CHAPTER 3

Tension Members

3.1 INTRODUCTION

Tension members are structural elements that are subjected to axial tensile forces. They are used in various types of structures and include truss members, bracing for buildings and bridges, cables in suspended roof systems, and cables in suspension and cable-stayed bridges. Any cross-sectional configuration may be used, because for any given material, the only determinant of the strength of a tension member is the cross-sectional area. Circular rods and rolled angle shapes are frequently used. Built-up shapes, either from plates, rolled shapes, or a combination of plates and rolled shapes, are sometimes used when large loads must be resisted. The most common built-up configuration is probably the double-angle section, shown in Figure 3.1, along with other typical cross sections. Because the use of this section is so widespread, tables of properties of various combinations of angles are included in the AISC *Steel Construction Manual*.

The stress in an axially loaded tension member is given by

$$f = \frac{P}{A}$$

where P is the magnitude of the load and A is the cross-sectional area (the area normal to the load). The stress as given by this equation is exact, provided that the cross section under consideration is not adjacent to the point of application of the load, where the distribution of stress is not uniform.

If the cross-sectional area of a tension member varies along its length, the stress is a function of the particular section under consideration. The presence of holes in a member will influence the stress at a cross section through the hole or holes. At these locations, the cross-sectional area will be reduced by an amount equal to the area removed by the holes. Tension members are frequently connected at their ends with bolts, as illustrated in Figure 3.2. The tension member shown, a $\frac{1}{2} \times 8$ plate, is connected to a *gusset plate*, which is a connection element whose purpose is to transfer the load from the member to a support or to another member. The area of the bar at section *a–a* is $(\frac{1}{2})(8) = 4 \text{ in.}^2$, but the area at section *b–b* is only $4 - (2)(\frac{1}{2})(\frac{7}{8}) = 3.13 \text{ in.}^2$

FIGURE 3.1

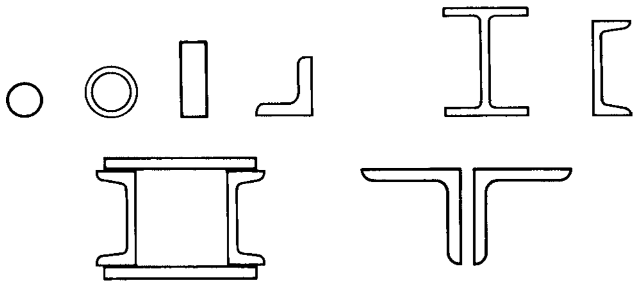
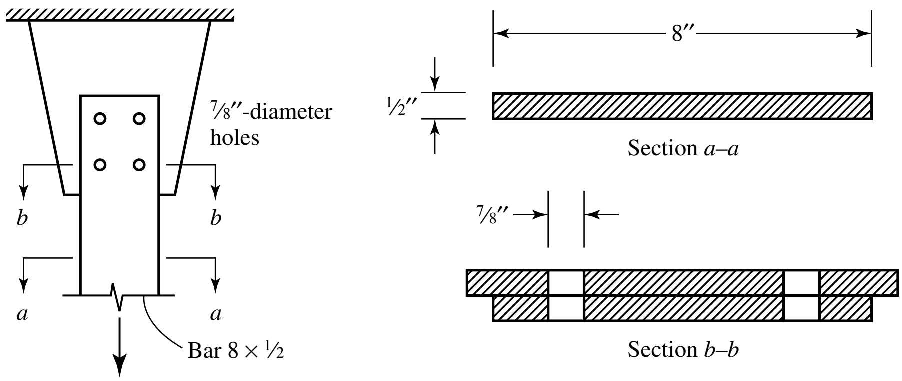


FIGURE 3.2



and will be more highly stressed. This reduced area is referred to as the *net area,* or *net section*, and the unreduced area is the *gross area*.

The typical design problem is to select a member with sufficient cross-sectional area to resist the loads. A closely related problem is that of analysis, or review, of a given member, where in the strength is computed and compared with the load. In general, analysis is a direct procedure, but design is an iterative process and may require some trial and error.

Tension members are covered in Chapter D of the Specification. Requirements that are common with other types of members are covered in Chapter B, "Design Requirements."

3.2 TENSILE STRENGTH

A tension member can fail by reaching one of two limit states: excessive deformation or fracture. To prevent excessive deformation, initiated by yielding, the load on the gross section must be small enough that the stress on the gross section is less than the yield stress F_y. To prevent fracture, the stress on the net section must be less than the tensile strength F_u. In each case, the stress P/A must be less than a limiting stress F or

$$\frac{P}{A} < F$$

Thus, the load P must be less than FA, or

$$P < FA$$

The *nominal* strength in yielding is

$$P_n = F_y A_g$$

and the nominal strength in fracture is

$$P_n = F_u A_e$$

where A_e is the *effective* net area, which may be equal to either the net area or, in some cases, a smaller area. We discuss effective net area in Section 3.3.

Although yielding will first occur on the net cross section, the deformation within the length of the connection will generally be smaller than the deformation in the remainder of the tension member. The reason is that the net section exists over a relatively small length of the member, and the total elongation is a product of the length and the strain (a function of the stress). Most of the member will have an unreduced cross section, so attainment of the yield stress on the gross area will result in larger total elongation. It is this larger deformation, not the first yield, that is the limit state.

LRFD: In load and resistance factor design, the factored tensile load is compared to the design strength. The design strength is the resistance factor times the nominal strength. Equation 2.6,

$$R_u = \phi R_n$$

can be written for tension members as

$$P_u \le \phi_t P_n$$

where P_u is the governing combination of factored loads. The resistance factor ϕ_t is smaller for fracture than for yielding, reflecting the more serious nature of fracture.

For yielding, $\phi_t = 0.90$
For fracture, $\phi_t = 0.75$

Because there are two limit states, both of the following conditions must be satisfied:

$$P_u \le 0.90 F_y A_g$$
$$P_u \le 0.75 F_u A_e$$

The smaller of these is the design strength of the member.

ASD: In allowable strength design, the total service load is compared to the allowable strength (allowable load):

$$P_a \le \frac{P_n}{\Omega_t}$$

where P_a is the required strength (applied load), and P_n/Ω_t is the allowable strength. The subscript "a" indicates that the required strength is for "allowable strength design," but you can think of it as standing for "applied" load.

For yielding of the gross section, the safety factor Ω_t is 1.67, and the allowable load is

$$\frac{P_n}{\Omega_t} = \frac{F_y A_g}{1.67} = 0.6 F_y A_g$$

(The factor 0.6 appears to be a rounded value, but recall that 1.67 is a rounded value. If $\Omega_t = 5/3$ is used, the allowable load is exactly 0.6 $F_y A_g$.)

For fracture of the net section, the safety factor is 2.00 and the allowable load is

$$\frac{P_n}{\Omega_t} = \frac{F_u A_e}{2.00} = 0.5 F_u A_e$$

Alternatively, the service load stress can be compared to the allowable stress. This can be expressed as

$$f_t \leq F_t$$

where f_t is the applied stress and F_t is the allowable stress. For yielding of the gross section,

$$f_t = \frac{P_a}{A_g} \quad \text{and} \quad F_t = \frac{P_n/\Omega_t}{A_g} = \frac{0.6 F_y A_g}{A_g} = 0.6 F_y$$

For fracture of the net section,

$$f_t = \frac{P_a}{A_e} \quad \text{and} \quad F_t = \frac{P_n/\Omega_t}{A_e} = \frac{0.5 F_u A_e}{A_e} = 0.5 F_u$$

You can find values of F_y and F_u for various structural steels in Table 2-3 in the *Manual.* All of the steels that are available for various hot-rolled shapes are indicated by shaded areas. The black areas correspond to preferred materials, and the gray areas represent other steels that are available. Under the W heading, we see that A992 is the preferred material for W shapes, but other materials are available, usually at a higher cost. For some steels, there is more than one grade, with each grade having different values of F_y and F_u. In these cases, the grade must be specified along with the ASTM designation—for example, A572 Grade 50. For A242 steel, F_y and F_u depend on the thickness of the flange of the cross-sectional shape. This relationship is given in footnotes in the table. For example, to determine the properties of a W33 × 221 of ASTM A242 steel, first refer to the dimensions and properties table in Part 1 of the *Manual* and determine that the flange thickness t_f is equal to 1.28 inches. This matches the thickness range indicated in footnote *1*; therefore, $F_y = 50$ ksi and $F_u = 70$ ksi. Values of F_y and F_u for plates and bars are given in the *Manual* Table 2-4, and information on structural fasteners, including bolts and rods, can be found in Table 2-5.

The exact amount of area to be deducted from the gross area to account for the presence of bolt holes depends on the fabrication procedure. The usual practice is to drill or punch standard holes (i.e., not oversized) with a diameter 1⁄16 inch larger than the fastener diameter. To account for possible roughness around the edges of the hole, Section B4.3 of the AISC Specification (in the remainder of this book, references to the Specification will usually be in the form AISC B4.3) requires the addition of 1⁄16 inch to the actual hole diameter. This amounts to using an effective hole diameter 1⁄8 inch larger than the fastener diameter. In the case of slotted holes, 1⁄16 inch should be added to the actual *width* of the hole. You can find details related to standard, oversized, and slotted holes in AISC J3.2, "Size and Use of Holes" (in Chapter J, "Design of Connections").

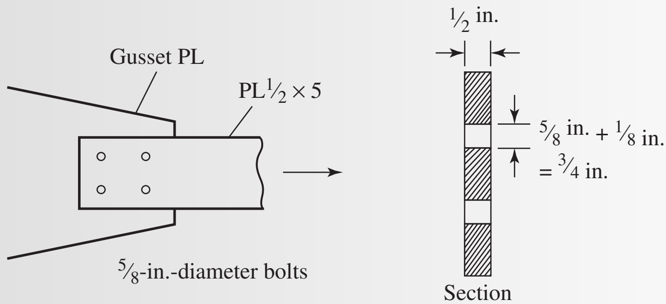
EXAMPLE 3.1

A ½ × 5 plate of A36 steel is used as a tension member. It is connected to a gusset plate with four ⅝-inch-diameter bolts as shown in Figure 3.3. Assume that the effective net area A_e equals the actual net area A_n (we cover computation of effective net area in Section 3.3).

a. What is the design strength for LRFD?

b. What is the allowable strength for ASD?

FIGURE 3.3



SOLUTION

For yielding of the gross section,

$$A_g = 5(1/2) = 2.5 \text{ in.}^2$$

and the nominal strength is

$$P_n = F_y A_g = 36(2.5) = 90.0 \text{ kips}$$

For fracture of the net section,

$$A_n = A_g - A_{holes} = 2.5 - (\tfrac{1}{2})(\tfrac{3}{4}) \times 2 \text{ holes}$$
$$= 2.5 - 0.75 = 1.75 \text{ in.}^2$$

$A_e = A_n = 1.75 \text{ in.}^2$ (This is true for this example, but A_e does not always equal A_n.)

The nominal strength is

$$P_n = F_u A_e = 58(1.75) = 101.5 \text{ kips}$$

a. The design strength based on yielding is

$$\phi_t P_n = 0.90(90) = 81.0 \text{ kips}$$

The design strength based on fracture is

$$\phi_t P_n = 0.75(101.5) = 76.1 \text{ kips}$$

ANSWER The design strength for LRFD is the smaller value: $\phi_t P_n = 76.1$ kips.

b. The allowable strength based on yielding is

$$\frac{P_n}{\Omega_t} = \frac{90}{1.67} = 53.9 \text{ kips}$$

The allowable strength based on fracture is

$$\frac{P_n}{\Omega_t} = \frac{101.5}{2.00} = 50.8 \text{ kips}$$

ANSWER The allowable service load is the smaller value = 50.8 kips.

