Centrifugal Tension

m = Mass of belt per unit length in kg,

v = Linear velocity of belt in m/s,

r = Radius of pulley over which the belt runs in metres, and

T_C = Centrifugal tension acting tangentially at P and Q in newtons.

$$T_C = m.v^2$$

Notes : 1. When centrifugal tension is taken into account, then total tension in the tight side,

$$T_{t1} = T_1 + T_C$$

and total tension in the slack side,

$$T_{t2} = T_2 + T_C$$

2. Power transmitted,

$P = (T_{t1} - T_{t2})\ v$ ...(in watts)

$= [(T_1 + T_C) - (T_2 + T_C)]\ v = (T_1 - T_2)\ v$ ... (same as before)

Thus we see that the centrifugal tension has no effect on the power transmitted.

3. The ratio of driving tensions may also be written as

$$2.3 \log\left(\frac{T_{t1} - T_C}{T_{t2} - T_C}\right) = \mu.\theta$$

T_{t1} = Maximum or total tension in the belt.

Maximum Tension in the Belt

σ = Maximum safe stress,

b = Width of the belt, and

t = Thickness of the belt.

We know that the maximum tension in the belt,

T = Maximum stress × Cross-sectional area of belt = $\sigma.b.t$

When centrifugal tension is neglected, then

T (or T_{t1}) = T_1, *i.e.* Tension in the tight side of the belt.

When centrifugal tension is considered, then

T (or T_{t1}) = $T_1 + T_C$

Condition for the Transmission of Maximum Power

We know that the power transmitted by a belt,

$$P = (T_1 - T_2)\, v$$

Where T_1 = Tension in the tight side in newtons,

T_2 = Tension in the slack side in newtons, and

v = Velocity of the belt in m/s.

$$T = 3T_C \quad , \quad m.v^2 = T_C$$

find that the velocity of the belt for maximum power

$$v = \sqrt{\frac{T}{3m}}$$

Problem 3

A leather belt 9 mm × 250 mm is used to drive a cast iron pulley 900 mm in diameter at 336 r.p.m. If the active arc on the smaller pulley is 120° and the stress in tight side is 2 MPa, find the power capacity of the belt. The density of leather may be taken as 980 kg/m3, and the coefficient of friction of leather on cast iron is 0.35

Solution

Given: t = 9 mm = 0.009 m ; b = 250 mm = 0.25 m; d = 900 mm = 0.9 m ; N = 336 r.p.m ; θ = 120° = 120 π /180 = 2.1 rad ; σ = 2 MPa = 2 N/mm^2 ; ρ = 980 kg/m^3 ; μ = 0.35

The velocity of the belt

$$v = \frac{\pi\, d.N}{60} = \frac{\pi \times 0.9 \times 336}{60} = 15.8 \text{ m/s}$$

Cross-sectional area of the belt,

$$a = b.t = 9 \times 250 = 2250 \text{ mm}^2$$

∴ Maximum or total tension in the tight side of the belt,

$$T = T_{t1} = \sigma.a = 2 \times 2250 = 4500 \text{ N}$$

We know that mass of the belt per metre length,

$$m = \text{Area} \times \text{length} \times \text{density} = b.t.l.\rho = 0.25 \times 0.009 \times 1 \times 980 \text{ kg/m}$$

$$= 2.2 \text{ kg/m}$$

∴ Centrifugal tension,

$$*T_C = m.v^2 = 2.2\,(15.8)^2 = 550 \text{ N}$$

and tension in the tight side of the belt,

$$T_1 = T - T_C = 4500 - 550 = 3950 \text{ N}$$

Let T_2 = Tension in the slack side of the belt

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.35 \times 2.1 = 0.735$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{0.735}{2.3} = 0.3196 \quad \text{or} \quad \frac{T_1}{T_2} = 2.085 \qquad \text{... (Taking antilog of 0.3196)}$$

$$T_2 = \frac{T_1}{2.085} = \frac{3950}{2.085} = 1895 \text{ N}$$

The power capacity of the belt,

$$P = (T_1 - T_2)\, v = (3950 - 1895)\, 15.8 = 32\,470 \text{ W} = 32.47 \text{ kW}$$

Notes : The power capacity of the belt, when centrifugal tension is taken into account, may also be obtained as discussed below :

