Trusses

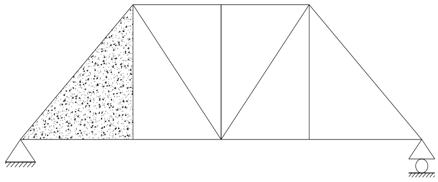
Trusses are used in bridges and ceilings with large spaces such as (factory ceilings, train stations, cinemas and Sports halls). Where they are in such cases are more economical than other construction systems.

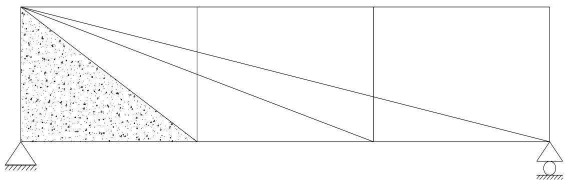
Types of statically determinate trusses

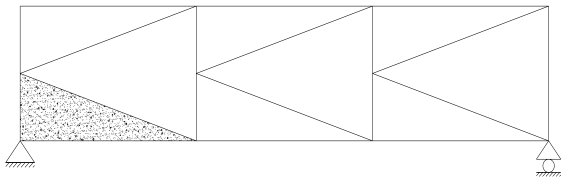
Where $(b+r = 2j)$

1- Simple trusses

Consists a base triangle (3 members meeting at 3 joints) and any additional 2 members meet at an additional joint.



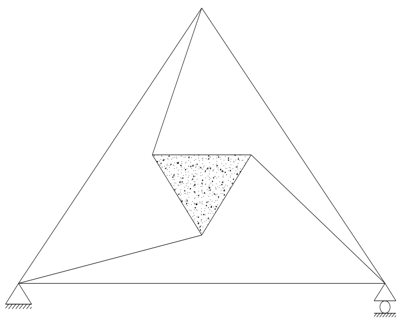


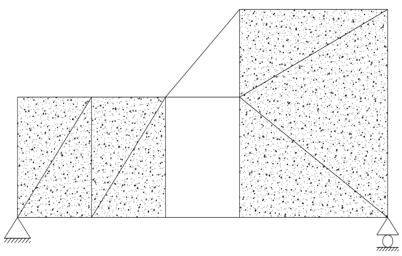


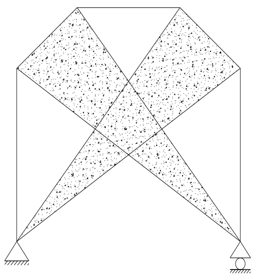
2- Compound trusses

Two or more simple trusses connected together by different means; such as

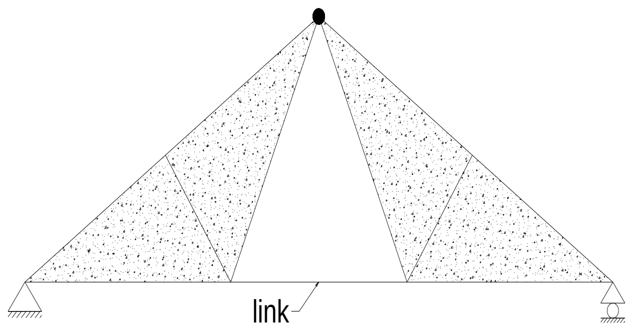
i- Three links: that is neither parallel nor concurrent.