Alternative Solution Using Allowable Stress: For yielding,

$$F_t = 0.6F_y = 0.6(36) = 21.6 \text{ ksi}$$

and the allowable load is

$$F_t A_g = 21.6(2.5) = 54.0 \text{ kips}$$

(The slight difference between this value and the one based on allowable strength is because the value of Ω in the allowable strength approach has been rounded from 5/3 to 1.67; the value based on the allowable stress is the more accurate one.)
For fracture,

$$F_t = 0.5F_u = 0.5(58) = 29.0 \text{ ksi}$$

and the allowable load is

$$F_t A_e = 29.0(1.75) = 50.8 \text{ kips}$$

ANSWER The allowable service load is the smaller value = 50.8 kips.

Because of the relationship given by Equation 2.8, the allowable strength will always be equal to the design strength divided by 1.5. In this book, however, we will do the complete computation of allowable strength even when the design strength is available.

The effects of stress concentrations at holes appear to have been overlooked. In reality, stresses at holes can be as high as three times the average stress on the net section, and at fillets of rolled shapes they can be more than twice the average (McGuire, 1968). Because of the ductile nature of structural steel, the usual design practice is to neglect such localized overstress. After yielding begins at a point of stress concentration, additional stress is transferred to adjacent areas of the cross section. This stress redistribution is responsible for the "forgiving" nature of structural steel. Its ductility permits the initially yielded zone to deform without fracture as the stress on the remainder of the cross section continues to increase. Under certain conditions, however, steel may lose its ductility and stress concentrations can precipitate brittle fracture. These situations include fatigue loading and extremely low temperature.

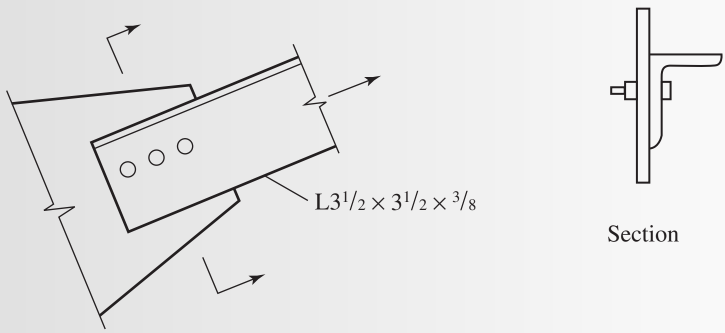
EXAMPLE 3.2

A single-angle tension member, an L3½ × 3½ × ⅜, is connected to a gusset plate with ⅞-inch-diameter bolts as shown in Figure 3.4. A36 steel is used. The service loads are 35 kips dead load and 15 kips live load. Investigate this member for compliance with the AISC Specification. Assume that the effective net area is 85% of the computed net area.

a. Use LRFD.

b. Use ASD.

FIGURE 3.4



SOLUTION

First, compute the nominal strengths.

Gross section:

$$A_g = 2.50 \text{ in.}^2 \qquad \text{(from Part 1 of the } \textit{Manual}\text{)}$$

$$P_n = F_y A_g = 36(2.50) = 90 \text{ kips}$$

Net section:

$$A_n = 2.50 - \left(\frac{3}{8}\right)\left(\frac{7}{8}+\frac{1}{8}\right) = 2.125 \text{ in.}^2$$

$$A_e = 0.85A_n = 0.85(2.125) = 1.806 \text{ in.}^2 \quad \text{(in } \textit{this} \text{ example)}$$

$$P_n = F_u A_e = 58(1.806) = 104.7 \text{ kips}$$

a. The design strength based on yielding is

$$\phi_t P_n = 0.90(90) = 81 \text{ kips}$$

The design strength based on fracture is

$$\phi_t P_n = 0.75(104.7) = 78.5 \text{ kips}$$

The design strength is the smaller value: $\phi_t P_n = 78.5$ kips

Factored load:

When only dead load and live load are present, the only load combinations with a chance of controlling are combinations 1 and 2.

Combination 1: $1.4D = 1.4(35) = 49$ kips

Combination 2: $1.2D + 1.6L = 1.2(35) + 1.6(15) = 66$ kips

The second combination controls; $P_u = 66$ kips.

(When only dead load and live load are present, combination 2 will always control when the dead load is less than eight times the live load. In future examples, we will not check combination 1 [$1.4D$] when it obviously does not control.)

ANSWER Since $P_u < \phi_t P_n$, (66 kips < 78.5 kips), the member is satisfactory.

b. For the gross section, The allowable strength is

$$\frac{P_n}{\Omega_t} = \frac{90}{1.67} = 53.9 \text{ kips}$$

For the net section, the allowable strength is

$$\frac{P_n}{\Omega_t} = \frac{104.7}{2.00} = 52.4 \text{ kips}$$

The smaller value controls; the allowable strength is 52.4 kips. When the only loads are dead load and live load, ASD load combination 2 will always control:

$$P_a = D + L = 35 + 15 = 50 \text{ kips}$$

ANSWER Since 50 kips < 52.4 kips, the member is satisfactory.

Alternative Solution Using Allowable Stress

For the gross area, the applied stress is

$$f_t = \frac{P_a}{A_g} = \frac{50}{2.50} = 20 \text{ ksi}$$

and the allowable stress is

$$F_t = 0.6F_y = 0.6(36) = 21.6 \text{ ksi}$$

For this limit state, $f_t < F_t$ (OK)

For the net section,

$$f_t = \frac{P_a}{A_e} = \frac{50}{1.806} = 27.7 \text{ ksi}$$

$$F_t = 0.5F_u = 0.5(58) = 29.0 \text{ ksi} > 27.7 \text{ ksi} \quad \text{(OK)}$$

ANSWER Since $f_t < F_t$ for both limit states, the member is satisfactory.

What is the difference in computational effort for the two different approaches? Regardless of the method used, the two nominal strengths must be computed (if a stress approach is used with ASD, an equivalent computation must be made). With LRFD, the nominal strengths are multiplied by resistance factors. With ASD, the nominal strengths are divided by load factors. Up to this point, the number of steps is the same. The difference in effort between the two methods involves the load side of the relationships. In LRFD, the loads are factored before adding. In ASD, in most cases the loads are simply added. Therefore, for tension members LRFD requires slightly more computation.

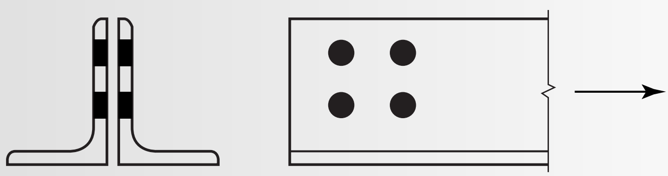
EXAMPLE 3.3

A double-angle shape is shown in Figure 3.5. The steel is A36, and the holes are for ½-inch-diameter bolts. Assume that $A_e = 0.75A_n$.

a. Determine the design tensile strength for LRFD.

b. Determine the allowable strength for ASD.

FIGURE 3.5



SOLUTION

Figure 3.5 illustrates the notation for unequal-leg double-angle shapes. The notation LLBB means "long-legs back-to-back," and SLBB indicates "short-legs back-to-back."

When a double-shape section is used, two approaches are possible: (1) consider a single shape and double everything, or (2) consider two shapes from the outset. (Properties of the double-angle shape are given in Part 1 of the *Manual.*) In this example, we consider one angle and double the result. For one angle, the nominal strength based on the gross area is

$$P_n = F_y A_g = 36(2.41) = 86.76 \text{ kips}$$

There are two holes in each angle, so the net area of one angle is

$$A_n = 2.41 - \left(\frac{5}{16}\right)\left(\frac{1}{2} + \frac{1}{8}\right) \times 2 = 2.019 \text{ in.}^2$$

The effective net area is

$$A_e = 0.75(2.019) = 1.514 \text{ in.}^2$$

The nominal strength based on the net area is

$$P_n = F_u A_e = 58(1.514) = 87.81 \text{ kips}$$

a. The design strength based on yielding of the gross area is

$$\phi_t P_n = 0.90(86.76) = 78.08 \text{ kips}$$

The design strength based on fracture of the net area is

$$\phi_t P_n = 0.75(87.81) = 65.86 \text{ kips}$$

ANSWER Because 65.86 kips < 78.08 kips, fracture of the net section controls, and the design strength for the two angles is $2 \times 65.86 = 132$ kips.

b. The allowable stress approach will be used. For the gross section,

$$F_t = 0.6F_y = 0.6(36) = 21.6 \text{ ksi}$$

The corresponding allowable load is

$$F_t A_g = 21.6(2.41) = 52.06 \text{ kips}$$

For the net section,

$$F_t = 0.5F_u = 0.5(58) = 29 \text{ ksi}$$

The corresponding allowable load is

$$F_t A_e = 29(1.514) = 43.91 \text{ kips}$$

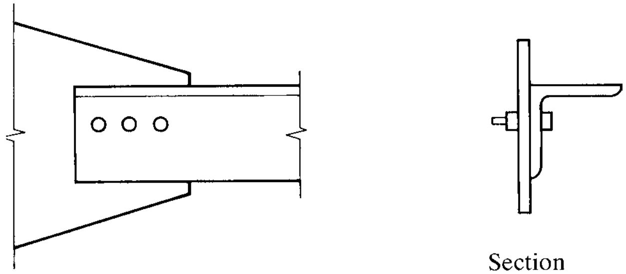
ANSWER Because 43.91 kips < 52.06 kips, fracture of the net section controls, and the allowable strength for the two angles is $2 \times 43.91 = 87.8$ kips.

3.3 EFFECTIVE AREA

Of the several factors influencing the performance of a tension member, the manner in which it is connected is the most important. A connection almost always weakens the member, and the measure of its influence is called the *joint efficiency*. This factor is a function of the ductility of the material, fastener spacing, stress concentrations at holes, fabrication procedure, and a phenomenon known as *shear lag*. All contribute to reducing the effectiveness of the member, but shear lag is the most important.

Shear lag occurs when some elements of the cross section are not connected, as when only one leg of an angle is bolted to a gusset plate, as shown in Figure 3.6. The consequence of this partial connection is that the connected element becomes overloaded and the unconnected part is not fully stressed. Lengthening the connected region will reduce this effect. Research reported by Munse and Chesson (1963)

FIGURE 3.6



suggests that shear lag be accounted for by using a reduced, or effective, net area. Because shear lag affects both bolted and welded connections, the effective net area concept applies to both types of connections.

