

Error Function.



$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} du$$

دالة error

$$\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-u^2} du$$

Complementary
مكملة - لـ error

* دالة error تحتوي على الكثير من الخصائص وكل خاصية لديها برهان اليكيم بعض الخصائص اثباتها.

ex prove $\text{erf}(x) = \frac{1}{\sqrt{\pi}} \int_0^{x^2} e^{-t} t^{-\frac{1}{2}} dt$

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} du$$

* ازالة الجذر من error

نقطة

let $u^2 = t$
الحودود

$$u = \sqrt{t}$$

نقطة

$$\frac{dt}{du} = 2u \rightarrow 2du = dt$$

$$dt = 2u du$$

$$du = \frac{dt}{2u}$$

$$du = \frac{dt}{2\sqrt{t}}$$

$t: 0 \rightarrow x^2$
 $t: 0 \rightarrow x^2$

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^{x^2} e^{-t} \cdot \frac{dt}{2\sqrt{t}}$$

$$\sqrt{t} = t^{\frac{1}{2}}$$

$$= \frac{1}{\sqrt{\pi}} \int_0^{x^2} e^{-t} \cdot t^{-\frac{1}{2}} dt$$

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② Prove : $\text{erf}(\infty) = 1$

Solution

$$\text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} du = \frac{1}{\sqrt{\pi}} \int_0^x e^{-t} \cdot t^{-\frac{1}{2}} dt$$

أثبتنا في السؤال السابق

$$\text{erfc}(\infty) = \frac{1}{\sqrt{\pi}} \int_0^{\infty} e^{-t} \cdot t^{-\frac{1}{2}} dt$$

$$= \frac{1}{\sqrt{\pi}} * \sqrt{\pi} \Gamma(n)$$

* نقل دالة غاما

$$n-1 = -\frac{1}{2}$$

$$n = \frac{1}{2}$$

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$$

$\text{erfc}(\infty) = 1$

 #

③ $\text{erf}(0) = 0$

$$\text{erf}(0) = \frac{2}{\sqrt{\pi}} \int_0^0 e^{-u^2} du = 0$$

④ $\text{erf}(x) + \text{erfc}(x) = 1$

نصف الكون الاخرى

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} du \quad \text{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^{\infty} e^{-u^2} du$$

$$\text{erf}(x) + \text{erfc}(x) = \frac{2}{\sqrt{\pi}} \left[\int_0^x e^{-u^2} du + \int_x^{\infty} e^{-u^2} du \right]$$

$$\frac{2}{\sqrt{\pi}} \int_0^{\infty} e^{-u^2} du = \text{erf}(\infty) = 1$$

... = 1 #

Prove the error function is an odd function
 * Function $f(x)$ is said to be odd if $f(-x) = -f(x)$

$$\text{erf}(-x) = -\text{erf}(x)$$

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-u^2} du$$

$$\text{erf}(-x) = \frac{2}{\sqrt{\pi}} \int_0^{-x} e^{-u^2} du$$

نعرّف

Put

$$u = -t$$

نشتق

$$du = -dt$$

الحدود

ed

$$\rightarrow \int_{-\infty}^x \frac{1}{\sqrt{\pi}} e^{-t^2} dt$$

-t

$$f(-x) = -\operatorname{erf}(x) \neq$$

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