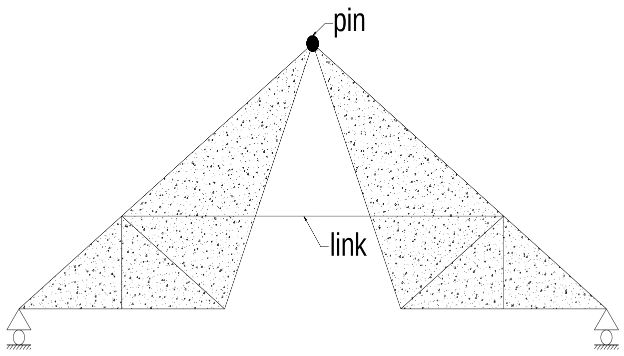




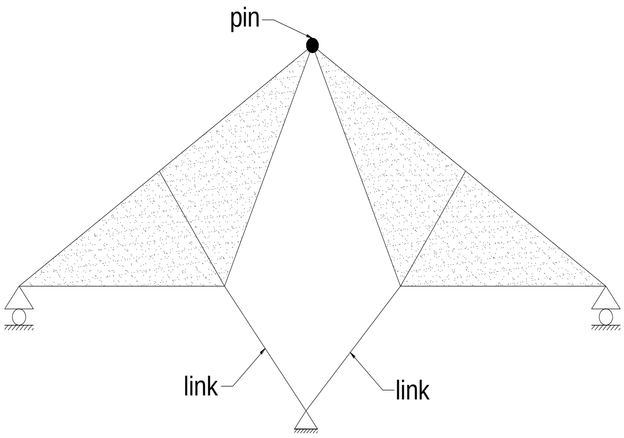


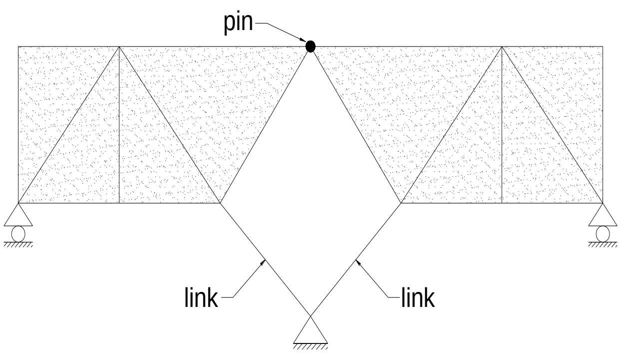
ii- A pin and link



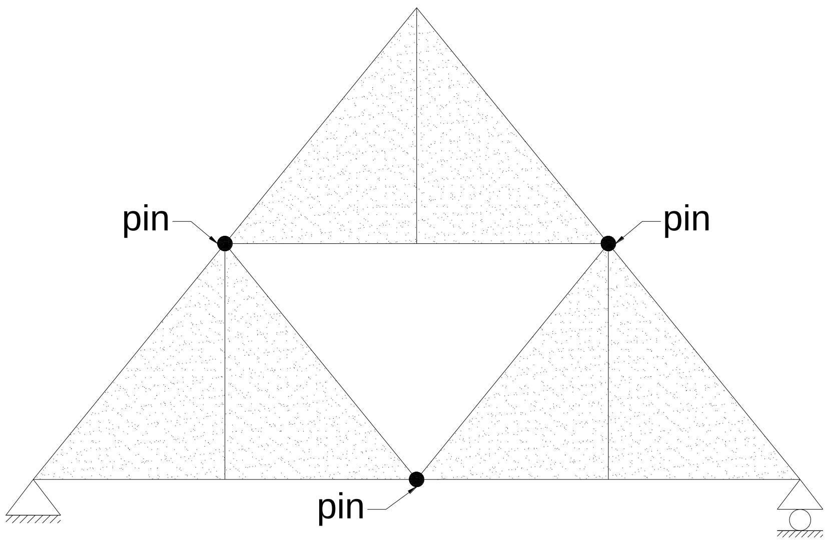


iii- A pin and two links intersecting at support



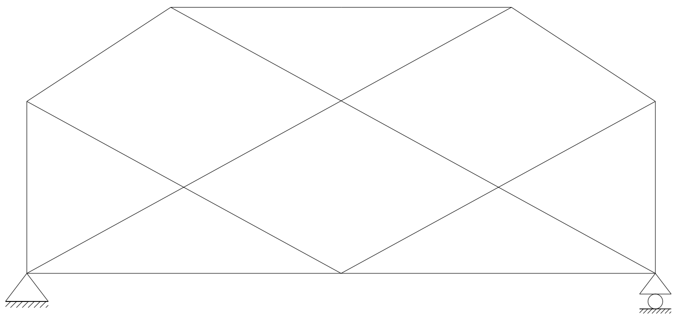


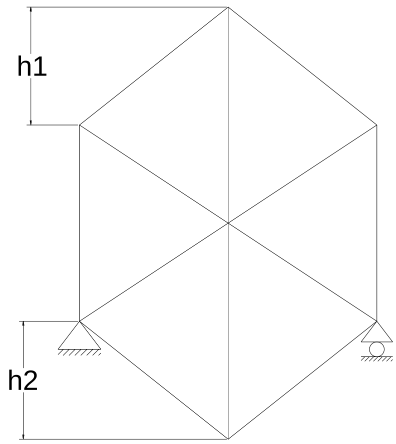
iv- Three pins



3-complex trusses

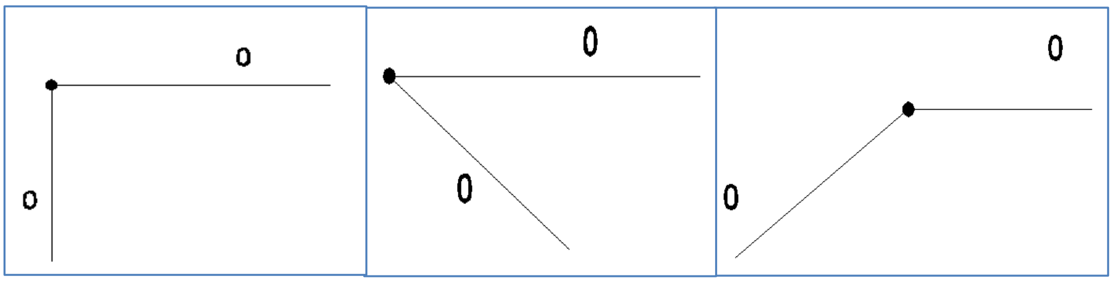
Each joint has at least 3 members. If a section is taken it will intersect so many members there force the truss cannot be easily (complex truss).



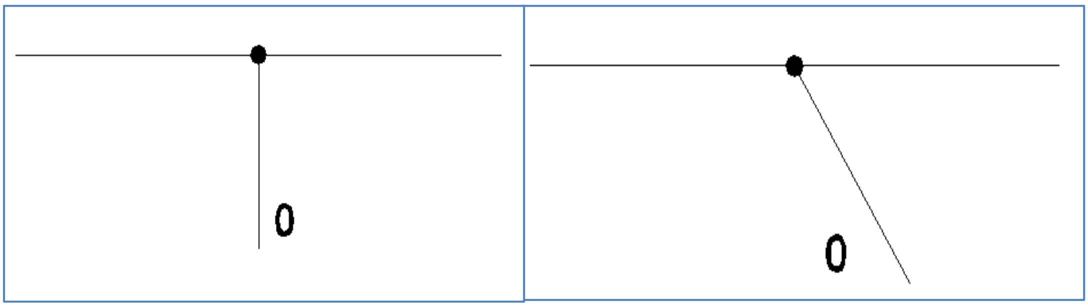


Important Notes:

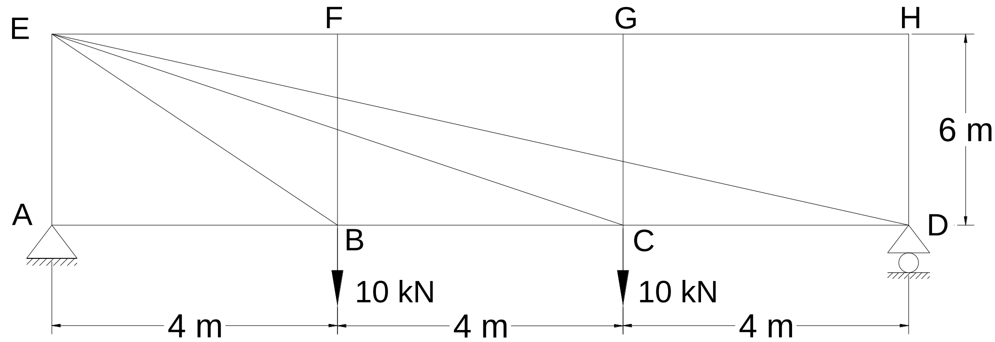
1- When met two bars in joint not on one straightness and there is no external force on this joint, we conclude that the force in both bars equal to zero.