For bolted connections, the effective net area is

$$A_e = A_n U \qquad \text{(AISC Equation D3-1)}$$

For welded connections, we refer to this reduced area as the *effective area* (rather than the effective *net* area), and it is given by

$$A_e = A_g U$$

where the reduction factor U is given in AISC D3, Table D3.1. The table gives a general equation that will cover most situations as well as alternative numerical values for specific cases. These definitions of U will be presented here in a different format from that in the Specification. The rules for determining U fall into five categories:

1. A general category for any type of tension member except plates and round HSS with $\ell \geq 1.3D$ (See Figure 3.7e.)
2. Plates
3. Round HSS with $\ell \geq 1.3\,D$
4. Alternative values for single and double angles
5. Alternative values for W, M, S, and HP shapes

1. For any type of tension member except plates and round HSS with $\ell \geq 1.3D$

$$U = 1 - \frac{\bar{x}}{\ell} \qquad (3.1)$$

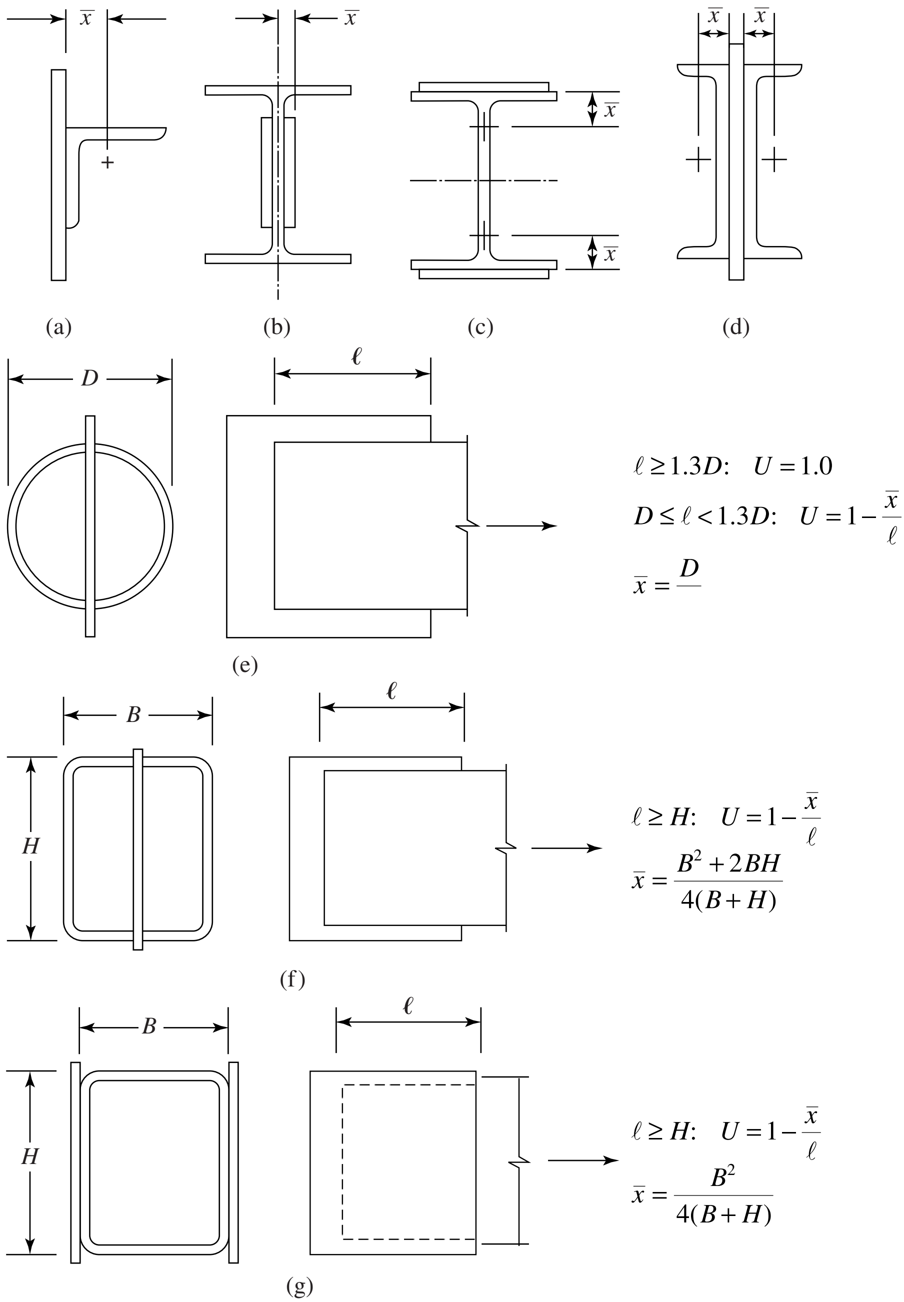
where

$\bar{x}$ = distance from centroid of connected area to the plane of the connection
ℓ = length of the connection

This definition of $\bar{x}$ was formulated by Munse and Chesson (1963). If a member has two symmetrically located planes of connection, $\bar{x}$ is measured from the centroid of the nearest one-half of the area. Figure 3.7 illustrates $\bar{x}$ for various types of connections.

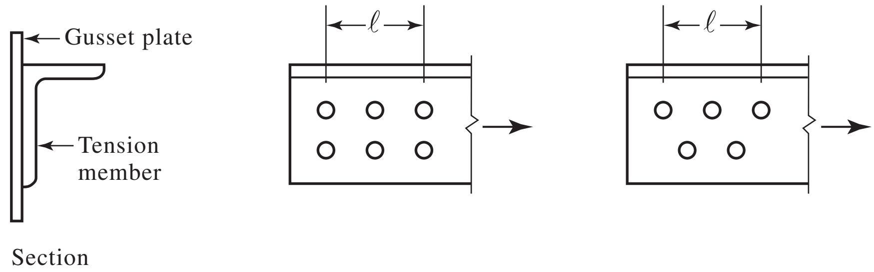
The length ℓ in Equation 3.1 is the length of the connection in the direction of the load, as shown in Figure 3.8. For bolted connections, it is measured from the center of the bolt at one end of the connection to the center of the bolt at the other end. For welds, it is measured from one end of the weld to the other. If there are weld segments of different lengths in the direction of the load, use the average length.

FIGURE 3.7

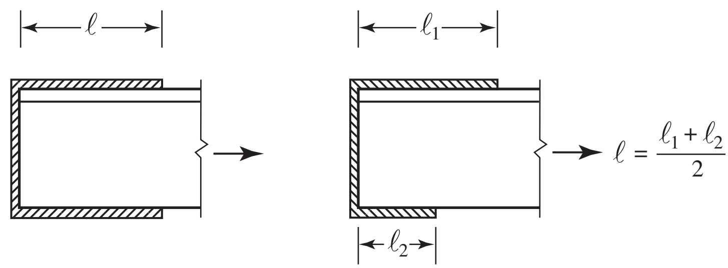


$\bar{x}$
(a)
(b)
(c)
(d)
D
ℓ
$\ell \geq 1.3D$: $U = 1.0$
$D \leq \ell < 1.3D$: $U = 1 - \frac{\bar{x}}{\ell}$
$\bar{x} = \frac{D}{\quad}$
(e)
B
H
ℓ
$\ell \geq H$: $U = 1 - \frac{\bar{x}}{\ell}$
$\bar{x} = \frac{B^2 + 2BH}{4(B+H)}$
(f)
B
H
ℓ
$\ell \geq H$: $U = 1 - \frac{\bar{x}}{\ell}$
$\bar{x} = \frac{B^2}{4(B+H)}$
(g)

FIGURE 3.8



(a) Bolted



(b) Welded

The Commentary of the AISC Specification further illustrates $\bar{x}$ and ℓ. Figure C-D3.2 shows some special cases for $\bar{x}$, including channels and I-shaped members connected through their webs. To compute $\bar{x}$ for these cases, the Commentary uses the concept of the plastic neutral axis to explain the procedure. Since this concept is not covered until Chapter 5 of this book, we will use $\bar{x}$ for channels as shown in Case 2 of Specification Table D3.1 and in Figure 3.7b of this book. For I-shaped members and tees connected through the web, we can use Case 2 or Case 7 of Specification Table D3.1.

2. Plates

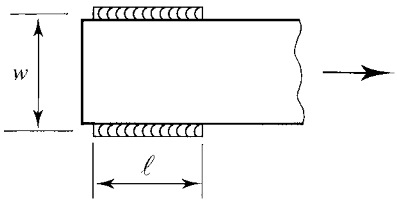
In general, $U = 1.0$ for plates, since the cross section has only one element and it is connected. There is one exception for welded plates, however. If the member is connected with longitudinal welds on each side with no transverse weld (as in Figure 3.9), the following values apply:

- For $\ell \geq 2w$ $\quad U = 1.0$
- For $1.5w \leq \ell < 2w$, $\quad U = 0.87$
- For $w \leq \ell < 1.5w$, $\quad U = 0.75$

3. Round HSS with $\ell \geq 1.3D$ (see Figure 3.7e):

$U = 1.0$

FIGURE 3.9



4. Alternatives to Equation 3.1 for Single and Double Angles:

The following values may be used in lieu of Equation 3.1.

- For four or more fasteners in the direction of loading, $U = 0.80$.
- For three fasteners in the direction of loading, $U = 0.60$.

5. Alternatives to Equation 3.1 for W, M, S, HP, or Tees Cut from These Shapes:

If the following conditions are satisfied, the corresponding values may be used in lieu of Equation 3.1.

- Connected through the flange with three or more fasteners in the direction of loading, with a width at least ⅔ of the depth: $U = 0.90$.
- Connected through the flange with three or more fasteners in the direction of loading, with a width less than ⅔ of the depth: $U = 0.85$.
- Connected through the web with four or more fasteners in the direction of loading: $U = 0.70$.

Figure 3.10 illustrates the alternative values of U for various connections.

If a tension member is connected with only transverse welds, $U = 1.0$, and A_n is the area of the connected element. Figure 3.11 illustrates the difference between transverse and longtitudinal welds. Connections by transverse welds alone are not common.

There are some limiting values for the effective area:

- For bolted *splice plates,* $A_e = A_n \le 0.85A_g$. This limit is given in a user note and is from a requirement in Chapter J of the Specification "Design of Connections."
- For open cross-sectional shapes (such as W, M, S, C, HP, WT, and ST) and (angles), the value of U need not be less than the ratio of the connected element gross area to the total gross area.

EXAMPLE 3.4

Determine the effective net area for the tension member shown in Figure 3.12.

