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Flat Belt Drives

Coefficient of Friction Between Belt and Pulley

The coefficient of friction (μ)

$$\mu = 0.54 - \frac{42.6}{152.6 + v}$$

where v = Speed of the belt in metres per minute.The following table shows the values of coefficient of friction for various materials of belt and pulley.

Velocity Ratio of a Belt Drive

It is the ratio between the velocities of the driver and the follower or driven. It may be

expressed, mathematically, as discussed below:

Let d_1 = Diameter of the driver,

d_2 = Diameter of the follower,

N_1 = Speed of the driver in r.p.m.,

N_2 = Speed of the follower in r.p.m.,

∴ Length of the belt that passes over the driver, in one minute

$= \pi d_1 N_1$

Similarly, length of the belt that passes over the follower, in one minute

$= \pi d_2 N_2$

Since the length of belt that passes over the driver in one minute is equal to the length of belt that

passes over the follower in one minute, therefore

$\pi d_1 N_1 = \pi d_2 N_2$

velocity ratio, $$\frac{N_2}{N_1} = \frac{d_1}{d_2}$$

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When thickness of the belt (*t*) is considered, then velocity ratio

$$\frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}$$

Notes :

1. The velocity ratio of a belt drive may also be obtained as discussed below:

We know that the peripheral velocity of the belt on the driving pulley

$$v_1 = \frac{\pi d_1 N_1}{60} \text{ m/s}$$

and peripheral velocity of the belt on the driven pulley,

$$v_2 = \frac{\pi d_2 N_2}{60} \text{ m/s}$$

When there is no slip, then $v_1 = v_2$.

$$\frac{\pi d_1 N_1}{60} = \frac{\pi d_2 N_2}{60} \text{ or } \frac{N_2}{N_1} = \frac{d_1}{d_2}$$

2. In case of a compound belt drive the velocity ratio is given by

$$\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4}$$

Slip of the Belt

s_1 % = Slip between the driver and the belt, and

s_2 % = Slip between the belt and follower,

Velocity of the belt passing over the driver per second,

$$v = \frac{\pi\, d_1\, N_1}{60} - \frac{\pi\, d_1\, N_1}{60} \times \frac{s_1}{100} = \frac{\pi\, d_1\, N_1}{60}\left(1 - \frac{s_1}{100}\right)$$

and velocity of the belt passing over the follower per second

$$\frac{\pi\, d_2\, N_2}{60} = v - v\left(\frac{s_2}{100}\right) = v\left(1 - \frac{s_2}{100}\right)$$

Substituting the value of v from equation

$$\frac{\pi\, d_2\, N_2}{60} = \frac{\pi\, d_1\, N_1}{60}\left(1 - \frac{s_1}{100}\right)\left(1 - \frac{s_2}{100}\right)$$

$$\frac{N_2}{N_1} = \frac{d_1}{d_2}\left(1 - \frac{s_1}{100} - \frac{s_2}{100}\right)$$

$$= \frac{d_1}{d_2}\left[1 - \left(\frac{s_1 + s_2}{100}\right)\right] = \frac{d_1}{d_2}\left(1 - \frac{s}{100}\right)$$

...(where $s = s_1 + s_2$ *i.e.* total percentage of slip)

If thickness of the belt (t) is considered, then

$$\frac{N_2}{N_1} = \frac{d_1 + t}{d_2 + t}\left(1 - \frac{s}{100}\right)$$

Creep of Belt

$$\frac{N_2}{N_1} = \frac{d_1}{d_2} \times \frac{E + \sqrt{\sigma_2}}{E + \sqrt{\sigma_1}}$$

σ_1 and σ_2 = Stress in the belt on the tight and slack side respectively, and

E = Young's modulus for the material of the belt.

Problem 1

An engine running at 150 r.p.m. drives a line shaft by means of a belt. The engine pulley is 750 mm diameter and the pulley on the line shaft is 450 mm. A 900 mm diameter pulley on the line shaft drives a 150 mm diameter pulley keyed to a dynamo shaft. Fine the speed of dynamo shaft, when 1. there is no slip, and 2. there is a slip of 2% at each drive.