2- When met three bars in joint two of them on one straightness and there is no external force on this joint, we conclude that the force in third bar equal to zero.



Ex1: for the simple truss shown in fig. find the axial force in bars CD, DE, CE & BE.



Solution:

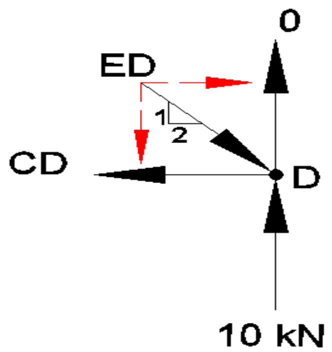
Due to symmetry of loading and distances

Dy= Ay = 10 kN ↑

Ax = 0

FB = GC = HD = HG = GF = FE = AB = 0

For CD & DE use joint D



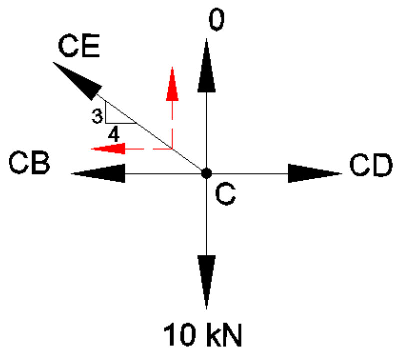
$$\uparrow \sum f_y = 0$$

$$10 - ED * \frac{1}{\sqrt{5}} = 0 => ED = 10\sqrt{5}\ \text{kN (Comp.)}$$

$$\rightarrow \sum f_x = 0$$

$$10\sqrt{5} * \frac{2}{\sqrt{5}} - CD = 0 => CD = 20\ \text{kN (Ten.)}$$

For CE use joint C



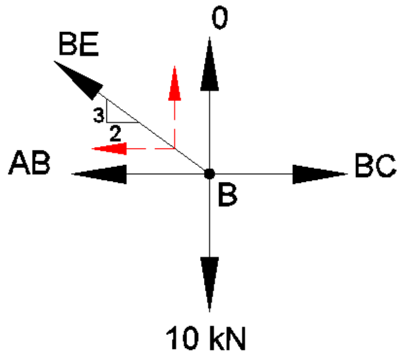
$$\uparrow \sum f_y = 0$$

$$CE * \frac{3}{5} - 10 = 0 => CE = 16.67\ \text{kN (Ten.)}$$

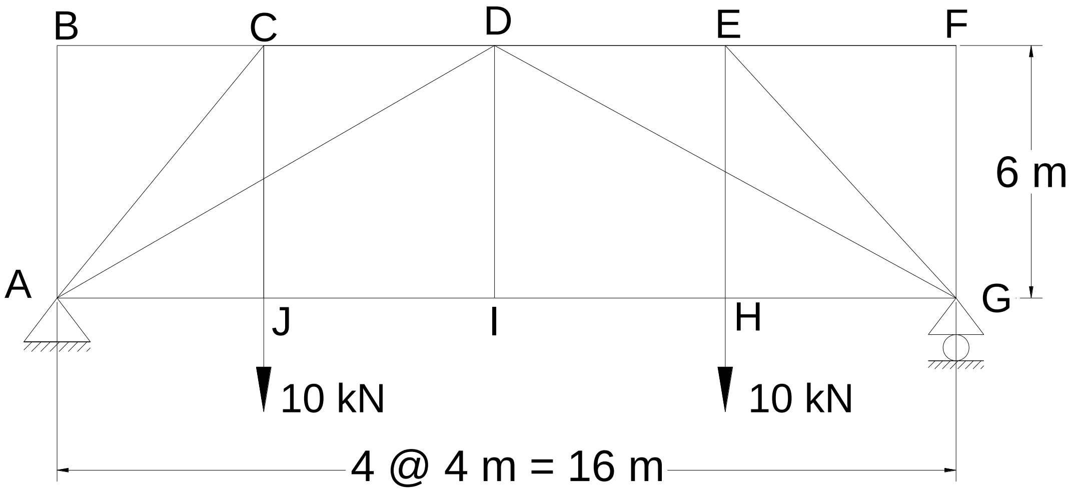
For BE use joint B

$\uparrow \sum f_y = 0$

$$BE * \frac{3}{\sqrt{13}} - 10 = 0 \Rightarrow BE = 12 \text{ kN (Ten.)}$$



Ex2: For the simple truss shown in fig. find the axial force in bars AC, AD & AJ.



Solution:

Due to symmetry of loading and distances

Ay= Gy = 10 kN ↑

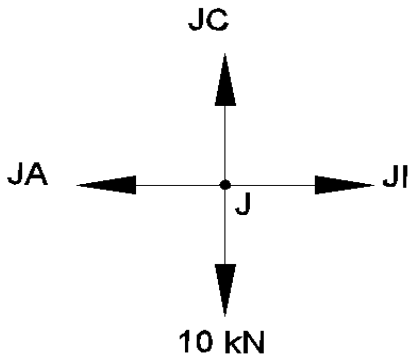
Ax = 0

BC, BA, FE, FG & DI = 0

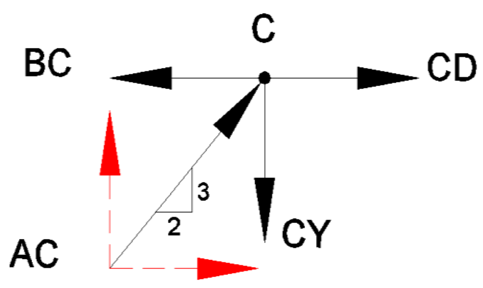
Use joint J

$\uparrow \sum f_y = 0$

JC – 10 = 0 => JC = 10 kN (ten.)



