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Problem 6

Design a rubber belt to drive a dynamo generating 20 kW at 2250 r.p.m. and fitted with a pulley 200 mm diameter. Assume dynamo efficiency to be 85%. Allowable stress for belt = 2.1 MPa, Density of rubber = 1000 kg / m3, Angle of contact for dynamo pulley = 165° , Coefficient of friction between belt and pulley = 0.3

Solution

P = 20 kW = 20 × 10^3 W ; N = 2250 r.p.m. ; d = 200 mm = 0.2 m ; η_d = 85% = 0.85 ; σ = 2.1 MPa = 2.1 × 106 N/m^2 ; ρ = 1000 kg/m^3 ; θ = 165° = 165 × π/180 = 2.88 rad ; μ = 0.3

$$v = \frac{\pi\, d.N}{60} = \frac{\pi \times 0.2 \times 2250}{60} = 23.6 \text{ m/s}$$

power transmitted (P),

$20 \times 10^3 = (T_1 - T_2)\, v.\eta_d$

$= (T_1 - T_2)\, 23.6 \times 0.85$

$= 20.1\,(T_1 - T_2)$

$\therefore T_1 - T_2 = 20 \times 10^3 / 20.1 = 995$ N

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.3 \times 2.88 = 0.864$$

$$\therefore \quad \log\left(\frac{T_1}{T_2}\right) = \frac{0.864}{2.3} = 0.3756$$

or $$\frac{T_1}{T_2} = 2.375$$

... (Taking antilog of 0.3756)

$T1$ = 1719 N ; and $T2$ = 724 N

Let b = Width of the belt in metres, and

t = Thickness of the belt in metres.

Assuming thickness of the belt, t = 10 mm = 0.01 m, we have

Cross-sectional area of the belt

$= b \times t = b \times 0.01 = 0.01\ b$ m2

We know that mass of the belt per metre length,

m = Area × length × density = 0.01 b × 1 × 1000 = 10 b kg / m

∴ Centrifugal tension,

$T_C = m.v^2 = 10\ b\ (23.6)^2 = 5570\ b$ N

We know that maximum tension in the belt,

$T = \sigma.b.t = 2.1 \times 106 \times b \times 0.01 = 21\ 000\ b$ N

and tension in the tight side of belt ($T1$),

$1719 = T - T_C = 21\ 000\ b - 5570\ b = 15\ 430\ b$

∴ b = 1719 / 15 430 = 0.1114 m = 111.4 mm

The standard width of the belt (b) is 112 mm

Problem 7

Design a belt drive to transmit 110 kW for a system consisting of two pulleys of diameters 0.9 m and 1.2 m, centre distance of 3.6 m, a belt speed 20 m / s, coefficient of friction 0.3, a slip of 1.2% at each pulley and 5% friction loss at each shaft, 20% over load.

Solution

P = 110 kW = 110 × 10^3 W ; $d1$ = 0.9 m or $r1$ = 0.45 m ; $d2$ = 1.2 m or $r2$ = 0.6 m ; x = 3.6 m ; v = 20 m/s ; μ = 0.3 ; $s1 = s2$ = 1.2%

N_1 = Speed of the smaller or driving pulley in r.p.m., and

And N_2 = Speed of the larger or driven pulley in r.p.m.