Solution

Given : N_1 = 150 r.p.m. ; d_1 = 750 mm ; d_2 = 450 mm ; d_3 = 900 mm ; d_4 = 150 mm ; $s_1 = s_2$ = 2%

N_4 = Speed of the dynamo shaft.

1. *When there is no slip*

$$\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4} \text{ or } \frac{N_4}{150} = \frac{750 \times 900}{450 \times 150} = 10$$

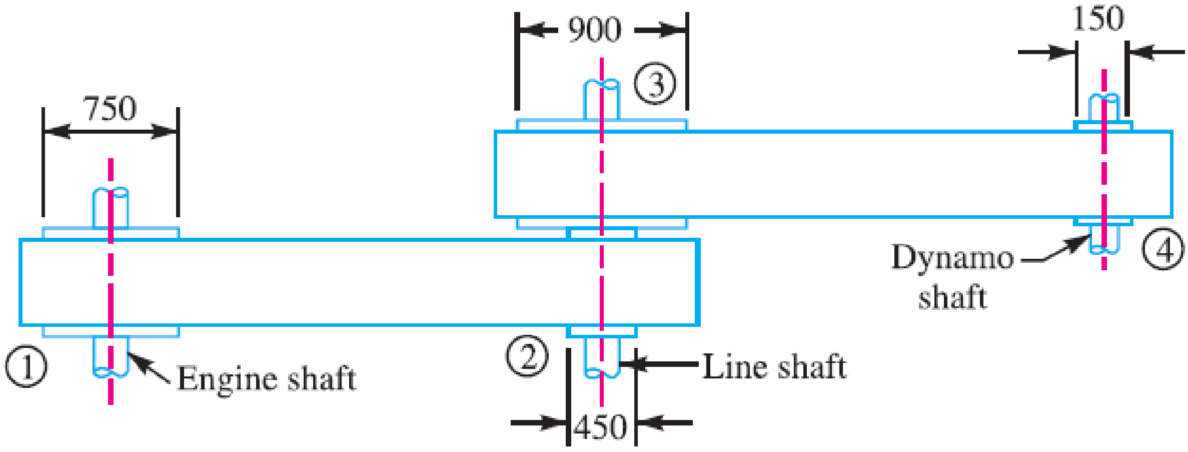
$$N_4 = 150 \times 10 = 1500 \text{ r.p.m. } \textbf{Ans.}$$

2. *When there is a slip of 2% at each drive*

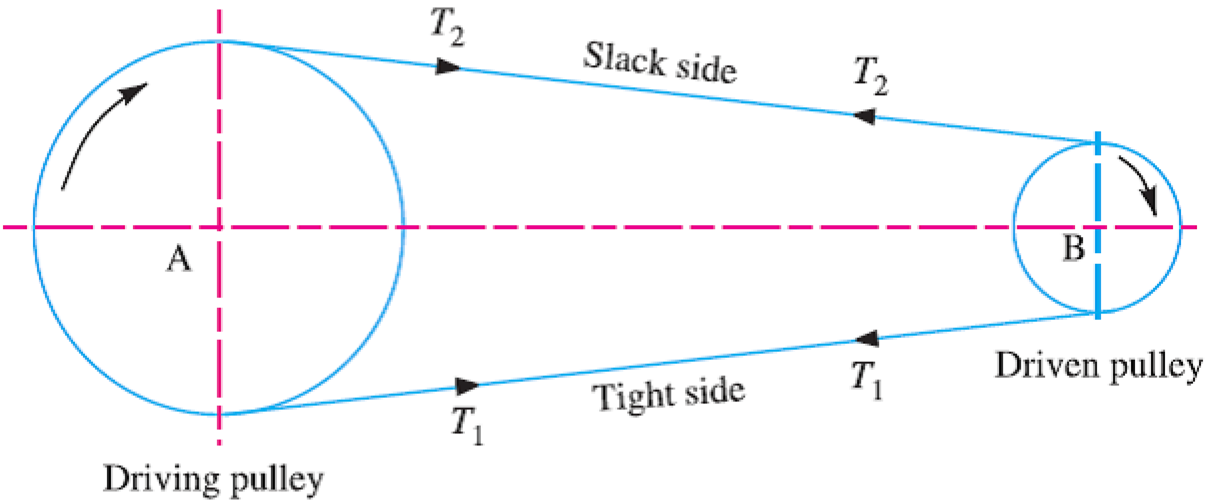
$$\frac{N_4}{N_1} = \frac{d_1 \times d_3}{d_2 \times d_4}\left(1 - \frac{s_1}{100}\right)\left(1 - \frac{s_2}{100}\right)$$