Use joint C



$\uparrow \sum f_y = 0$

$AC * \frac{3}{\sqrt{13}} - 10 = 0 \Rightarrow AC = 12.018 \text{ kN (Comp.)}$

Use joint A

$\uparrow \sum f_y = 0$

$$10 - AC * \frac{3}{\sqrt{13}} + AD * \frac{3}{5} = 0$$

$$10 - 12.018 * \frac{3}{\sqrt{13}} + AD * \frac{3}{5} = 0$$

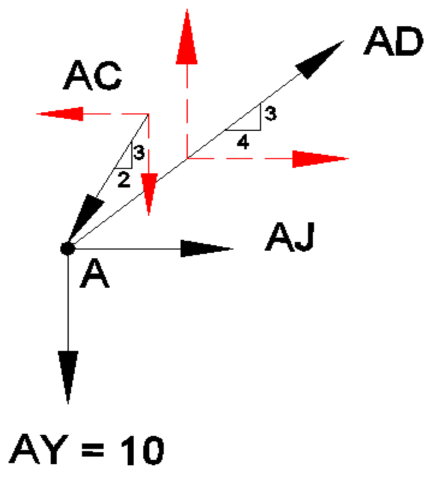
$AD = 0$

$\rightarrow \sum f_x = 0$

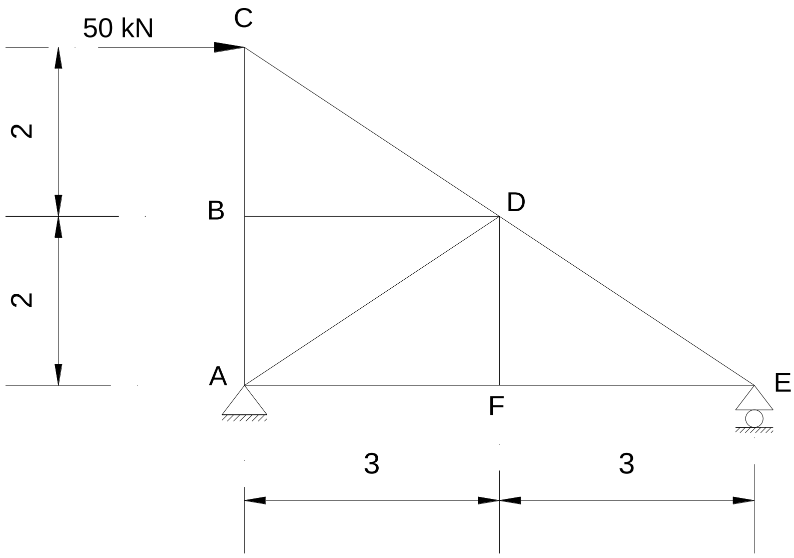
$$AJ - AC * \frac{2}{\sqrt{13}} = 0$$

$$AJ - 12.018 * \frac{2}{\sqrt{13}} = 0$$

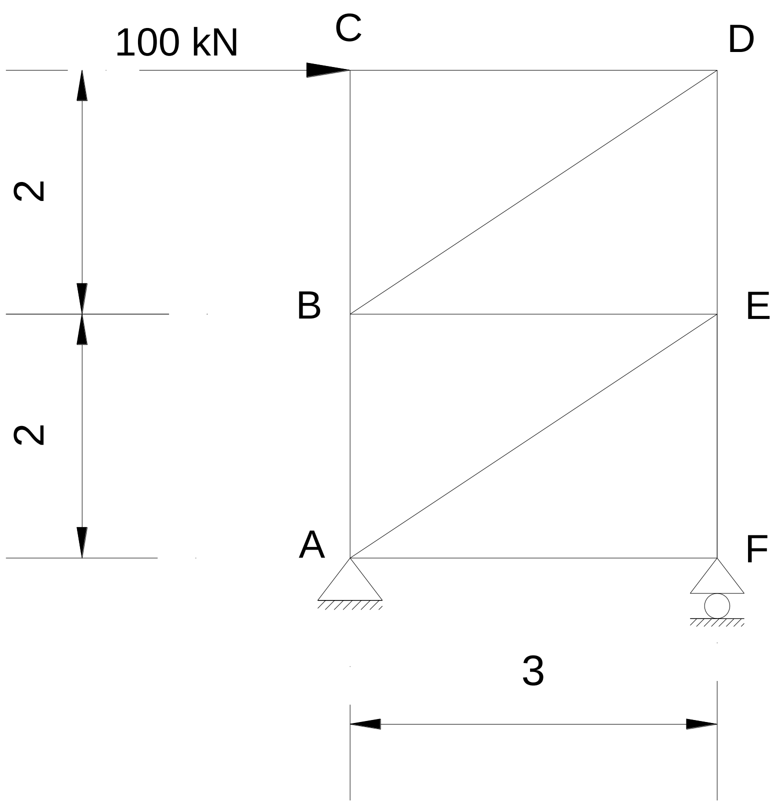
$AJ = 6.67 \text{ kN (Ten.)}$



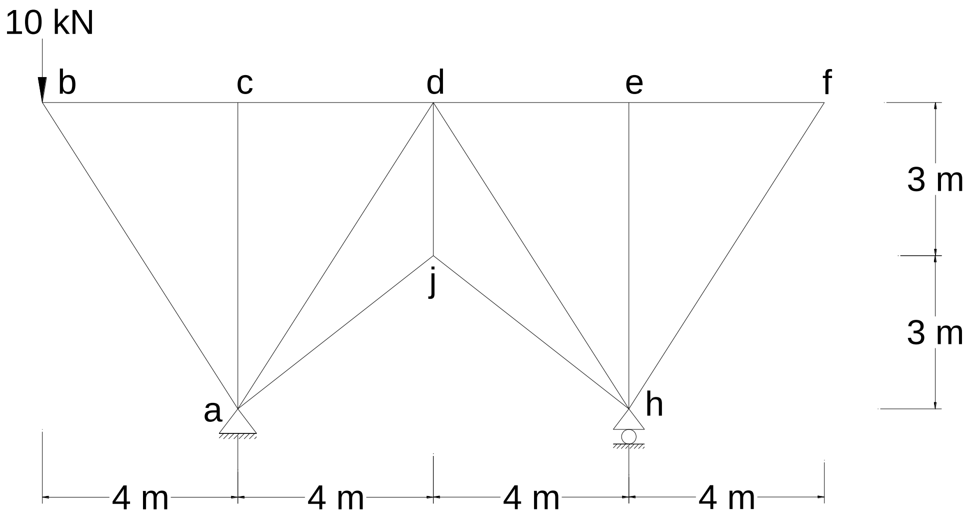
H.W1 :For the simple truss shown in fig. find the axial force in bars AB, AF,ED & CD.