1. We know that the maximum tension in the tight side of the belt,

$$T_{t1} = T = 4500 \text{ N}$$

Centrifugal tension, $T_C = 550$ N

and tension in the slack side of the belt,

$$T_2 = 1895 \text{ N}$$

∴ Total tension in the slack side of the belt,

$$T_{t2} = T_2 + T_C = 1895 + 550 = 2445 \text{ N}$$

We know that the power capacity of the belt,

$$P = (T_{t1} - T_{t2})\, v = (4500 - 2445)\, 15.8 = 32\,470 \text{ W} = 32.47 \text{ kW}$$

2. The value of total tension in the slack side of the belt ($Tt2$)

$$2.3 \log\left(\frac{T_{t1} - T_C}{T_{t2} - T_C}\right) = \mu.\theta$$

Problem 4

A flat belt is required to transmit 30 kW from a pulley of 1.5 m effective diameter running at 300 r.p.m. The angle of contact is spread over 11/24 of the circumference. The coefficient of friction between the belt and pulley surface is 0.3. Determine, taking centrifugal tension into account, width of the belt required. It is given that the belt thickness is 9.5 mm, density of its material is 1100 kg / m3 and the related permissible working stress is 2.5 MPa.

Solution

P = 30 kW = 30 × 103 W ; d = 1.5 m ; N = 300 r.p.m. ; θ =11/24 × 360 = 165° = 165 × π / 180 = 2.88 rad ; μ = 0.3 ; t = 9.5 mm = 0.0095 m ; ρ = 1100 kg/m^3 ; σ = 2.5 MPa = 2.5 × 10^6 N/m^2

T_1 = Tension in the tight side of the belt in newtons, and

T_2 = Tension in the slack side of the belt in newtons.

The velocity of the belt

$$v = \frac{\pi \, d \, N}{60} = \frac{\pi \times 1.5 \times 300}{60} = 23.57 \text{ m/s}$$

power transmitted (P),

$$30 \times 10^3 = (T_1 - T_2)\, v = (T_1 - T_2)\, 23.57$$

$$\therefore \quad T_1 - T_2 = 30 \times 10^3 / 23.57 = 1273 \text{ N}$$

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.3 \times 2.88 = 0.864$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{0.864}{2.3} = 0.3756 \quad \text{or} \quad \frac{T_1}{T_2} = 2.375$$

... (Taking antilog of 0.3756)

T_1 = 2199 N ; and T_2 = 926 N

Let b = Width of the belt required in metres.

We know that mass of the belt per metre length,

$$m = \text{Area} \times \text{length} \times \text{density} = b \times t \times l \times \rho$$

$$= b \times 0.0095 \times 1 \times 1100 = 10.45\, b \text{ kg/m}$$

and centrifugal tension,

$$T_C = m.v^2 = 10.45\, b\, (23.57)^2 = 5805\, b \text{ N}$$

We know that maximum tension in the belt,

$$T = T_1 + T_C = \text{Stress} \times \text{Area} = \sigma.b.t$$

or $$2199 + 5805\, b = 2.5 \times 106 \times b \times 0.0095 = 23\,750\, b$$

∴ $$23\,750\, b - 5805\, b = 2199 \text{ or } b = 0.122 \text{ m or } 122 \text{ mm}$$

The standard width of the belt is 125 mm.