SOLUTION

$$A_n = A_g - A_{\text{holes}}$$

$$= 5.77 - \frac{1}{2}\left(\frac{5}{8} + \frac{1}{8}\right)(2) = 5.02 \text{ in.}^2$$

Only one element (one leg) of the cross section is connected, so the net area must be reduced. From the properties tables in Part 1 of the *Manual,* the distance from the centroid to the outside face of the leg of an L6 × 6 × ½ is

$$\bar{x} = 1.67 \text{ in.}$$

FIGURE 3.10

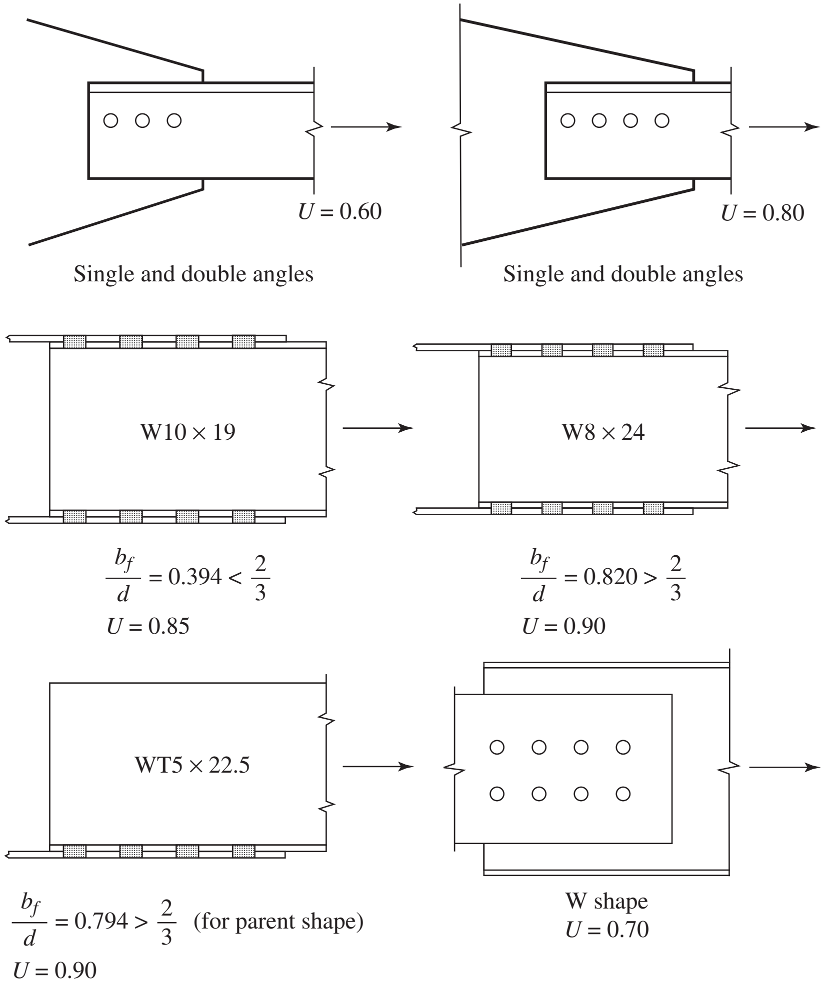


FIGURE 3.11

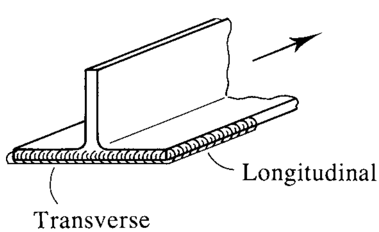
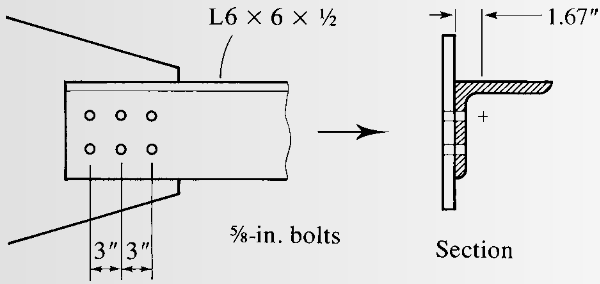


FIGURE 3.12



The length of the connection is

$$\ell = 3 + 3 = 6 \text{ in.}$$

$$\therefore U = 1 - \left(\frac{\bar{x}}{\ell}\right) = 1 - \left(\frac{1.67}{6}\right) = 0.7217$$

$$A_e = A_n U = 5.02(0.7217) = 3.623 \text{ in.}^2$$

The alternative value of U could also be used. Because this angle has three bolts in the direction of the load, the reduction factor U can be taken as 0.60, and

$$A_e = A_n U = 5.02(0.60) = 3.012 \text{ in.}^2$$

Either U value is acceptable, and the Specification permits the larger one to be used. However, the value obtained from Equation 3.1 is more accurate. The alternative values of U can be useful during preliminary design, when actual section properties and connection details are not known.

EXAMPLE 3.5

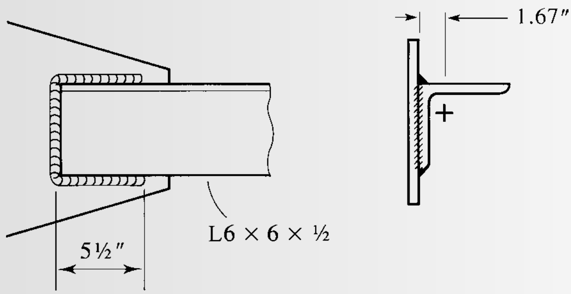
If the tension member of Example 3.4 is welded as shown in Figure 3.13, determine the effective area.

SOLUTION As in Example 3.4, only part of the cross section is connected and a reduced effective area must be used.

$$U = 1 - \left(\frac{\bar{x}}{\ell}\right) = 1 - \left(\frac{1.67}{5.5}\right) = 0.6964$$

ANSWER $A_e = A_g U = 5.77(0.6964) = 4.02 \text{ in.}^2$

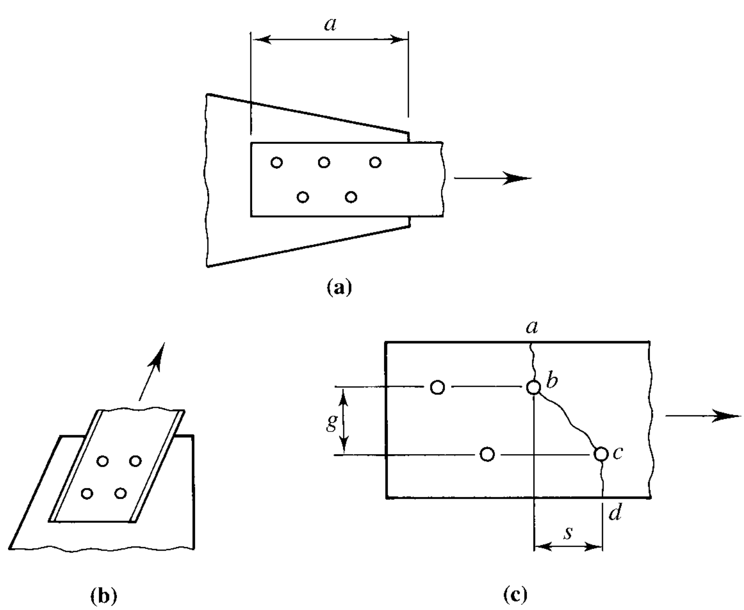
FIGURE 3.13



3.4 STAGGERED FASTENERS

If a tension member connection is made with bolts, the net area will be maximized if the fasteners are placed in a single line. Sometimes space limitations, such as a limit on dimension *a* in Figure 3.14a, necessitate using more than one line. If so, the reduction in cross-sectional area is minimized if the fasteners are arranged in a staggered pattern, as shown. Sometimes staggered fasteners are required by the geometry of a connection, such as the one shown in Figure 3.14b. In either case, any cross section passing through holes will pass through fewer holes than if the fasteners are not staggered.

FIGURE 3.14



If the amount of stagger is small enough, the influence of an offset hole may be felt by a nearby cross section, and fracture along an inclined path such as *abcd* in Figure 3.14c is possible. In such a case, the relationship $f = P/A$ does not apply, and stresses on the inclined portion *b–c* are a combination of tensile and shearing stresses. Several approximate methods have been proposed to account for the effects of staggered holes. Cochrane (1922) proposed that when deducting the area corresponding to a staggered hole, use a reduced diameter, given by

$$d' = d - \frac{s^2}{4g} \tag{3.2}$$

where d is the hole diameter, s is the stagger, or pitch, of the bolts (spacing in the direction of the load), and g is the gage (transverse spacing). This means that in a failure pattern consisting of both staggered and unstaggered holes, use d for holes at the end of a transverse line between holes ($s = 0$) and use d' for holes at the end of an inclined line between holes.

The AISC Specification, in Section B4.3b, uses this approach, but in a modified form. If the net area is treated as the product of a thickness times a net width, and the diameter from Equation 3.2 is used for all holes (since $d' = d$ when the stagger $s = 0$), the net width in a failure line consisting of both staggered and unstaggered holes is

$$\begin{aligned} w_n &= w_g - \Sigma d' \\ &= w_g - \Sigma\left(d - \frac{s^2}{4g}\right) \\ &= w_g - \Sigma d + \Sigma \frac{s^2}{4g} \end{aligned}$$

where w_n is the net width and w_g is the gross width. The second term is the sum of all hole diameters, and the third term is the sum of $s^2/4g$ for all inclined lines in the failure pattern.

When more than one failure pattern is conceivable, all possibilities should be investigated, and the one corresponding to the smallest load capacity should be used. Note that this method will not accommodate failure patterns with lines parallel to the applied load.

EXAMPLE 3.6

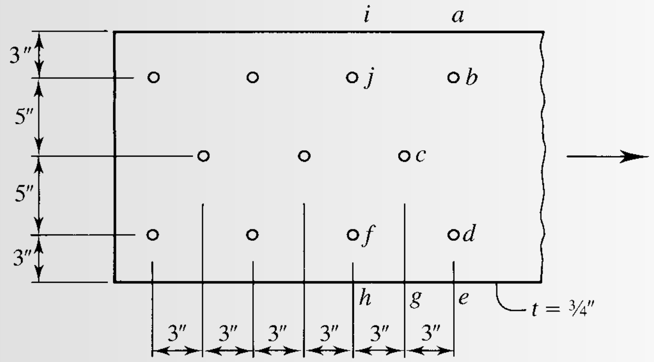
Compute the smallest net area for the plate shown in Figure 3.15. The holes are for 1-inch-diameter bolts.

SOLUTION

The effective hole diameter is $1 + \frac{1}{8} = 1\frac{1}{8}$ in. For line *abde,*

$$w_n = 16 - 2(1.125) = 13.75 \text{ in.}$$

FIGURE 3.15



For line *abcde,*

$$w_n = 16 - 3(1.125) + \frac{2(3)^2}{4(5)} = 13.52 \text{ in.}$$

The second condition will give the smallest net area:

ANSWER $A_n = tw_n = 0.75(13.52) = 10.1 \text{ in.}^2$

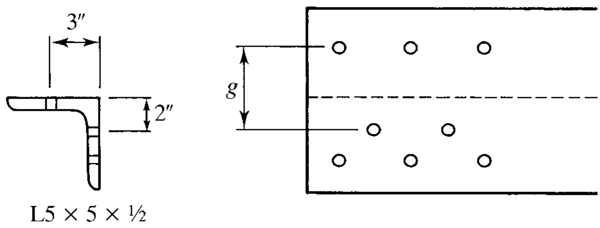
Equation 3.2 can be used directly when staggered holes are present. In the computation of the net area for line *abcde* in Example 3.6,

$$\begin{aligned} A_n &= A_g - \sum t \times (d \text{ or } d') \\ &= 0.75(16) - 0.75(1.125) - 0.75\left[1.125 - \frac{(3)^2}{4(5)}\right] \times 2 = 10.1 \text{ in.}^2 \end{aligned}$$

As each fastener resists an equal share of the load (an assumption used in the design of simple connections; see Chapter 7), different potential failure lines may be subjected to different loads. For example, line *abcde* in Figure 3.15 must resist the full load, whereas *ijfh* will be subjected to $8/11$ of the applied load. The reason is that $3/11$ of the load will have been transferred from the member before *ijfh* receives any load.

When lines of bolts are present in more than one element of the cross section of a rolled shape, and the bolts in these lines are staggered with respect to one another, the use of areas and Equation 3.2 is preferable to the net-width approach of the AISC Specification. If the shape is an angle, it can be visualized as a plate formed by “unfolding” the legs to more clearly identify the pitch and gage distances. AISC B4.3b specifies that any gage line crossing the heel of the angle be reduced by an amount that equals the angle thickness. Thus, the distance g in Figure 3.16, to be used in the $s^2/4g$ term, would be $3 + 2 - ½ = 4½$ inches.

FIGURE 3.16



EXAMPLE 3.7

An angle with staggered fasteners in each leg is shown in Figure 3.17. A36 steel is used, and holes are for ⅞-inch-diameter bolts.

a. Determine the design strength for LRFD.
b. Determine the allowable strength for ASD.