H.W2 : For the simple truss shown in fig. find the axial force in bars AB, AF,AE, BC & CD.



Ex3: For the simple truss shown in fig. find the axial force in bars HD & HJ.



Solution

$\sum M_a = 0$

(hy *8) – (10*4) = 0 => hy = 5 kN

$\uparrow \sum f_y = 0$

ay – 10 – 5 = 0 => ay= 15 kN

ca, eh, fh, fe & ed = 0

For hd & hj use joint h

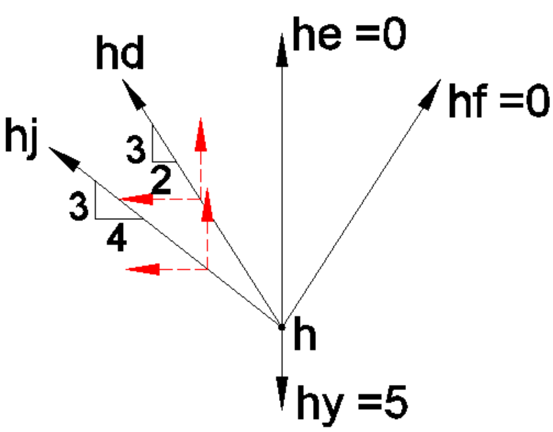
$\uparrow \sum f_y = 0$

$hd * \frac{3}{\sqrt{13}} + hj * \frac{3}{5} = 5 \quad \dots\dots\dots\dots\dots 1$

$\rightarrow \sum f_x = 0$

$$-hd * \frac{2}{\sqrt{13}} - hj * \frac{4}{5} = 0$$

$$hd = -\frac{\sqrt{13}}{2} * \frac{4}{5} * hj \quad \dots\dots\dots\dots\dots 2$$



Sub equation 2 in equation 1

$$\left(-\frac{\sqrt{13}}{2} * \frac{4}{5} * \text{hj}\right) * \frac{3}{\sqrt{13}} + \left(\text{hj} * \frac{3}{5}\right) = 5$$

hj = -8.34 kN = 8.34 kN (Comp.)

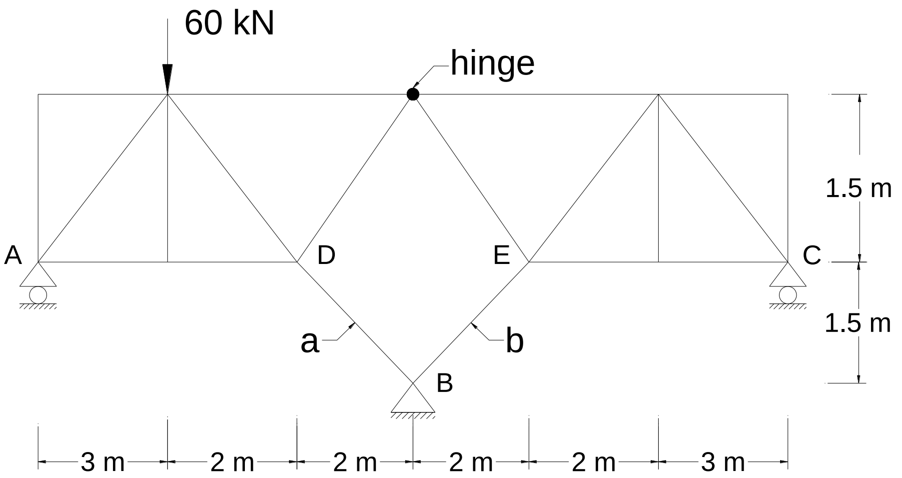
$$\text{hd} = -\frac{\sqrt{13}}{2} * \frac{4}{5} * (-8.34) = 12 \text{ kN (Ten.)}$$

H.W: For the same example find axial force in bars aj & ad

Ex4: For the compound truss shown in fig. Find

i- Reaction at supports.