$$\frac{N_4}{150} = \frac{750 \times 900}{450 \times 150}\left(1 - \frac{2}{100}\right)\left(1 - \frac{2}{100}\right) = 9.6$$

$$N_4 = 150 \times 9.6 = 1440 \text{ r.p.m. } \textbf{Ans.}$$



Power Transmitted by a Belt



Let T_1 and T_2 = Tensions in the tight side and slack side of the belt respectively in newtons,

r_1 and r_2 = Radii of the driving and driven pulleys respectively in metres,

and v = Velocity of the belt in m/s.

The effective turning (driving) force at the circumference of the driven pulley or follower is the difference between the two tensions (*i.e.* $T_1 - T_2$).

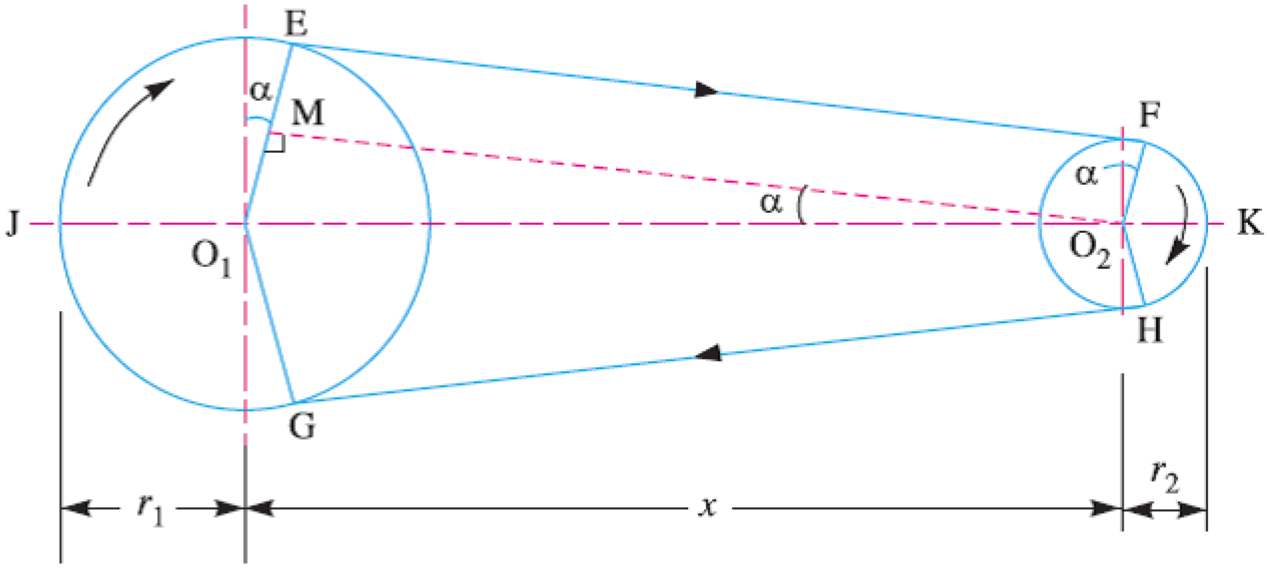
∴ Work done per second = $(T_1 - T_2)\, v$ N-m/s

and power transmitted = $(T_1 - T_2)\, v$ W ... (1 N-m/s = 1W)

A little consideration will show that torque exerted on the driving pulley is $(T_1 - T_2)\, r_1$.