Problem 5

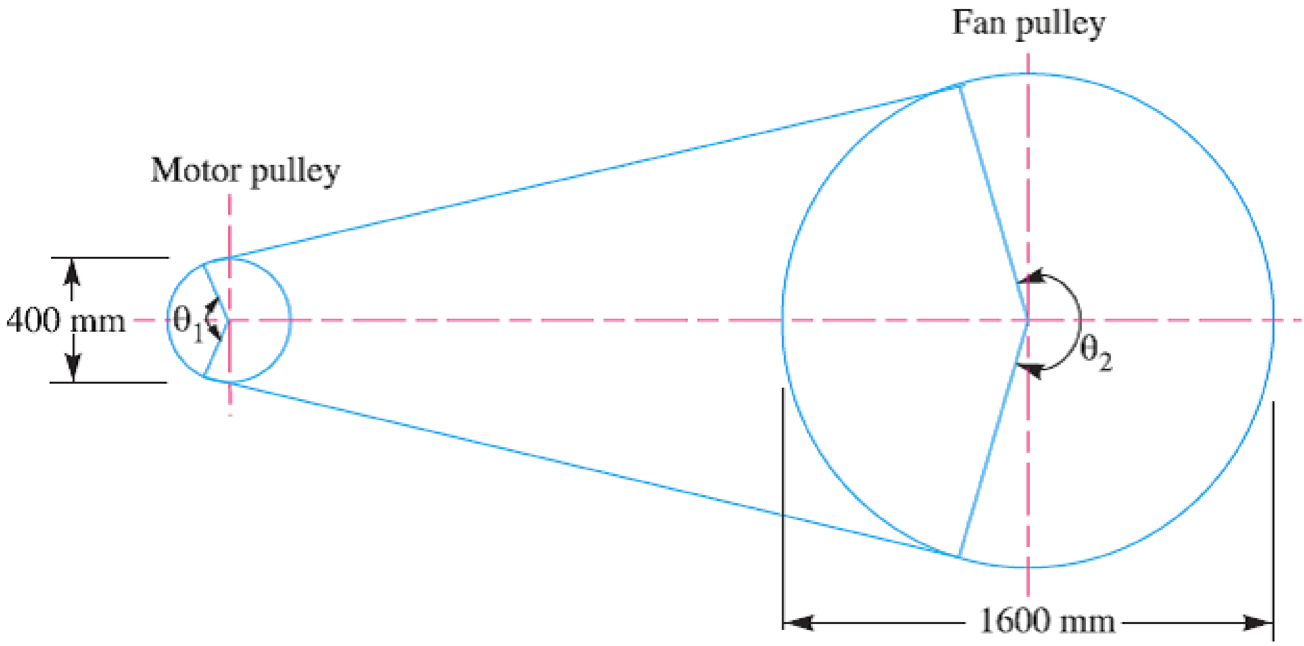
An electric motor drives an exhaust fan. Following data are provided :

	Motor pulley	*Fan pulley*
Diameter	*400 mm*	*1600 mm*
Angle of warp	*2.5 radians*	*3.78 radians*
Coefficient of friction	*0.3*	*0.25*
Speed	*700 r.p.m.*	—
Power transmitted	*22.5 kW*	—

Calculate the width of 5 mm thick flat belt. Take permissible stress for the belt material as 2.3 MPa.

Solution

d_1 = 400 mm or r_1 = 200 mm ; d_2 = 1600 mm or r_2 = 800 mm ; θ_1 = 2.5 rad ; θ_2 = 3.78 rad ; μ_1 = 0.3; μ_2 = 0.25 ; N_1 = 700 r.p.m. ; P = 22.5 kW = 22.5 × 103 W ; t = 5 mm = 0.005 m ; σ = 2.3 MPa = 2.3×10^6 N/m^2



For motor pulley, $\mu_1.\theta_1 = 0.3 \times 2.5 = 0.75$

and for fan pulley, $\mu_2.\theta_2 = 0.25 \times 3.78 = 0.945$

the velocity of the belt

$$v = \frac{\pi\, d_1.N_1}{60} = \frac{\pi \times 0.4 \times 700}{60} = 14.7 \text{ m/s}$$

the power transmitted (P),

$$22.5 \times 10^3 = (T_1 - T_2)\, v = (T_1 - T_2)\, 14.7$$

$\therefore \quad T_1 - T_2 = 22.5 \times 10^3 / 14.7 = 1530$ N

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu_1.\theta_1 = 0.3 \times 2.5 = 0.75$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{0.75}{2.3} = 0.3261 \quad \text{or} \quad \frac{T_1}{T_2} = 2.12$$

... (Taking antilog of 0.3261)

$T_1 = 2896$ N ; and $T_2 = 1366$ N

Let b = Width of the belt in metres.

Since the velocity of the belt is more than 10 m/s, therefore centrifugal tension must be taken into consideration. Assuming a leather belt for which the density may be taken as 1000 kg / m3.

∴ Mass of the belt per metre length,

$$m = \text{Area} \times \text{length} \times \text{density} = b \times t \times l \times \rho$$

$$= b \times 0.005 \times 1 \times 1000 = 5\, b \text{ kg/m}$$

and centrifugal tension,

$$T_C = m.v^2 = 5\, b\, (14.7)^2 = 1080\, b \text{ N}$$

We know that the maximum (or total) tension in the belt,

$$T = T_1 + T_C = \text{Stress} \times \text{Area} = \sigma.b.t$$

Or $\quad 2896 + 1080\, b = 2.3 \times 106\, b \times 0.005 = 11\,500\, b$

$\therefore$ 11 500 b – 1080 b = 2896 or $\quad b = 0.278$ say 0.28 m or 280 mm