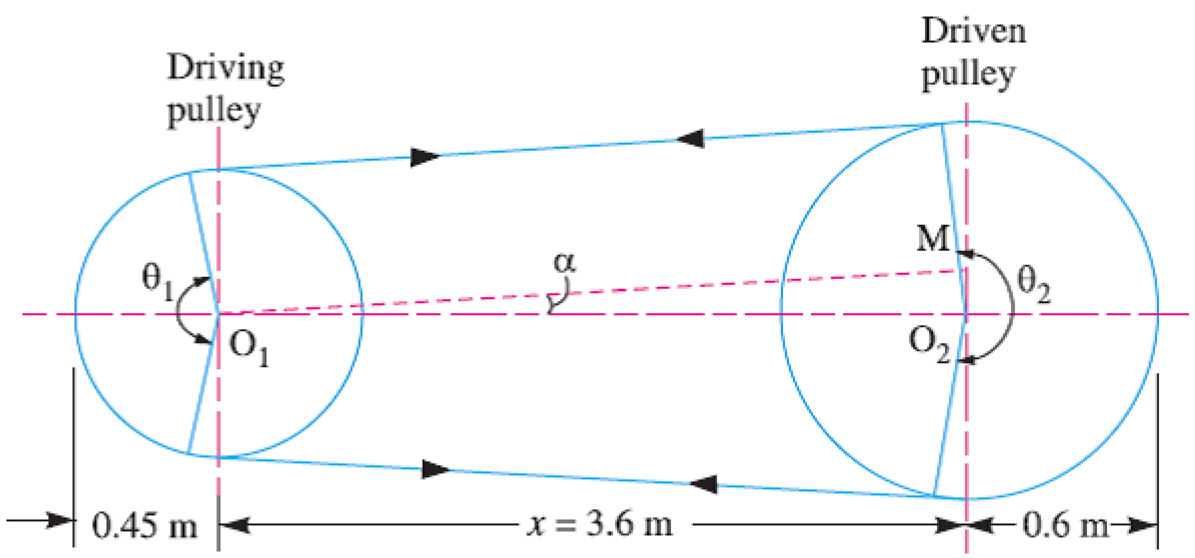
$$20 = \frac{\pi\, d_1 . N_1}{60}\left(1 - \frac{s_1}{100}\right) = \frac{\pi \times 0.9\ N_1}{60}\left(1 - \frac{1.2}{100}\right) = 0.0466\ N_1$$

$$N_1 = 20 / 0.0466 = 430 \text{ r.p.m.}$$

$$\frac{\pi\, d_2 . N_2}{60} = \text{Belt speed in m/s}\left(1 - \frac{s_2}{100}\right) = v\left(1 - \frac{s_2}{100}\right)$$

$$\frac{\pi \times 1.2 \times N_2}{60} = 20\left(1 - \frac{1.2}{100}\right) = 19.76$$

$$N_2 = \frac{19.76 \times 60}{\pi \times 1.2} = 315 \text{ r.p.m.}$$



the torque acting on the driven shaft

$$= \frac{\text{Power transmitted} \times 60}{2\,\pi\, N_2} = \frac{110 \times 10^3 \times 60}{2\,\pi \times 315} = 3334 \text{ N-m}$$

Since there is a 5% friction loss at each shaft, therefore torque acting on the belt

$= 1.05 \times 3334 = 3500$ N-m

Since the belt is to be designed for 20% overload, therefore design torque

$= 1.2 \times 3500 = 4200$ N-m

Let $T1$ = Tension in the tight side of the belt, and

$T2$ = Tension in the slack side of the belt.

We know that the torque exerted on the driven pulley

$= (T1 - T2)\, r2 = (T1 - T2)\, 0.6 = 0.6\,(T1 - T2)$ N-m

Equating this to the design torque, we have

$0.6\,(T1 - T2) = 4200$ or $T1 - T2 = 4200 / 0.6 = 7000$ N

Now let us find out the angle of contact (θ_1) of the belt on the smaller or driving pulley. From the geometry of the Figure, we find that

$$\sin\alpha = \frac{O_2M}{O_1O_2} = \frac{r_2 - r_1}{x} = \frac{0.6 - 0.45}{3.6} = 0.0417 \quad \text{or} \quad \alpha = 2.4^\circ$$

$$\theta_1 = 180^\circ - 2\alpha = 180 - 2 \times 2.4 = 175.2^\circ = 175.2 \times \frac{\pi}{180} = 3.06 \text{ rad}$$

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta_1 = 0.3 \times 3.06 = 0.918$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{0.918}{2.3} = 0.3991 \quad \text{or} \quad \frac{T_1}{T_2} = 2.51 \ \text{... (Taking antilog of 0.3991)}$$

$T1$ = 11 636 N ; and $T2$ = 4636 N

Let σ = Safe stress for the belt = 2.5 MPa = 2.5 × 106 N/m2 ...(Assume)

t = Thickness of the belt = 15 mm = 0.015 m, and ...(Assume)

b = Width of the belt in metres.