ii- Axial force in bars a & b



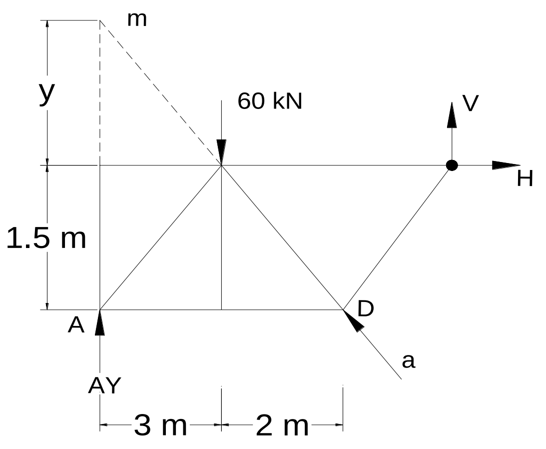


fig. 1

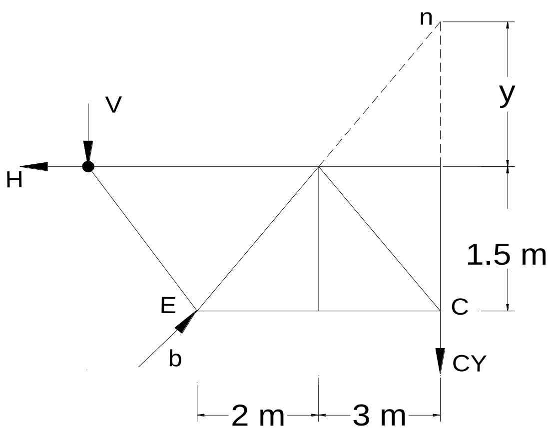


fig. 2

i) Reaction at supports

$$\frac{y}{3} = \frac{1.5}{2} \Rightarrow y = 2.25$$

From fig 1

$$\sum M_m = 0$$

(H*2.25) + (V*7) – (60*3) = 0

2.25H + 7V -180 = 0 ………………..1

From fig 2

$\sum M_n = 0$

H *2.25 – V*7 = 0

$H = \frac{7V}{2.25}$ ……………………..2

Sub equation 2 in equation 1

$2.25 * \frac{7V}{2.25} + 7 * V - 180 = 0$

V = 12.86 kN

$H = \frac{7 * 12.86}{2.25} = 40\ kN$

From fig 1

$\sum M_D = 0$

(Ay*5) + (40*1.5) – (12.86*2) – (60*2) = 0

Ay = 17.14 kN ↑

From fig 2

$\sum M_E = 0$

(Cy*5) – (12.86*2) – (40*1.5) = 0

Cy = 17.14 kN ↓

From the whole truss

$\sum F_X = 0$, => Bx = 0

↑ $\sum F_y = 0$

17.14 – 17.14 – 60 + By = 0

By = 60 kN ↓

ii) Axial force in bars a&b

For a& b use joint B

$\uparrow \sum F_y = 0$

$60 - a * \frac{3}{5} - b * \frac{3}{5} = 0$ 1

$\rightarrow \sum F_x = 0$

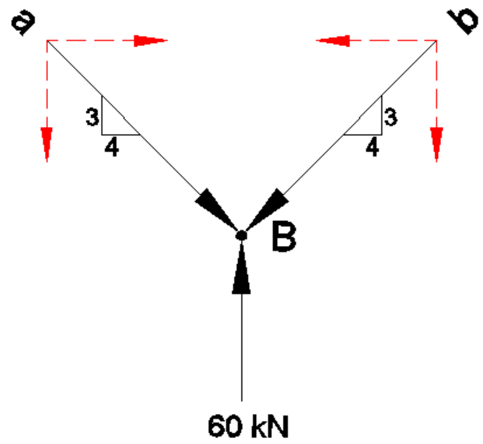
$a * \frac{4}{5} - b * \frac{4}{5} = 0$

a = b ……………2, sub in equation 1

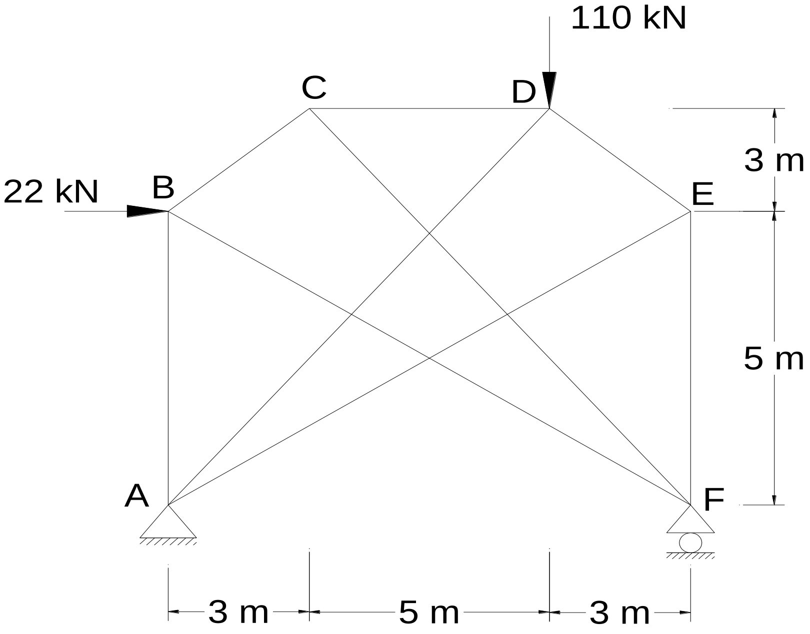
$60 - a * \frac{3}{5} - a * \frac{3}{5} = 0$

a= 50 kN (comp.)

b = 50 kN (comp.)



Ex5: For the compound truss shown in fig. find the axial force in bars a, b &c



$\sum M_A = 0$

(22*5) + (110*8) – (By*11) = 0

By = 90 kN ↑

↑ $\sum F_y = 0$, => Ay + 90 – 110 = 0 => Ay = 20 kN ↑

→ $\sum F_X = 0$, => 22 - Ax = 0 => Ax = 22 kN ←

$\sum M_O = 0$

C*11 – 22*3 = 0 => C =6 kN (comp.)

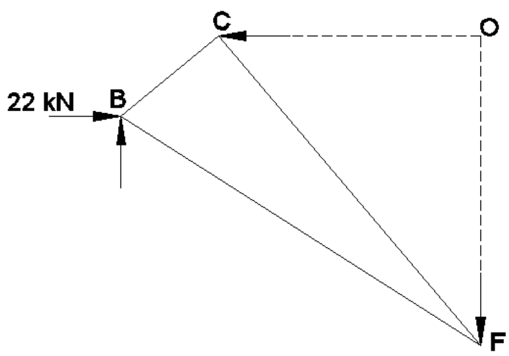
By = 90 kN

↑ $\sum F_y = 0$

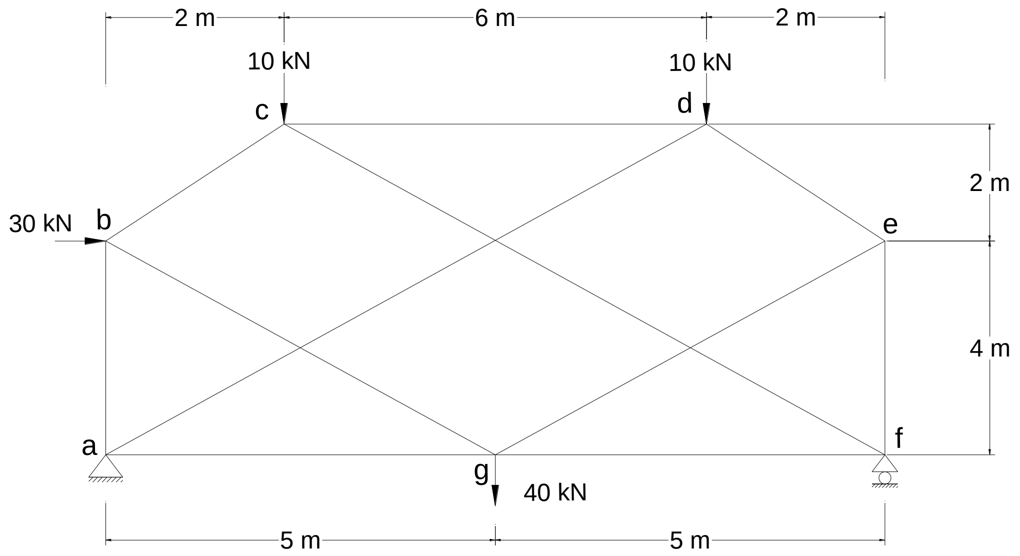
90 + 6 – b = 0 => b = 96 kN (comp.)

