EXPERMINT NO.3

Response ***O***f ***Second*** ***O***rder ***S***ystem ***W***ith ***SS***

will generate plots of unit-step responses (t in the step command is the user-specified time.) For a control system defined in a state-space form, where state matrix A, control matrix B, output matrix C, and direct transmission matrix D of state-space equations are known, the command

step(A,B,C,D) OR step(A,B,C,D,t)

$$\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}$$
$$\mathbf{y} = \mathbf{Cx} + \mathbf{Du}$$

Example:1

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 6.5 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$
$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Ans:

```
clear all
close all
clc

% Enter matrices A, B, C, and D

A = [-1 -1;6.5 0];
B = [1 1;1 0];
C = [1 0;0 1];
D = [0 0;0 0];

%creat the state space

g=ss(A,B,C,D)

%step respone of 2nd order system with ss

step(g)
```

Example:2

$$A = \begin{bmatrix} -1 & -1 \\ 6.5 & 0 \end{bmatrix}; B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}; C[1 \quad 1] = ; D = 0$$

Ans:

```
clear all
close all
clc
% Enter  A, B, C, and D
A = [-1 -1;6.5 0];
B = [1;0];
C = [1 1];
D = 0;
%creat the state space
g=ss(A,B,C,D)
%step respone of 2nd order system with ss
step(g)
```

Example:3 The system has a sample time of 0.2 s and by the following state-space matrices.

$$A= \begin{bmatrix} 1.6 & -0.7 \\ 1 & 0 \end{bmatrix}; B=\begin{bmatrix} 0.5 \\ 0 \end{bmatrix}; C= [0.1 \quad 0.1];D=0;$$

Ans:

```
clear all
close all
clc

%enter value of sample time
t=0.2

% Enter  A, B, C, and D
A = [1.6 -0.7;1  0];
B = [0.5; 0];
C = [0.1 0.1];
D = 0;

%creat the state space
g = ss(A,B,C,D,t);

%step respone of 2nd order system with ss
step(g)
```

Example:4 A system having two inputs (u1 and u2) with following state-space matrices.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -1 & -1 \\ 6.5 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Ans:

```
clear all
close all
clc
% Enter matrices A, B, C, and D
A = [-1 -1;6.5 0];
B = [1 1;1 0];
C = [1 0;0 1];
D = 0;
g=ss(A,B,C,D)
step(g)
%To plot step-response curves when the input is u1
step(A,B,C,D,1)
%To plot step-response curves when the input is u2
step(A,B,C,D,2)
```

Example:5 convert the following transfer function to state space with time 0:0.01:10

$$G(s) = \frac{9}{s^2+2s+9}$$

Ans:

```
clear all
close all
clc
%enter specific time value
t=0:0.01:10
n=[9];
d=[1 2 9];
%creat the transfer function
g=tf(n,d)
%convert transfer function to state space
[A,B,C,D]=tf2ss(n,d)
%step respone of 2nd order system with ss
step(g,t)
```

Example:6 convert the following state space to transfer function with time 0:0.003:6 then determine rise time,peak time,maxovershoot and settling time

$$A=\begin{bmatrix}-6 & -36\\ 1 & 0\end{bmatrix};\ B=\begin{bmatrix}1\\0\end{bmatrix};\ C=[0 \quad 36];D=0$$

Ans:

```
clear all
close all
clc

%enter specific time value
t=0:0.003:6;
% Enter  A, B, C, and D
A=[-6 -36;1 0];
B=[1;0];
C=[0 36];
D=0;
%convert state space to transfer function
[n,d]=ss2tf(A,B,C,D)
n=[36];
d=[1 6 36];
g=tf(n,d)
step(g,t)
[y,x,t]=step(g,t);

%determnation of rise time
r=1;
while y(r)<1.00001
    r=r+1;
end
rise_time=(r-1)*0.003
%determnation of peaktime & maxovershoot
[ymax,tp]=max(y)
peak_time=(tp-1)*0.003
max_overshoot=ymax-1

%determination of the setteling time
s=2001;
while y(s)>0.98 &y(s)<1.02
    s=s-1;
end

settling_time=(s-1)*0.003
```