SOLUTION

From the dimensions and properties tables, the gross area is A_g = 6.80 in.². The effective hole diameter is ⅞ + ⅛ = 1 in.

For line *abdf*, the net area is

$$A_n = A_g - \Sigma t_w \times (d \text{ or } d')$$

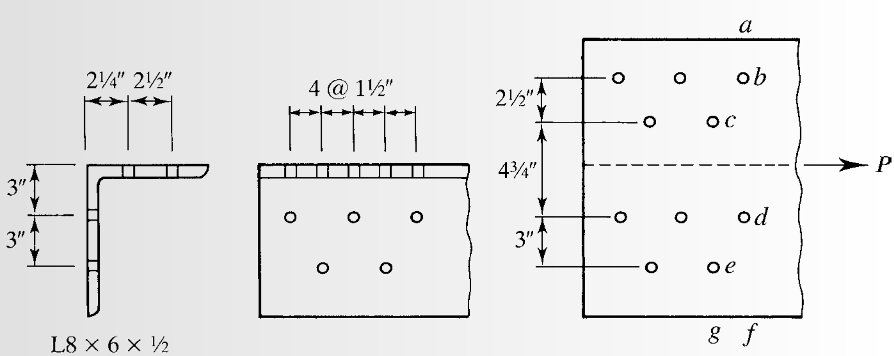
$$= 6.80 - 0.5(1.0) \times 2 = 5.80 \text{ in.}^2$$

For line *abceg*,

$$A_n = 6.80 - 0.5(1.0) - 0.5\left[1.0 - \frac{(1.5)^2}{4(2.5)}\right] - 0.5(1.0) = 5.413 \text{ in.}^2$$

Because 1⁄10 of the load has been transferred from the member by the fastener at *d*, this potential failure line must resist only 9⁄10 of the load. Therefore, the net area

FIGURE 3.17



of 5.413 in.^{2} should be multiplied by $^{10}/_9$ to obtain a net area that can be compared with those lines that resist the full load. Use $A_n = 5.413(^{10}/_9) = 6.014$ in.^{2} For line *abcdeg,*

$$g_{cd} = 3 + 2.25 - 0.5 = 4.75 \text{ in.}$$

$$A_n = 6.80 - 0.5(1.0) - 0.5\left[1.0 - \frac{(1.5)^2}{4(2.5)}\right] - 0.5\left[1.0 - \frac{(1.5)^2}{4(4.75)}\right] - 0.5\left[1.0 - \frac{(1.5)^2}{4(3)}\right]$$

$$= 5.065 \text{ in.}^2$$

The last case controls; use

$$A_n = 5.065 \text{ in.}^2$$

Both legs of the angle are connected, so

$$A_e = A_n = 5.065 \text{ in.}^2$$

The nominal strength based on fracture is

$$P_n = F_u A_e = 58(5.065) = 293.8 \text{ kips}$$

The nominal strength based on yielding is

$$P_n = F_y A_g = 36(6.80) = 244.8 \text{ kips}$$

a. The design strength based on fracture is

$$\phi_t P_n = 0.75(293.8) = 220 \text{ kips}$$

The design strength based on yielding is

$$\phi_t P_n = 0.90(244.8) = 220 \text{ kips}$$

ANSWER Design strength = 220 kips.

b. For the limit state of fracture, the allowable stress is

$$F_t = 0.5F_u = 0.5(58) = 29.0 \text{ ksi}$$

and the allowable strength is

$$F_t A_e = 29.0(5.065) = 147 \text{ kips}$$

For yielding,

$$F_t = 0.6F_y = 0.6(36) = 21.6 \text{ ksi}$$

$$F_t A_g = 21.6(6.80) = 147 \text{ kips}$$

ANSWER Allowable strength = 147 kips.

EXAMPLE 3.8

Determine the smallest net area for the American Standard Channel shown in Figure 3.18. The holes are for ⅝-inch-diameter bolts.

SOLUTION

$$A_n = A_g - \sum t_w \times (d \text{ or } d')$$

$$d = \text{bolt diameter} + \frac{1}{8} = \frac{5}{8} + \frac{1}{8} = \frac{3}{4} \text{ in.}$$

Line *abe:*

$$A_n = A_g - t_w d = 3.82 - 0.437\left(\frac{3}{4}\right) = 3.49 \text{ in.}^2$$

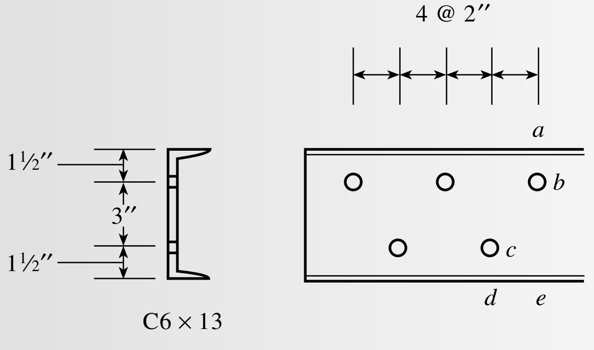
Line *abcd:*

$$A_n = A_g - t_w(d \text{ for hole at } b) - t_w(d' \text{ for hole at } c)$$

$$= 3.82 - 0.437\left(\frac{3}{4}\right) - 0.437\left[\frac{3}{4} - \frac{(2)^2}{4(3)}\right] = 3.31 \text{ in.}^2$$

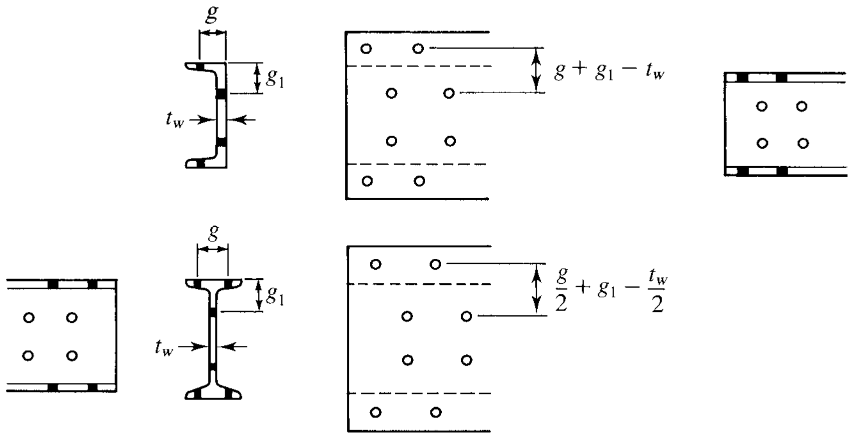
ANSWER Smallest net area = 3.31 in.^{2}

FIGURE 3.18



When staggered holes are present in shapes other than angles, and the holes are in different elements of the cross section, the shape can still be visualized as a plate, even if it is an I-shape. The AISC Specification furnishes no guidance for gage lines crossing a "fold" when the different elements have different thicknesses. A method for handling this case is illustrated in Figure 3.19. In Example 3.8, all of the holes are in one element of the cross section, so this difficulty does not arise. Example 3.9 illustrates the case of staggered holes in different elements of an S-shape.

FIGURE 3.19



EXAMPLE 3.9

Find the available strength of the S-shape shown in Figure 3.20. The holes are for ¾-inch-diameter bolts. Use A36 steel.

SOLUTION Compute the net area:

$$A_n = A_g - \sum t \times (d \text{ or } d')$$

$$\text{Effective hole diameter} = \frac{3}{4} + \frac{1}{8} = \frac{7}{8}$$

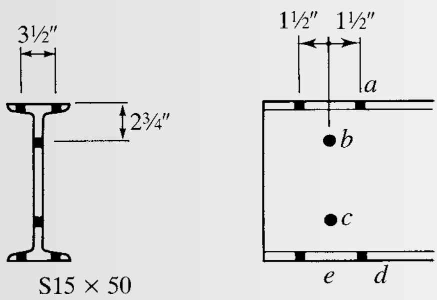
For line *ad*,

$$A_n = 14.7 - 4\left(\frac{7}{8}\right)(0.622) = 12.52 \text{ in.}^2$$

For line *abcd*, the gage distance for use in the $s^2/4g$ term is

$$\frac{g}{2} + g_1 - \frac{t_w}{2} = \frac{3.5}{2} + 2.75 - \frac{0.550}{2} = 4.225 \text{ in.}$$

FIGURE 3.20



Starting at a and treating the holes at b and d as the staggered holes gives

$$A_n = A_g - \sum t \times (d \text{ or } d')$$

$$= 14.7 - 2(0.622)\left(\frac{7}{8}\right) - (0.550)\left[\frac{7}{8} - \frac{(1.5)^2}{4(4.225)}\right]$$

$$-(0.550)\left(\frac{7}{8}\right) - 2(0.622)\left[\frac{7}{8} - \frac{(1.5)^2}{4(4.225)}\right] = 11.73 \text{ in.}^2$$

Line *abcd* controls. As all elements of the cross section are connected,

$$A_e = A_n = 11.73 \text{ in.}^2$$

For the net section, the nominal strength is

$$P_n = F_u A_e = 58(11.73) = 680.3 \text{ kips}$$

For the gross section,

$$P_n = F_y A_g = 36(14.7) = 529.2 \text{ kips}$$

LRFD SOLUTION

The design strength based on fracture is

$$\phi_t P_n = 0.75(680.3) = 510 \text{ kips}$$

The design strength based on yielding is

$$\phi_t P_n = 0.90(529.2) = 476 \text{ kips}$$

Yielding of the gross section controls.

ANSWER Design strength = 476 kips.

ASD SOLUTION

The allowable stress based on fracture is

$$F_t = 0.5F_u = 0.5(58) = 29.0 \text{ ksi}$$

and the corresponding allowable strength is $F_t A_e = 29.0(11.73) = 340$ kips.

The allowable stress based on yielding is

$$F_t = 0.6F_y = 0.6(36) = 21.6 \text{ ksi}$$

and the corresponding allowable strength is $F_t A_g = 21.6(14.7) = 318$ kips.

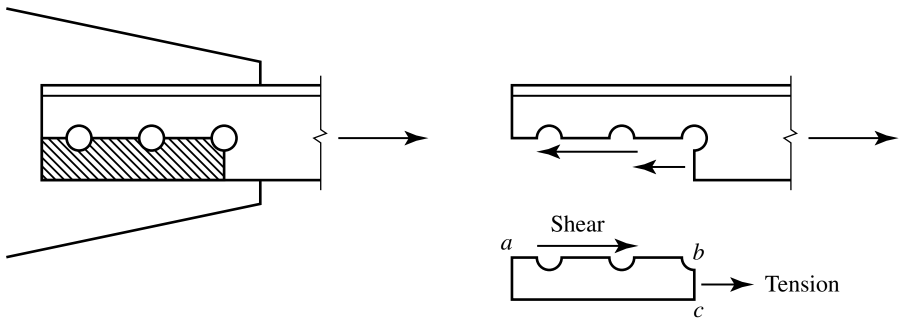
Yielding of the gross section controls.

ANSWER Allowable strength = 318 kips.

3.5 BLOCK SHEAR

For certain connection configurations, a segment or "block" of material at the end of the member can tear out. For example, the connection of the single-angle tension member shown in Figure 3.21 is susceptible to this phenomenon, called *block shear*.