Similarly, the torque exerted on the driven pulley is $(T_1 - T_2)\, r_2$.

Length of an Open Belt Drive



r_1 and r_2 = Radii of the larger and smaller pulleys,

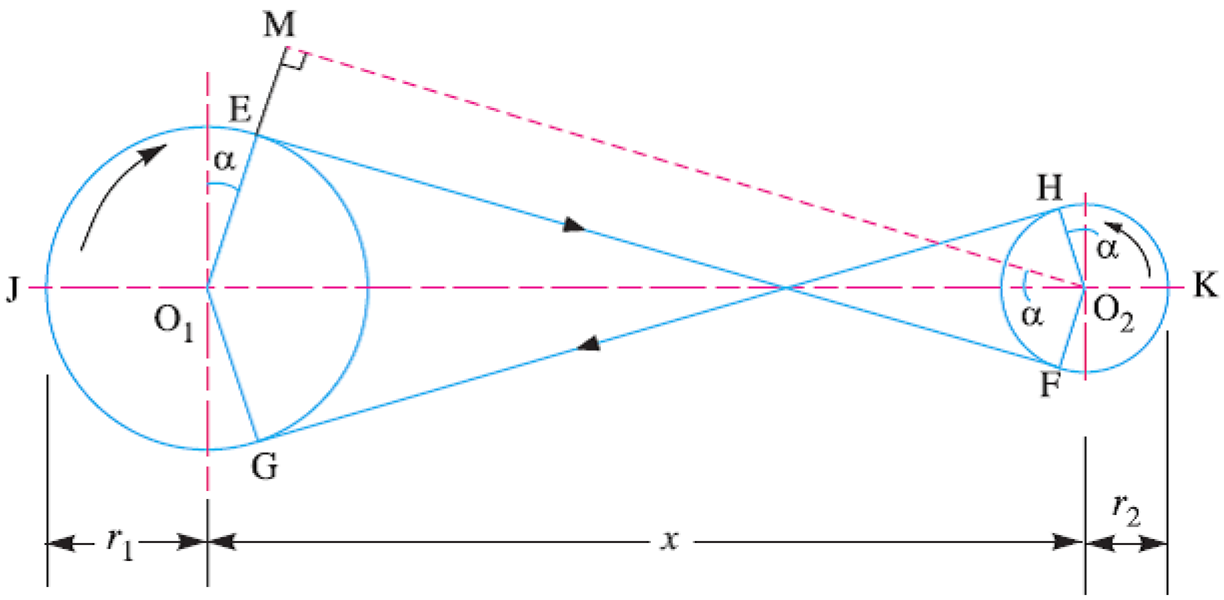
x = Distance between the centres of two pulleys (*i.e.* $O1O2$), and

L = Total length of the belt.

$$= \pi (r_1 + r_2) + 2x + \frac{(r_1 - r_2)^2}{x} \quad \text{... (in terms of pulley radii)}$$

$$= \frac{\pi}{2} (d_1 + d_2) + 2x + \frac{(d_1 - d_2)^2}{4x} \quad \text{... (in terms of pulley diameters)}$$

Length of a Cross Belt Drive



L = Total length of the belt.

$$= \pi (r_1 + r_2) + 2x + \frac{(r_1 + r_2)^2}{x}$$ ... (in terms of pulley radii)

$$= \frac{\pi}{2} (d_1 + d_2) + 2x + \frac{(d_1 + d_2)^2}{4x}$$ ... (in terms of pulley diameters)