→ $\sum F_x = 0$

22 – a = 0 => a = 22kN (comp.)



Ex 6 Analysis the truss shown in fig. finds all bars forces.



Solution procedure

1- Assume the force in any bar for example (ad) and equal (R) in tension
2- Calculate the force in another bar for example (ab) in term of (R) by following two paths.
   a) First path: (joint d), (joint c) & (joint b). By applying equilibrium equations we find the force in bar (ad) in term of (R).
   b) Second path: (joint a), from it we find the value of force in the bar (ab) in term of (R).
3- We equal the forces outputs in bar (ab) which Obtained from the two paths, we solve the equation to find the value of (R).
4- All the forces of the bars can now be calculated after identifying the value of (R).

Solution:

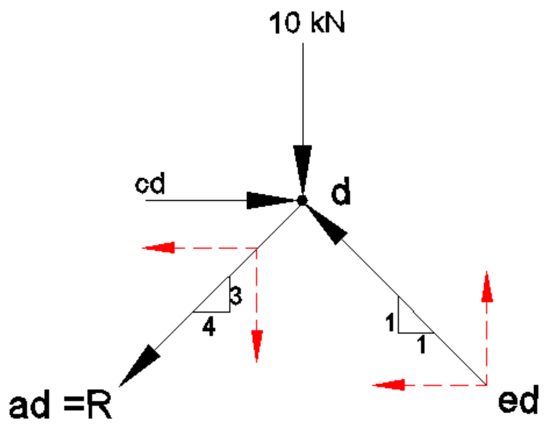
$\sum M_A = 0$ => fy = 42 kN $\uparrow$

$\sum F_y = 0$ => ay = 18 kN $\uparrow$

$\sum F_x = 0$ => ax = 30 kN $\leftarrow$

Let force in bar ad = R (tension)

Use joint d



$\uparrow \sum F_y = 0$

$$ed * \frac{1}{\sqrt{2}} - 10 - 0.6R = 0$$

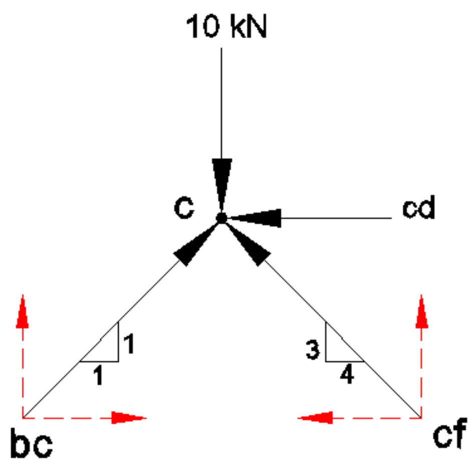
$$ed = \sqrt{2}\,(10 + 0.6R)$$

$\rightarrow \sum F_x = 0$

$$d - 0.8R - \sqrt{2}\,(10 + 0.6R) * \frac{1}{\sqrt{2}} = 0$$

$$cd = 10 + 1.4R$$

Use joint c



$\uparrow \sum F_y = 0$

$$bc * \frac{1}{\sqrt{2}} + cf * \frac{3}{5} - 10 = 0$$

$$bc = \sqrt{2}\,(10 - 0.6\,cf) \dots\dots\dots\dots 1$$

$\rightarrow \sum F_x = 0$

$$bc * \frac{1}{\sqrt{2}} - cf * \frac{4}{5} - 10 - 1.4R = 0 \dots\dots\dots..2$$

Sub equation 1 in equation 2

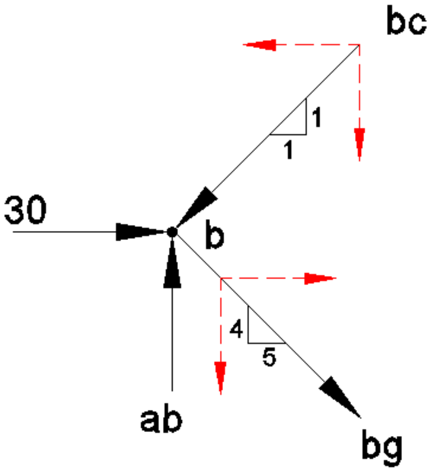
$$(\sqrt{2}\,(10 - 0.6\,cf) * \frac{1}{\sqrt{2}}) - (cf * \frac{4}{5}) - 10 - 1.4R = 0$$

$$cf = -R$$

$$bc = \sqrt{2}\,(10 - 0.6 * (-R))$$

$$bc = \sqrt{2}\,(10 + 0.6R)$$

Use joint b



$\rightarrow \sum F_x = 0, => 30 - 10 - 0.6R + bg * \frac{5}{\sqrt{41}} = 0$

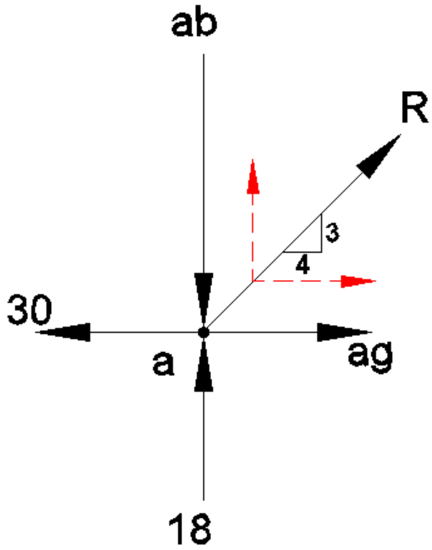
$bg = \frac{\sqrt{41}}{5}(0.6R - 20)$

$\uparrow \sum F_y = 0$

$ab - 10 - 0.6R - (0.6R - 20) * \frac{\sqrt{41}}{5} * \frac{4}{\sqrt{41}} = 0$

$ab = -6 + 1.08R$

Use joint a



$\rightarrow \sum F_x = 0$

ag + 0.8R - 30 = 0

ag= 30 – 0.8R

$\sum F_y = 0$

18 + 0.6R = -6 + 1.08R

R = 50 kN (ten.)