FIGURE 3.21

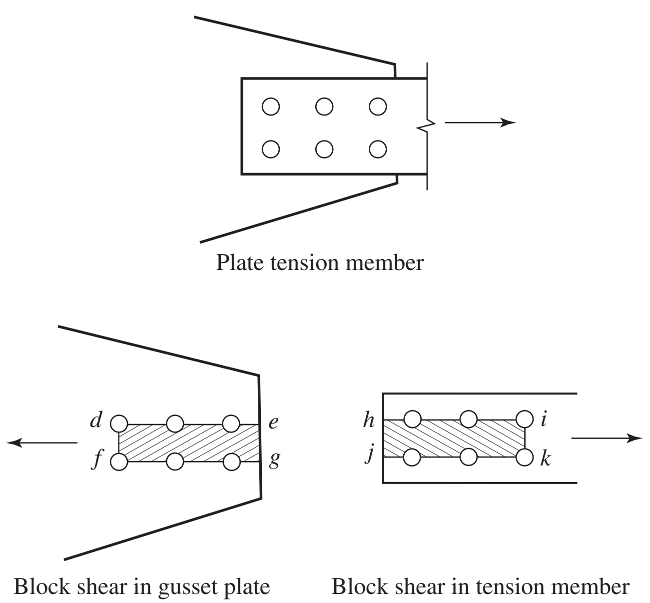


For the case illustrated, the shaded block would tend to fail by shear along the longitudinal section *ab* and by tension on the transverse section *bc*.

For certain arrangements of bolts, block shear can also occur in gusset plates. Figure 3.22 shows a plate tension member connected to a gusset plate. In this connection, block shear could occur in both the gusset plate and the tension member. For the gusset plate, tension failure would be along the transverse section *df,* and shear failure would occur on two longitudinal surfaces, *de* and *fg*. Block shear failure in the plate tension member would be tension on *ik* and shear on both *hi* and *jk.* This topic is not covered explicitly in AISC Chapter D ("Design of Members for Tension"), but the introductory user note directs you to Chapter J ("Design of Connections"), Section J4.3, "Block Shear Strength."

The model used in the AISC Specification assumes that failure occurs by rupture (fracture) on the shear area and rupture on the tension area. Both surfaces contribute to the total strength, and the resistance to block shear will be the sum of the strengths of the two surfaces. The shear rupture stress is taken as 60% of the tensile ultimate

FIGURE 3.22



stress, so the nominal strength in shear is $0.6F_uA_{nv}$ and the nominal strength in tension is F_uA_{nt},

where

A_{nv} = net area along the shear surface or surfaces
A_{nt} = net area along the tension surface

This gives a nominal strength of

$$R_n = 0.6F_uA_{nv} + F_uA_{nt} \tag{3.3}$$

The AISC Specification uses Equation 3.3 for angles and gusset plates, but for certain types of coped beam connections (to be covered in Chapter 5), the second term is reduced to account for nonuniform tensile stress. The tensile stress is nonuniform when some rotation of the block is required for failure to occur. For these cases,

$$R_n = 0.6F_uA_{nv} + 0.5F_uA_{nt} \tag{3.4}$$

The AISC Specification limits the $0.6F_uA_{nv}$ term to $0.6F_yA_{gv}$, where

$0.6F_y$ = shear yield stress

A_{gv} = gross area along the shear surface or surfaces

and gives one equation to cover all cases as follows:

$$R_n = 0.6F_uA_{nv} + U_{bs}F_uA_{nt} \le 0.6F_yA_{gv} + U_{bs}F_uA_{nt} \qquad \text{(AISC Equation J4-5)}$$

where U_{bs} = 1.0 when the tension stress is uniform (angles, gusset plates, and most coped beams) and U_{bs} = 0.5 when the tension stress is nonuniform. A nonuniform case is illustrated in the Commentary to the Specification.

For LRFD, the resistance factor ϕ is 0.75, and for ASD, the safety factor Ω is 2.00. Recall that these are the factors used for the fracture—or rupture—limit state, and block shear is a rupture limit state.

Although AISC Equation J4-5 is expressed in terms of bolted connections, block shear can also occur in welded connections, especially in gusset plates.

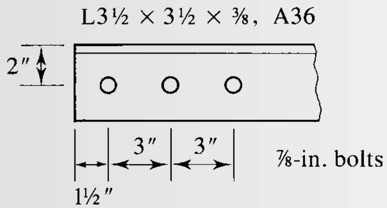
EXAMPLE 3.10

Compute the block shear strength of the tension member shown in Figure 3.23. The holes are for ⅞-inch-diameter bolts, and A36 steel is used.

a. Use LRFD.

b. Use ASD.

FIGURE 3.23



SOLUTION The shear areas are

$$A_{gv} = \frac{3}{8}(7.5) = 2.813 \text{ in.}^2$$

and, since there are 2.5 hole diameters,

$$A_{nv} = \frac{3}{8}\left[7.5 - 2.5\left(\frac{7}{8} + \frac{1}{8}\right)\right] = 1.875 \text{ in.}^2$$

The tension area is

$$A_{nt} = \frac{3}{8}\left[1.5 - 0.5\left(\frac{7}{8} + \frac{1}{8}\right)\right] = 0.3750 \text{ in.}^2$$

(The factor of 0.5 is used because there is one-half of a hole diameter in the tension section.)

Since the block shear will occur in an angle, $U_{bs} = 1.0$, and from AISC Equation J4-5,

$$\begin{aligned} R_n &= 0.6F_uA_{nv} + U_{bs}F_uA_{nt} \\ &= 0.6(58)(1.875) + 1.0(58)(0.3750) = 87.00 \text{ kips} \end{aligned}$$

with an upper limit of

$$0.6F_yA_{gv} + U_{bs}F_uA_{nt} = 0.6(36)(2.813) + 1.0(58)(0.3750) = 82.51 \text{ kips}$$

The nominal block shear strength is therefore 82.51 kips.

ANSWER a. The design strength for LRFD is $\phi R_n = 0.75(82.51) = 61.9$ kips.

b. The allowable strength for ASD is $\dfrac{R_n}{\Omega} = \dfrac{82.51}{2.00} = 41.3$ kips.

3.6 DESIGN OF TENSION MEMBERS

The design of a tension member involves finding a member with adequate gross and net areas. If the member has a bolted connection, the selection of a suitable cross section requires an accounting for the area lost because of holes. For a member with a rectangular cross section, the calculations are relatively straightforward. If a rolled shape is to be used, however, the area to be deducted cannot be predicted in advance because the member•s thickness at the location of the holes is not known.

A secondary consideration in the design of tension members is slenderness. If a structural member has a small cross section in relation to its length, it is said to be *slender*. A more precise measure is the slenderness ratio, L/r, where L is the member length and r is the minimum radius of gyration of the cross-sectional area. The minimum radius of gyration is the one corresponding to the minor principal axis of the cross section. This value is tabulated for all rolled shapes in the properties tables in Part 1 of the *Manual*.

Although slenderness is critical to the strength of a compression member, it is inconsequential for a tension member. In many situations, however, it is good practice to limit the slenderness of tension members. If the axial load in a slender tension member is removed and small transverse loads are applied, undesirable vibrations or deflections might occur. These conditions could occur, for example, in a slack bracing rod subjected to wind loads. For this reason, the user note in AISC D1 suggests a maximum slenderness ratio of 300. It is only a recommended value because slenderness has no structural significance for tension members, and the limit may be exceeded when special circumstances warrant it. This limit does not apply to cables, and the user note explicitly excludes rods.

The central problem of all member design, including tension member design, is to find a cross section for which the required strength does not exceed the available strength. For tension members designed by LRFD, the requirement is

$$P_u \le \phi_t P_n \quad \text{or} \quad \phi_t P_n \ge P_u$$

where P_u is the sum of the factored loads. To prevent yielding,

$$0.90 F_y A_g \ge P_u \quad \text{or} \quad A_g \ge \frac{P_u}{0.90 F_y}$$

To avoid fracture,

$$0.75 F_u A_e \ge P_u \quad \text{or} \quad A_e \ge \frac{P_u}{0.75 F_u}$$

For allowable strength design, if we use the allowable *stress* form, the requirement corresponding to yielding is

$$P_a \le F_t A_g$$

and the required gross area is

$$A_g \ge \frac{P_a}{F_t} \quad \text{or} \quad A_g \ge \frac{P_a}{0.6 F_y}$$

For the limit state of fracture, the required effective area is

$$A_e \ge \frac{P_a}{F_t} \quad \text{or} \quad A_e \ge \frac{P_a}{0.5 F_u}$$

The slenderness ratio limitation will be satisfied if

$$r \ge \frac{L}{300}$$

where r is the minimum radius of gyration of the cross section and L is the member length.

EXAMPLE 3.11

A tension member with a length of 5 feet 9 inches must resist a service dead load of 18 kips and a service live load of 52 kips. Select a member with a rectangular cross section. Use A36 steel and assume a connection with one line of ⅞-inch-diameter bolts.

LRFD SOLUTION

$$P_u = 1.2D + 1.6L = 1.2(18) + 1.6(52) = 104.8 \text{ kips}$$

$$\text{Required } A_g = \frac{P_u}{\phi_t F_y} = \frac{P_u}{0.90F_y} = \frac{104.8}{0.90(36)} = 3.235 \text{ in.}^2$$

$$\text{Required } A_e = \frac{P_u}{\phi_t F_u} = \frac{P_u}{0.75F_u} = \frac{104.8}{0.75(58)} = 2.409 \text{ in.}^2$$

Try $t = 1$ in.

$$\text{Required } w_g = \frac{\text{required } A_g}{t} = \frac{3.235}{1} = 3.235 \text{ in.}$$

Try a $1 \times 3\frac{1}{2}$ cross section.

$$A_e = A_n = A_g - A_{\text{hole}}$$

$$= (1 \times 3.5) - \left(\frac{7}{8} + \frac{1}{8}\right)(1) = 2.5 \text{ in.}^2 > 2.409 \text{ in.}^2 \qquad \text{(OK)}$$

Check the slenderness ratio:

$$I_{\min} = \frac{3.5(1)^3}{12} = 0.2917 \text{ in.}^4$$

$$A = 1(3.5) = 3.5 \text{ in.}^2$$

From $I = Ar^2$, we obtain

$$r_{\min} = \sqrt{\frac{I_{\min}}{A}} = \sqrt{\frac{0.2917}{3.5}} = 0.2887 \text{ in.}^2$$

$$\text{Maximum } \frac{L}{r} = \frac{5.75(12)}{0.2887} = 239 < 300 \qquad \text{(OK)}$$

ANSWER Use a PL $1 \times 3\frac{1}{2}$.