Ratio of Driving Tensions for Flat Belt Drive

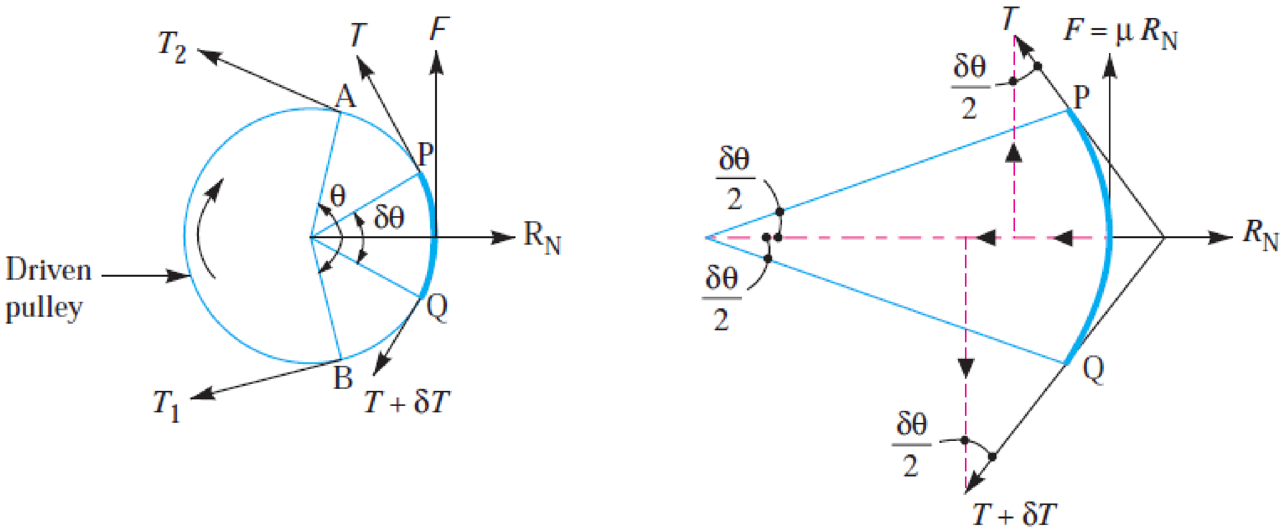
T_1 = Tension in the belt on the tight side,

T_2 = Tension in the belt on the slack side, and

θ = Angle of contact in radians (*i.e.* angle subtended by the arc *AB*, along which the belt touches the pulley, at the centre).

Normal reaction R_N, and

Frictional force $F = \mu \times R_N$, where μ is the coefficient of friction between the belt and pulley



$$2.3 \log \left(\frac{T_1}{T_2} \right) = \mu.\theta$$

The above expression gives the relation between the tight side and slack side tensions, in terms of coefficient of friction and the angle of contact

$$\sin \alpha = \frac{r_1 - r_2}{x} \quad \text{... (for open belt drive)}$$

$$= \frac{r_1 + r_2}{x} \quad \text{... (for cross-belt drive)}$$

Angle of contact or lap,

$$\theta = (180^\circ - 2\alpha) \frac{\pi}{180} \text{ rad} \quad \text{...(for open belt drive)}$$

$$= (180^\circ + 2\alpha) \frac{\pi}{180} \text{ rad} \quad \text{... (for cross-belt drive)}$$

