Chain rule

1-If Fx&Fy are contain &w=F(x,y),y&x are function of (t) only.

$$\frac{\partial w}{\partial t}=\frac{\partial w}{\partial x}.\frac{\partial x}{\partial t}+\frac{\partial w}{\partial y}.\frac{\partial y}{\partial t}$$

2-If W is a function of (x,y&y) in the same time

x=F(r,s),y=F(r,s)&z=F(r,s)then

$$\frac{\partial w}{\partial r}=\frac{\partial w}{\partial x}\frac{\partial x}{\partial r}+\frac{\partial w}{\partial y}\frac{\partial y}{\partial r}+\frac{\partial w}{\partial z}\frac{\partial z}{\partial r}$$

$$\frac{\partial w}{\partial s}=\frac{\partial w}{\partial x}\frac{\partial x}{\partial s}+\frac{\partial w}{\partial y}\frac{\partial y}{\partial s}+\frac{\partial w}{\partial z}\frac{\partial z}{\partial s}$$

Example1: Find the derivative of w=3xy if x=cos t &y=sin t

Solution//

$$\frac{\partial w}{\partial t}=\frac{\partial w}{\partial x}.\frac{\partial x}{\partial t}+\frac{\partial w}{\partial y}.\frac{\partial y}{\partial t}$$

$$\frac{dw}{dy} = 3x. \qquad \frac{\partial w}{\partial x} = 3y$$

$$\frac{\partial x}{\partial t} = -\sin t. \qquad \frac{\partial y}{\partial t} = \cos t$$

$$\frac{\partial w}{\partial t} = 3y * (-\sin t) + 3x * \cos t$$

$$\frac{\partial w}{\partial t} = -3 * \sin^2 t + 3\cos^2 t$$

Example2: Find the derivative of w=xy+z if x=cos t ,y=sin t &z=2t^2

Solution//

$$\frac{\partial w}{\partial t} = \frac{\partial w}{\partial x}.\frac{\partial x}{\partial t} + \frac{\partial w}{\partial y}.\frac{\partial y}{\partial t} + \frac{\partial w}{\partial z}.\frac{\partial z}{\partial t}$$

=y.(-sin t)+x*cos t+1*4t

= - $\sin^2 t$ +$\cos^2 t$ +4t

Exercise: Find $\frac{\partial w}{\partial t}$ $in\ terms\ of\ w = x^2 + y^2. x = \cos t. y = sint$

Example 3: Find $\frac{\partial w}{\partial t}$ .w=2ye^x-ln z , x=ln($t^2 + 1$) ,y=$\tan^{-1} t$ &z=e^t

Solution//

$$\frac{\partial w}{\partial t}=\frac{\partial w}{\partial x}.\frac{\partial x}{\partial t}+\frac{\partial w}{\partial y}.\frac{\partial y}{\partial t}+\frac{\partial w}{\partial z}.\frac{\partial z}{\partial t}$$

$$\frac{\partial w}{\partial t}=2y\,e^{x}.\frac{2t}{t^2+1}+2e^{x}.\frac{1}{t^2+1}+\frac{-1}{z}.e^{t}$$

$$\frac{\partial w}{\partial t}=2\,tan^{-1}t\ \ e^{ln(t^2+1)}.\frac{2t}{t^2+1}+2e^{ln(t^2+1)}.\frac{1}{t^2+1}-\frac{1}{e^{t}}.e^{t}$$

Example 4: Find $\frac{\partial w}{\partial r}$ & $\frac{\partial w}{\partial s}$ $.in\ terms\ of\ r\&s\ if$ **w=x+4y+z^2 , x=$\frac{r}{s}$,y=$2r^2+ln\,2s$ &z=2r**

Solution//

$$\frac{\partial w}{\partial r}=\frac{\partial w}{\partial x}.\frac{\partial x}{\partial r}+\frac{\partial w}{\partial y}.\frac{\partial y}{\partial r}+\frac{\partial w}{\partial z}.\frac{\partial z}{\partial r}$$

$$\frac{\partial w}{\partial r}=\frac{1}{s}+16r+8r=\frac{1}{s}+24\,r$$

$$\frac{\partial w}{\partial s}=\frac{\partial w}{\partial x}.\frac{\partial x}{\partial s}+\frac{\partial w}{\partial y}.\frac{\partial y}{\partial s}+\frac{\partial w}{\partial z}.\frac{\partial z}{\partial s}$$

$$\frac{\partial w}{\partial s}=\frac{-1}{s^2}+\frac{4}{s}$$

Example 5: Find $\frac{\partial w}{\partial r}$ & $\frac{\partial w}{\partial s}$. $in\ terms\ of\ r\&s\ if$ w=x^2 +y^2 , x=$r+s$, y=$r-s$

Solution//

$$\frac{\partial w}{\partial r}=\frac{\partial w}{\partial x}.\frac{\partial x}{\partial r}+\frac{\partial w}{\partial y}.\frac{\partial y}{\partial r}$$

$$\frac{\partial w}{\partial r}=2x.1+2y=4r$$

$$\frac{\partial w}{\partial s}=\frac{\partial w}{\partial x}.\frac{\partial x}{\partial s}+\frac{\partial w}{\partial y}.\frac{\partial y}{\partial s}$$

$$\frac{\partial w}{\partial s}=2x.1+2y.(-1)=2(r+s)-2(r-s)=4s$$