ASD SOLUTION

$$P_a = D + L = 18 + 52 = 70.0 \text{ kips}$$

For yielding, $F_t = 0.6F_y = 0.6(36) = 21.6$ ksi, and

$$\text{Required } A_g = \frac{P_a}{F_t} = \frac{70}{21.6} = 3.24 \text{ in.}^2$$

For fracture, $F_t = 0.5F_u = 0.5(58) = 29.0$ ksi, and

$$\text{Required } A_e = \frac{P_a}{F_t} = \frac{70}{29.0} = 2.414 \text{ in.}^2$$

(The rest of the design *procedure* is the same as for LRFD. The numerical results may be different)

Try $t = 1$ in.

$$\text{Required } w_g = \frac{\text{required } A_g}{t} = \frac{3.241}{1} = 3.241 \text{ in.}$$

Try a 1 × 3 ½ cross section.

$$A_e = A_n = A_g - A_{\text{hole}}$$
$$= (1 \times 3.5) - \left(\frac{7}{8} + \frac{1}{8}\right)(1) = 2.5 \text{ in.}^2 > 2.414 \text{ in.}^2 \qquad \text{(OK)}$$

Check the slenderness ratio:

$$I_{\min} = \frac{3.5(1)^3}{12} = 0.2917 \text{ in.}^4$$
$$A = 1(3.5) = 3.5 \text{ in.}^2$$

From $I = Ar^2$, we obtain

$$r_{\min} = \sqrt{\frac{I_{\min}}{A}} = \sqrt{\frac{0.2917}{3.5}} = 0.2887 \text{ in.}^2$$
$$\text{Maximum } \frac{L}{r} = \frac{5.75(12)}{0.2887} = 239 < 300 \qquad \text{(OK)}$$

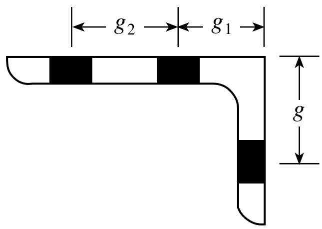
ANSWER Use a PL 1 × 3½.

Example 3.11 illustrates that once the required area has been determined, the procedure is the same for both LRFD and ASD. Note also that in this example, the required areas are virtually the same for LRFD and ASD. This is because the ratio of live load to dead load is approximately 3, and the two approaches will give the same results for this ratio.

The member in Example 3.11 is less than 8 inches wide and thus is classified as a bar rather than a plate. Bars should be specified to the nearest ¼ inch in width and to the nearest ⅛ inch in thickness (the precise classification system is given in Part 1 of the *Manual* under the heading “Plate Products”). It is common practice to use the PL (Plate) designation for both bars and plates.

If an angle shape is used as a tension member and the connection is made by bolting, there must be enough room for the bolts. Space will be a problem only when there are two lines of bolts in a leg. The usual fabrication practice is to punch or drill holes in standard locations in angle legs. These hole locations are given in Table 1-7A in Part 1 of the *Manual*. This table is located at the end of the dimensions and properties table for angles. Figure 3.24 presents this same information. Gage distance g applies when there is one line of bolts, and g_1 and g_2 apply when there are two lines. Figure 3.24 shows that an angle leg must be at least 5 inches long to accommodate two lines of bolts.

FIGURE 3.24



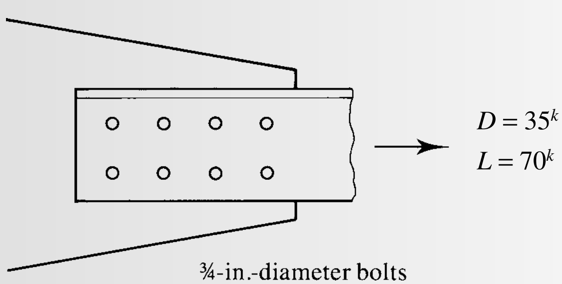
Usual Gages for Angles (inches)

Leg	8	7	6	5	4	3½	3	2½	2	1¾	1½	1⅜	1¼	1
g	4½	4	3½	3	2½	2	1¾	1⅜	1⅛	1	⅞	⅞	¾	⅝
g_1	3	2½	2¼	2										
g_2	3	3	2½	1¾										

EXAMPLE 3.12

Select an unequal-leg angle tension member 15 feet long to resist a service dead load of 35 kips and a service live load of 70 kips. Use A36 steel. The connection is shown in Figure 3.25.

FIGURE 3.25



LRFD SOLUTION

The factored load is

$$P_u = 1.2D + 1.6L = 1.2(35) + 1.6(70) = 154 \text{ kips}$$

$$\text{Required } A_g = \frac{P_u}{\phi_t F_y} = \frac{154}{0.90(36)} = 4.75 \text{ in.}^2$$

$$\text{Required } A_e = \frac{P_u}{\phi_t F_u} = \frac{154}{0.75(58)} = 3.54 \text{ in.}^2$$

The radius of gyration should be at least

$$\frac{L}{300} = \frac{15(12)}{300} = 0.6 \text{ in.}$$

To find the lightest shape that satisfies these criteria, we search the dimensions and properties table for the unequal-leg angle that has the smallest acceptable gross area and then check the effective net area. The radius of gyration can be checked by inspection. There are two lines of bolts, so the connected leg must be at least 5 inches long (see the usual gages for angles in Figure 3.24). Starting at either end of the table, we find that the shape with the smallest area that is at least equal to 4.75 in.^{2} is an L6 × 4 × ½ with an area of 4.75 in.^{2} and a minimum radius of gyration of 0.864 in.

Try L6 × 4 × ½.

$$A_n = A_g - A_{\text{holes}} = 4.75 - 2\left(\frac{3}{4} + \frac{1}{8}\right)\left(\frac{1}{2}\right) = 3.875 \text{ in.}^2$$

Because the length of the connection is not known, Equation 3.1 cannot be used to compute the shear lag factor U. Since there are four bolts in the direction of the load, we will use the alternative value of $U = 0.80$.

$$A_e = A_n U = 3.875(0.80) = 3.10 \text{ in.}^2 < 3.54 \text{ in.}^2 \qquad \text{(N.G.)}$$

Try the next larger shape from the dimensions and properties tables.

Try L5 × 3½ × ⅝ ($A_g = 4.93$ in.^{2} and $r_{min} = 0.746$ in.)

$$A_n = A_g - A_{\text{holes}} = 4.93 - 2\left(\frac{3}{4} + \frac{1}{8}\right)\left(\frac{5}{8}\right) = 3.836 \text{ in.}^2$$

$$A_e = A_n U = 3.836(0.80) = 3.07 \text{ in.}^2 < 3.54 \text{ in.}^2 \qquad \text{(N.G.)}$$

(Note that this shape has slightly more gross area than that produced by the previous trial shape, but because of the greater leg thickness, slightly more area is deducted for the holes.) Passing over the next few heavier shapes,

Try L8 × 4 × ½ ($A_g = 5.80$ in.^{2} and $r_{min} = 0.863$ in.)

$$A_n = A_g - A_{\text{holes}} = 5.80 - 2\left(\frac{3}{4} + \frac{1}{8}\right)\left(\frac{1}{2}\right) = 4.925 \text{ in.}^2$$

$$A_e = A_n U = 4.925(0.80) = 3.94 \text{ in.}^2 > 3.54 \text{ in.}^2 \qquad \text{(OK)}$$

*The notation N.G. means "No Good."

ANSWER This shape satisfies all requirements, so use an L8 × 4 × ½.

ASD SOLUTION The total service load is

$$P_a = D + L = 35 + 70 = 105 \text{ kips}$$

$$\text{Required } A_g = \frac{P_a}{F_t} = \frac{P_a}{0.6F_y} = \frac{105}{0.6(36)} = 4.86 \text{ in.}^2$$

$$\text{Required } A_e = \frac{P_a}{0.5F_u} = \frac{105}{0.5(58)} = 3.62 \text{ in.}^2$$

$$\text{Required } r_{\text{min}} = \frac{L}{300} = \frac{15(12)}{300} = 0.6 \text{ in.}$$

Try L8 × 4 × ½ ($A_g = 5.80$ in.^{2} and $r_{\text{min}} = 0.863$ in.). For a shear lag factor U of 0.80,

$$A_n = A_g - A_{\text{holes}} = 5.80 - 2\left(\frac{3}{4} + \frac{1}{8}\right)\left(\frac{1}{2}\right) = 4.925 \text{ in.}^2$$

$$A_e = A_nU = 4.925(0.80) = 3.94 \text{ in.}^2 > 3.62 \text{ in.}^2 \qquad \text{(OK)}$$

ANSWER This shape satisfies all requirements, so use an L8 × 4 × ½.

The ASD solution in Example 3.12 is somewhat condensed, in that some of the discussion in the LRFD solution is not repeated and only the final trial is shown. All essential computations are included, however.

Tables for the Design of Tension Members

Part 5 of the *Manual* contains tables to assist in the design of tension members of various cross-sectional shapes, including Table 5-2 for angles. The use of these tables will be illustrated in the following example.

EXAMPLE 3.13

Design the tension member of Example 3.12 with the aid of the tables in Part 5 of the *Manual*.

LRFD SOLUTION From Example 3.12,

$$P_u = 154 \text{ kips}$$

$$r_{\text{min}} \geq 0.600 \text{ in.}$$

The tables for design of tension members give values of A_g and A_e for various shapes based on the assumption that $A_e = 0.75A_g$. In addition, the corresponding available strengths based on yielding and rupture (fracture) are given. All values available for angles are for A36 steel. Starting with the lighter shapes (the ones with the smaller gross area), we find that an L6 × 4 × ½, with $\phi_t P_n = 154$ kips based on the gross section and $\phi_t P_n = 155$ kips based on the net section, is a possibility. From the dimensions and properties tables in Part 1 of the *Manual,* $r_{\min} = 0.864$ in. To check this selection, we must compute the actual net area. If we assume that $U = 0.80$,

$$A_n = A_g - A_{\text{holes}} = 4.75 - 2\left(\frac{3}{4} + \frac{1}{8}\right)\left(\frac{1}{2}\right) = 3.875 \text{ in.}^2$$

$$A_e = A_n U = 3.875(0.80) = 3.10 \text{ in.}^2$$

$$\phi_t P_n = \phi_t F_u A_e = 0.75(58)(3.10) = 135 \text{ kips} < 154 \text{ kips} \qquad \text{(N.G.)}$$

This shape did not work because the ratio of actual effective net area A_e to gross area A_g is not equal to 0.75. The ratio is closer to

$$\frac{3.10}{4.75} = 0.6526$$

This corresponds to a required $\phi_t P_n$ (based on rupture) of

$$\frac{0.75}{\text{actual ratio}} \times P_u = \frac{0.75}{0.6526}(154) = 177 \text{ kips}$$

Try an L8 × 4 × ½, with $\phi_t P_n = 188$ kips (based on yielding) and $\phi_t P_n = 189$ Kips (based on rupture strength, with $A_e = 0.75A_g = 4.31$ in.^{2}). From the dimensions and properties tables in Part 1 of the *Manual,* $r_{\min} = 0.863$ in. The actual effective net area and rupture strength are computed as follows:

$$A_n = A_g - A_{\text{holes}} = 5.80 - 2\left(\frac{3}{4} + \frac{1}{8}\right)\left(\frac{1}{2}\right) = 4.925 \text{ in.}^2$$

$$A_e = A_n U = 4.925(0.80) = 3.94 \text{ in.}^2$$

$$\phi_t P_n = \phi_t F_u A_e = 0.75(58)(3.94) = 171 > 154 \text{ kips} \qquad \text{(OK)}$$

ANSWER Use an L8 × 4 × ½, connected through the 8-inch leg.

ASD SOLUTION From Example 3.12,

$$P_a = 105 \text{ kips}$$

$$\text{Required } r_{\min} = 0.600 \text{ in.}$$

From *Manual* Table 5-2, try an L5 × 3½ × ⅝, with $P_n/\Omega_t = 106$ kips based on yielding of the gross section and $P_n/\Omega_t = 107$ kips based on rupture of the net section. From the dimensions and properties tables in Part 1 of the *Manual,* $r_{\min} = 0.746$ in.

