Omega} \mathbf{E}\cdot\mathrm{d}\mathbf{S} = 4\pi\iiint_{\Omega}\rho\,\mathrm{d}V$	$\nabla\cdot\mathbf{E} = 4\pi\rho$
Gauss's law for magnetism	$\oiint_{\partial\Omega} \mathbf{B}\cdot\mathrm{d}\mathbf{S} = 0$	$\nabla\cdot\mathbf{B} = 0$
Maxwell–Faraday equation (Faraday's law of induction)	$\oint_{\partial\Sigma} \mathbf{E}\cdot\mathrm{d}\boldsymbol{\ell} = -\frac{1}{c}\frac{\mathrm{d}}{\mathrm{d}t}\iint_{\Sigma}\mathbf{B}\cdot\mathrm{d}\mathbf{S}$	$\nabla\times\mathbf{E} = -\frac{1}{c}\frac{\partial\mathbf{B}}{\partial t}$
Ampère's circuital law (with Maxwell's addition)	$\oint_{\partial\Sigma} \mathbf{B}\cdot\mathrm{d}\boldsymbol{\ell} = \frac{1}{c}\left(4\pi\iint_{\Sigma}\mathbf{J}\cdot\mathrm{d}\mathbf{S} + \frac{\mathrm{d}}{\mathrm{d}t}\iint_{\Sigma}\mathbf{E}\cdot\mathrm{d}\mathbf{S}\right)$	$\nabla\times\mathbf{B} = \frac{1}{c}\left(4\pi\mathbf{J} + \frac{\partial\mathbf{E}}{\partial t}\right)$

Applications of Ampere's Law

Ampere's Law is used to :

- Determine the magnetic induction due to long current-carrying wire.
- Determine the magnetic field inside a toroid.
- Determine the magnetic field created by a long current carrying conducting cylinder.
- Determine the magnetic field inside the conductor
- Find forces between currents

Ampere's Law is one of the important law of Electromagnetism. It is finding the expression for any closed-loop path. It states that the sum of the length elements times the magnetic field in the direction of the length element will be equal to the permeability times the electric current. This law allows us to maintain a proper bridge to fulfil the gap between electricity and magnetism. It also provides the mathematical relation between magnetic fields and electric currents. Amperes law gives the way to calculate the magnetic field, produced due to the result of an electric current moving through a wire of any shape. This article will discuss the Amperes Law in a brief but systematic way.