Since the belt speed is more than 10 m/s, therefore centrifugal tension must be taken into consideration. Assuming a leather belt for which the density may be taken as 1000 kg / m^3.

∴ Mass of the belt per metre length,

$$m = \text{Area} \times \text{length} \times \text{density} = b \times t \times l \times \rho$$

$$= b \times 0.015 \times 1 \times 1000 = 15\, b \text{ kg/m}$$

and centrifugal tension,

$$T_C = m.v^2 = 15\, b\, (20)^2 = 6000\, b \text{ N}$$

We know that maximum tension in the belt,

$$T = T_1 + T_C = \sigma.b.t$$

or $$11\,636 + 6000\, b = 2.5 \times 106 \times b \times 0.015 = 37\,500\, b$$

∴ $$37\,500\, b - 6000\, b = 11\,636 \text{ or } b = 0.37 \text{ m or } 370 \text{ mm}$$

The standard width of the belt (b) is 400 mm

length of the belt

$$L = \pi(r_2 + r_1) + 2x + \frac{(r_2 - r_1)^2}{x}$$

$$= \pi(0.6 + 0.45) + 2 \times 3.6 + \frac{(0.6 - 0.45)^2}{3.6}$$

$$= 3.3 + 7.2 + 0.006 = 10.506 \text{ m}$$

Problem 8

A belt 100 mm wide and 10 mm thick is transmitting power at 1000 metres/min. The net driving tension is 1.8 times the tension on the slack side. If the safe permissible stress on the belt section in 1.6 MPa, calculate the maximum power, that can be transmitted at this speed. Assume density of the leather as 1000 kg/m3. Calculate the absolute maximum power that can be transmitted by this belt and the speed at which this can be transmitted.

Solution

b = 100 mm = 0.1 m ; t = 10 mm = 0.01 m ; v = 1000 m/min = 16.67 m/s ; $T_1 - T_2 = 1.8\, T_2$; σ = 1.6 MPa = 1.6 N/mm^2 ; ρ = 1000 kg/m^3

the maximum tension in the belt,

$$T = \sigma.b.t = 1.6 \times 100 \times 10 = 1600 \text{ N}$$

Mass of the belt per metre length,

$$m = \text{Area} \times \text{length} \times \text{density} = b \times t \times l \times \rho$$

$$= 0.1 \times 0.01 \times 1 \times 1000 = 1 \text{ kg/m}$$

∴ Centrifugal tension,

$$T_C = m.v^2 = 1\,(16.67)^2 = 278 \text{ N}$$

We know that

$$T_1 = T - T_C = 1600 - 278 = 1322 \text{ N}$$

$$T_1 - T_2 = 1.8\, T_2$$

$$T_2 = \frac{T_1}{2.8} = \frac{1322}{2.8} = 472 \text{ N}$$

The power transmitted.

$$P = (T_1 - T_2)\, v = (1322 - 472)\, 16.67 = 14\,170 \text{ W} = 14.17 \text{ kW}$$

the speed of the belt for maximum power

$$v = \sqrt{\frac{T}{3m}} = \sqrt{\frac{1600}{3 \times 1}} = 23.1 \text{ m/s}$$

Absolute maximum power, the centrifugal tension,

$T_C = T / 3 = 1600 / 3 = 533$ N

∴ Tension in the tight side,

$T_1 = T - T_C = 1600 - 533 = 1067$ N

and tension in the slack side,

$$T_2 = \frac{T_1}{2.8} = \frac{1067}{2.8} = 381 \text{ N}$$

Absolute maximum power transmitted,

$P = (T_1 - T_2)\, v = (1067 - 381)\, 23.1 = 15\,850$ W $= 15.85$ kW