Example:7 from problems B-5-2 on page 263

```
clear all
close all
clc

n=[1];        %cofficient of numerator
d=[1 1 1];    %cofficient of denominator
g=tf(n,d)      %creat the transfer function

%define damping ratio (zeta)
zeta=0.5;

%enter value of natural frequency wn
wn=1;

%calculation of damped frequency
wd=wn*sqrt(1-zeta^2)

%calculation of theta
theta=atan(sqrt(1-zeta^2)/zeta)

%calculation of Rise Time
tr=((pi-theta)/wd)

%calculation of Peak Time
tp=pi/wd

%calculation of time constant
tc=1/(zeat*wn)

%calculation of Settling Time for 2%
ts=4/(zeta*wn)

%calculation of Settling Time for 5%
ts=3/(zeta*wn)

%calculation of max overshoot
Mp=exp((-zeta*pi)/sqrt(1-zeta*zeta))*100

%step response of second order system
step(g)
```

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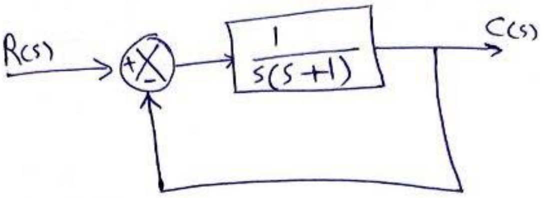
$$G(s) = \frac{1}{s(s+1)}$$

solution



$H(s) = 1$

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)\,H(s)} = \frac{\frac{1}{s(s+1)}}{1 + \frac{1}{s(s+1)}} = \frac{1}{s^2 + s + 1}$$

by comparision with standard form of the second order system

$$\frac{C(s)}{R_s} = \frac{Wn^2}{s^2 + 2\mathcal{E}Wn + Wn^2}$$

$Wn^2 = 1 \Rightarrow$ $Wn = 1$ rad/sec

$2\mathcal{E}Wn = 1 \Rightarrow$ $\mathcal{E} = 0.5$

$$\phi = \tan^{-1}\left(\frac{\sqrt{1-\mathcal{E}^2}}{\mathcal{E}}\right) * \frac{\pi}{180}$$

$$= \tan^{-1}\left(\frac{\sqrt{1-(0.5)^2}}{0.5}\right) * \frac{\pi}{180}$$

$\phi = 1.0472$ rad

$Wd = Wn\sqrt{1-\mathcal{E}^2} = 1\sqrt{1-(0.5)^2} \Rightarrow$ $Wd = 0.866$ rad/sec

*Rise Time: $T_r = \frac{\pi - \phi}{\omega_d} = \frac{\pi - 1.0472}{0.866} = 2.42$ sec

*Peak Time: $T_p = \frac{\pi}{\omega_d} = \frac{\pi}{0.866} = 3.63$ sec

*Time Constant: $T_c = \frac{1}{\sigma} = \frac{1}{\xi \omega_n} = \frac{1}{0.5 * 1} = 2$ sec

* Settling Time (for 2%): $T_s = 4 T_c$

$$T_s = \frac{4}{\sigma} = \frac{4}{0.5 * 1} = 8 \text{ sec}$$

* Settling Time (for 5%): $T_s = 3 T_c$

$$T_s = \frac{3}{\sigma} = \frac{3}{0.5 * 1} = 6 \text{ sec}$$

* Max over shoot (peak)%:

$$M_p\% = e^{\frac{-\pi \xi}{\sqrt{1-\xi^2}}} * 100\%$$

$$M_p\% = e^{\frac{-\pi (0.5)}{\sqrt{1-(0.5)^2}}} * 100\%$$

$$M_p\% = 0.163 * 100\%$$

$$M_p = 16.3\%$$