ad= 50 kN (ten.)

ab = -6 + (1.08*50) = -48 = 48 kN (comp.)

cd= 10+1.4 *50 =80 kN (ten.)

$bg = \frac{\sqrt{41}}{5}(0.6 * 50 - 20) \; => bg = 2\sqrt{41}$ kN (ten.)

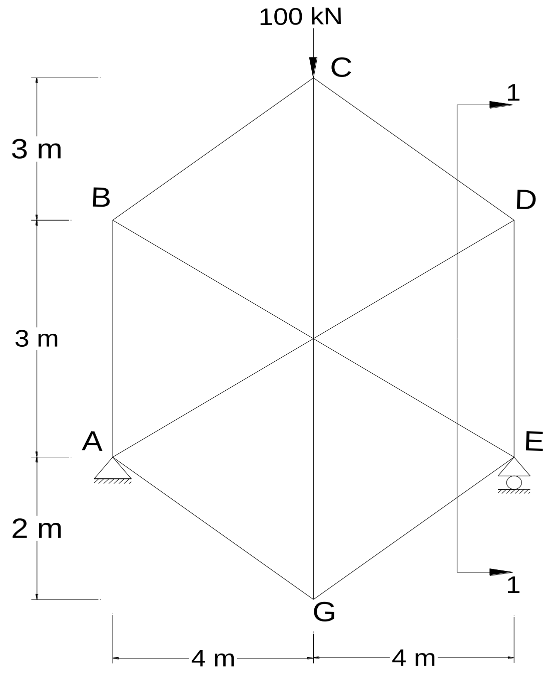
ag = 30 – 0.8*50 = 10 kN (ten.)

cf= -R = -50 = 50 kN (comp.)

$bc = \sqrt{2}\,(10 + 0.6 * 50) => \; bc = 40\sqrt{2}$ kN (comp.)

Ex 7: Analysis the truss shown in fig.

1- classify the truss
2- find the force in bar AD.



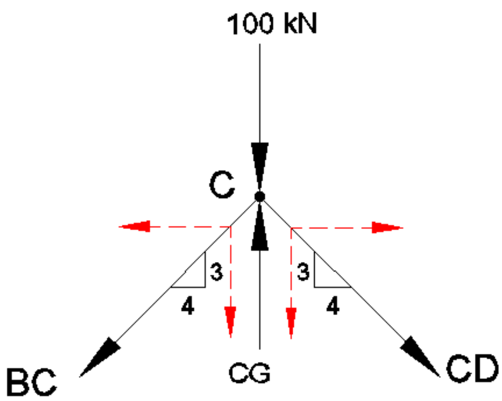
Sol:

Section 1 ------1

$\sum M_o = 0$

$$(50*4) + \left(\text{CD} * \frac{4}{5} * 4.5\right) - (GE * \frac{2}{\sqrt{5}} * 3.5) = 0 \ldots\ldots\ldots 1$$

Joint C as F.B.D



$\rightarrow \sum F_x = 0$

$$\text{CD} * \frac{4}{5} - BC * \frac{4}{5} = 0$$

CD = BC

$\uparrow \sum F_y = 0$

$$100 + 2 * \text{CD} * \frac{3}{5} = CG \ldots\ldots\ldots\ldots 2$$

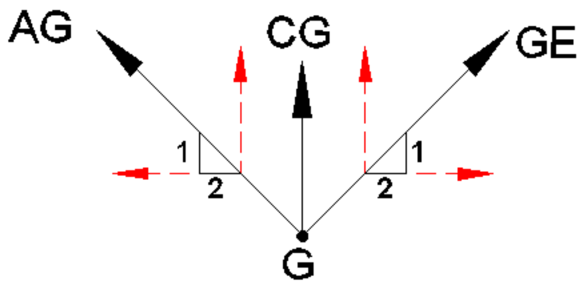
(Joint G) as F.B.D

$\rightarrow \sum F_x = 0$

$GE * \frac{2}{\sqrt{5}} - AG * \frac{2}{\sqrt{5}} = 0$

GE = AG

$\uparrow \sum F_y = 0$



$CG = 2GE * \frac{1}{\sqrt{5}} \dots\dots\dots\dots 3$

From equation 2&3

$100 + 2 * CD * \frac{3}{5} = 2GE * \frac{1}{\sqrt{5}} \dots\dots\dots 4$

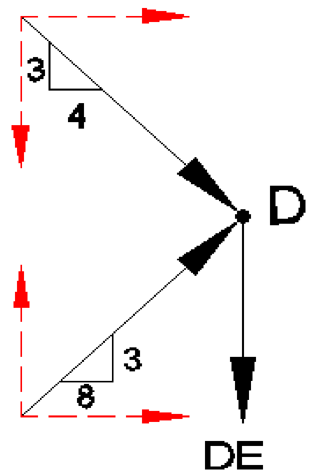
Solve equation 1 & equation 4similary to find

Cd = -249.6 kN (Comp.)

GE = -223.2 kN (Comp.)

Use joint D

$\rightarrow \sum F_x = 0$



$AD * \frac{8}{\sqrt{73}} + 249.6 * \frac{4}{5} = 0$

AD = -213.76 kN = 213.76 kN (Ten.)