Problem 2

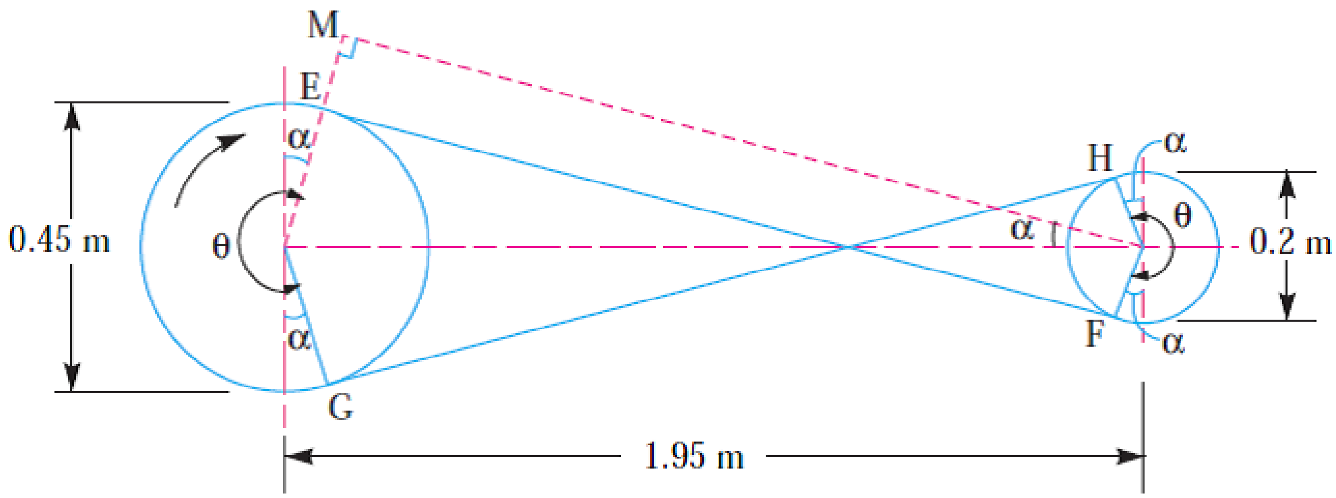
Two pulleys, one 450 mm diameter and the other 200 mm diameter, on parallel shafts 1.95 m apart are connected by a crossed belt. Find the length of the belt required and the angle of contact between the belt and each pulley.

What power can be transmitted by the belt when the larger pulley rotates at 200 rev/min, if the maximum permissible tension in the belt is 1 kN, and the coefficient of friction between the belt and pulley is 0.25?

Solution

Given : d_1 = 450 mm = 0.45 m or r_1 = 0.225 m ; d_2 = 200 mm = 0.2 m or r_2 = 0.1 m ; x = 1.95 m ; N_1 = 200 r.p.m. ; T_1 = 1 kN = 1000 N ; μ = 0.25

The arrangement of crossed belt drive



Length of the belt

We know that length of the belt,

$$L = \pi(r_1 + r_2) + 2x + \frac{(r_1 + r_2)^2}{x}$$

$$= \pi(0.225 + 0.1) + 2 \times 1.95 + \frac{(0.225 + 0.1)^2}{1.95}$$

$$= 1.02 + 3.9 + 0.054 = 4.974 \text{ m}$$

Angle of contact between the belt and each pulley

Let θ = Angle of contact between the belt and each pulley.

We know that for a crossed belt drive,

$$\sin \alpha = \frac{r_1 + r_2}{x} = \frac{0.225 + 0.1}{1.95} = 0.1667$$

$$\alpha = 9.6°$$

$$\theta = 180° + 2\alpha = 180 + 2 \times 9.6 = 199.2°$$

$$= 199.2 \times \frac{\pi}{180} = 3.477 \text{ rad}$$

Power transmitted

Let T_1 = Tension in the tight side of the belt, and

T_2 = Tension in the slack side of the belt.

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.25 \times 3.477 = 0.8693$$

$$\log\left(\frac{T_1}{T_2}\right) = \frac{0.8693}{2.3} = 0.378 \quad \text{or} \quad \frac{T_1}{T_2} = 2.387 \qquad \text{... (Taking antilog of 0.378)}$$

$$T_2 = \frac{T_1}{2.387} = \frac{1000}{2.387} = 419 \text{ N}$$

We know that the velocity of belt,

$$v = \frac{\pi\, d_1 N_1}{60} = \frac{\pi \times 0.45 \times 200}{60} = 4.713 \text{ m/s}$$

Power transmitted,

$$P = (T_1 - T_2)\, v = (1000 - 419)\, 4.713 = 2738 \text{ W} = 2.738 \text{ kW}$$