Using a shear lag factor U of 0.80, the actual effective net area is computed as follows:

$$A_n = A_g - A_{\text{holes}} = 4.93 - 2\left(\frac{3}{4} + \frac{1}{8}\right)\left(\frac{5}{8}\right) = 3.836 \text{ in.}^2$$

$$A_e = A_n U = 3.836(0.80) = 3.069 \text{ in.}^2$$

and the allowable strength based on rupture of the net section is

$$\frac{P_n}{\Omega_t} = \frac{F_u A_e}{\Omega_t} = \frac{58(3.069)}{2.00} = 89.0 \text{ kips} < 105 \text{ kips} \qquad \text{(N.G.)}$$

This shape did not work because the ratio of actual effective net area A_e to gross area A_g is not equal to 0.75. The ratio is closer to

$$\frac{3.069}{4.93} = 0.6225$$

This corresponds to a required P_n/Ω_t (based on rupture), for purposes of using Table 5-2, of

$$\frac{0.75}{0.6225}(105) = 127 \text{ kips}$$

Using this as a guide, try L6 × 4 × ⅝, with $P_n/\Omega_t = 126$ kips based on yielding of the gross section and $P_n/\Omega_t = 128$ kips based on rupture of the net section. From the dimensions and properties tables in Part 1 of the *Manual,* $r_{\text{min}} = 0.859$ in.

$$A_n = A_g - A_{\text{holes}} = 5.86 - 2\left(\frac{3}{4} + \frac{1}{8}\right)\left(\frac{5}{8}\right) = 4.766 \text{ in.}^2$$

$$A_e = A_n U = 4.766(0.80) = 3.81 \text{ in.}^2$$

$$\frac{P_n}{\Omega_t} = \frac{F_u A_e}{\Omega_t} = \frac{58(3.81)}{2.00} = 111 \text{ kips} > 105 \text{ kips} \qquad \text{(OK)}$$

ANSWER Use an L6 × 4 × ⅝, connected through the 6-inch leg.

Note that if the effective net area must be computed, the tables do not save much effort. In addition, you must still refer to the dimensions and properties tables to find the radius of gyration. The tables for design do, however, provide all other information in a compact form, and the search may go more quickely.

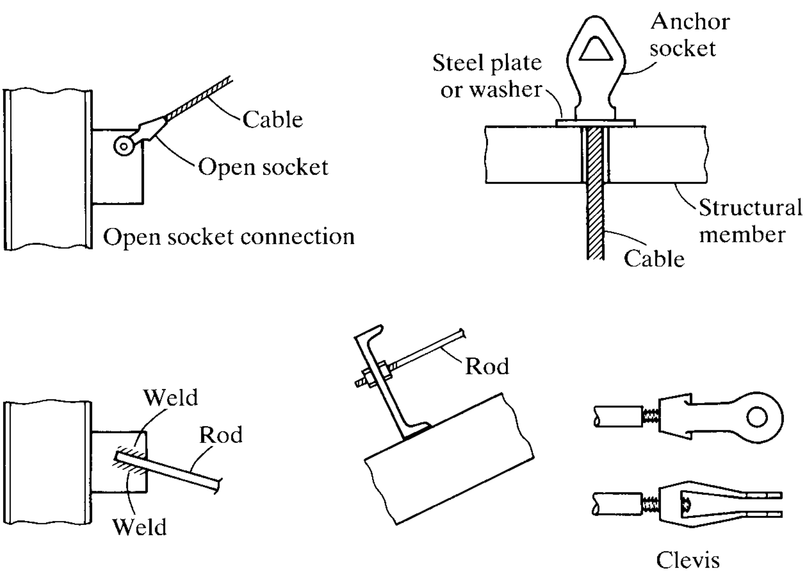
When structural shapes or plates are connected to form a built-up shape, they must be connected not only at the ends of the member but also at intervals along its length. A continuous connection is not required. This type of connection is called *stitching,* and the fasteners used are termed *stitch bolts*. The usual practice is to locate the points of stitching so that L/r for any component part does not exceed L/r for the built-up member. The user note in AISC D4 recommends that built-up shapes whose component parts are separated by intermittent fillers be connected at intervals such that the maximum L/r for any component does not exceed 300. Built-up shapes consisting of plates or a combination of plates and shapes are addressed in AISC Section J3.5 of Chapter J ("Design of Connections"). In general, the spacing of fasteners or welds should not exceed 24 times the thickness of the thinner plate, or 12 inches. If the member is of "weathering" steel subject to atmospheric corrosion, the maximum spacing is 14 times the thickness of the thinner part, or 7 inches.

3.7 THREADED RODS AND CABLES

When slenderness is not a consideration, rods with circular cross sections and cables are often used as tension members. The distinction between the two is that rods are solid and cables are made from individual strands wound together in ropelike fashion. Rods and cables are frequently used in suspended roof systems and as hangers or suspension members in bridges. Rods are also used in bracing systems; in some cases, they are pretensioned to prevent them from going slack when external loads are removed. Figure 3.26 illustrates typical rod and cable connection methods.

When the end of a rod is to be threaded, an upset end is sometimes used. This is an enlargement of the end in which the threads are to be cut. Threads reduce the cross-sectional area, and upsetting the end produces a larger gross area to start with. Standard upset ends with threads will actually have more net area in the threaded portion than in the unthreaded part. Upset ends are relatively expensive, however, and in most cases unnecessary.

FIGURE 3.26



The effective cross-sectional area in the threaded portion of a rod is called the *stress area* and is a function of the unthreaded diameter and the number of threads per inch. The ratio of stress area to nominal area varies but has a lower bound of approximately 0.75. The nominal tensile strength of the threaded rod can therefore be written as

$$P_n = A_s F_u = 0.75 A_b F_u \tag{3.5}$$

where

A_s = stress area

A_b = nominal (unthreaded) area

The AISC Specification, in Chapter J, presents the nominal strength in a somewhat different form:

$$R_n = F_n A_b \qquad \text{(AISC Equation J3-1)}$$

where R_n is the nominal strength and F_n is given in Table J3.2 as $F_{nt} = 0.75F_u$. This associates the 0.75 factor with the ultimate tensile stress rather than the area, but the result is the same as that given by Equation 3.5.

For LRFD, the resistance factor ϕ is 0.75, so the strength relationship is

$$P_u \le \phi_t P_n \quad \text{or} \quad P_u \le 0.75(0.75 A_b F_u)$$

and the required area is

$$A_b = \frac{P_u}{0.75(0.75F_u)} \tag{3.6}$$

For ASD, the safety factor Ω is 2.00, leading to the requirement

$$P_a \le \frac{P_n}{2.00} \quad \text{or} \quad P_a \le 0.5P_n$$

Using P_n from Equation 3.5, we get

$$P_a \le 0.5(0.75 A_b F_u)$$

If we divide both sides by the area A_b, we obtain the allowable stress

$$F_t = 0.5(0.75F_u) = 0.375F_u \tag{3.7}$$

EXAMPLE 3.14

A threaded rod is to be used as a bracing member that must resist a service tensile load of 2 kips dead load and 6 kips live load. What size rod is required if A36 steel is used?

LRFD SOLUTION

The factored load is

$$P_u = 1.2(2) + 1.6(6) = 12 \text{ kips}$$

From Equation 3.6,

$$\text{Required area } = A_b = \frac{P_u}{0.75(0.75F_u)} = \frac{12}{0.75(0.75)(58)} = 0.3678 \text{ in.}^2$$

From $A_b = \dfrac{d^2}{4}$,

$$\text{Required } d = \sqrt{\frac{4(0.3678)}{}} = 0.684 \text{ in.}$$

ANSWER Use a ¾-inch-diameter threaded rod ($A_b = 0.442$ in.^{2}).

ASD SOLUTION The required strength is

$$P_a = D + L = 2 + 6 = 8 \text{ kips}$$

From Equation 3.7, the allowable tensile stress is

$$F_t = 0.375F_u = 0.375(58) = 21.75 \text{ ksi}$$

and the required area is

$$A_b = \frac{P_a}{F_t} = \frac{8}{21.75} = 0.3678 \text{ in.}^2$$

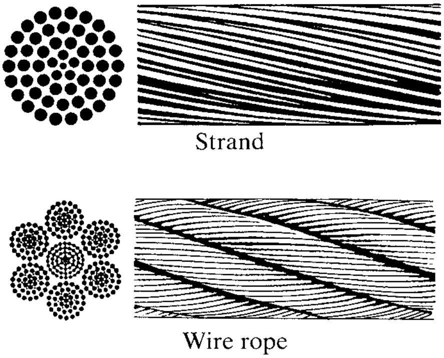
ANSWER Use a ¾-inch-diameter threaded rod ($A_b = 0.442$ in.^{2}).

To prevent damage during construction, rods should not be too slender. Although there is no specification requirement, a common practice is to use a minimum diameter of ⅝ inch.

Flexible cables, in the form of strands or wire rope, are used in applications where high strength is required and rigidity is unimportant. In addition to their use in bridges and cable roof systems, they are also used in hoists and derricks, as guy lines for towers, and as longitudinal bracing in metal building systems. The difference between strand and wire rope is illustrated in Figure 3.27. A strand consists of individual wires wound helically around a central core, and a wire rope is made of several strands laid helically around a core.

Selection of the correct cable for a given loading is usually based on both strength and deformation considerations. In addition to ordinary elastic elongation, an initial stretching is caused by seating or shifting of the individual wires, which results in a permanent stretch. For this reason, cables are often prestretched. Wire rope and strand are made from steels of much higher strength than structural steels and are not covered by the AISC Specification. The breaking strengths of various cables, as well as details of available fixtures for connections, can be obtained from manufacturers• literature.

FIGURE 3.27



3.8 TENSION MEMBERS IN ROOF TRUSSES

Many of the tension members that structural engineers design are components of trusses. For this reason, some general discussion of roof trusses is in order. A more comprehensive treatment of the subject is given by Lothars (1972).

When trusses are used in buildings, they usually function as the main supporting elements of roof systems where long spans are required. They are used when the cost and weight of a beam would be prohibitive. (A truss may be thought of as a deep beam with much of the web removed.) Roof trusses are often used in industrial or mill buildings, although construction of this type has largely given way to rigid frames. Typical roof construction with trusses supported by load-bearing walls is illustrated in Figure 3.28. In this type of construction, one end of the connection of the truss to the walls usually can be considered as pinned and the other as roller-supported. Thus the truss can be analyzed as an externally statically determinate structure. The supporting walls can be reinforced concrete, concrete block, brick, or a combination of these materials.

Roof trusses normally are spaced uniformly along the length of the building and are tied together by longitudinal beams called *purlins* and by x-bracing. The primary function of the purlins is to transfer loads to the top chord of the truss, but they can also act as part of the bracing system. Bracing is usually provided in the planes of both the top and bottom chords, but it is not required in every bay because lateral forces can be transferred from one braced bay to the other through the purlins.

Ideally, purlins are located at the truss joints so that the truss can be treated as a pin-connected structure loaded only at the joints. Sometimes, however, the roof deck cannot span the distance between joints, and intermediate purlins may be needed. In such cases, top chord members will be subjected to significant bending as well as axial compression and must be designed as beam–columns (Chapter 6).

Sag rods are tension members used to provide lateral support for the purlins. Most of the loads applied to the purlins are vertical, so there will be a component parallel to a sloping roof, which will cause the purlin to bend (sag) in that direction (Figure 3.29).

Sag rods can be located at the midpoint, the third points, or at more frequent intervals along the purlins, depending on the amount of support needed. The interval is a function of the truss spacing, the slope of the top chord, the resistance of the purlin

FIGURE 3.28

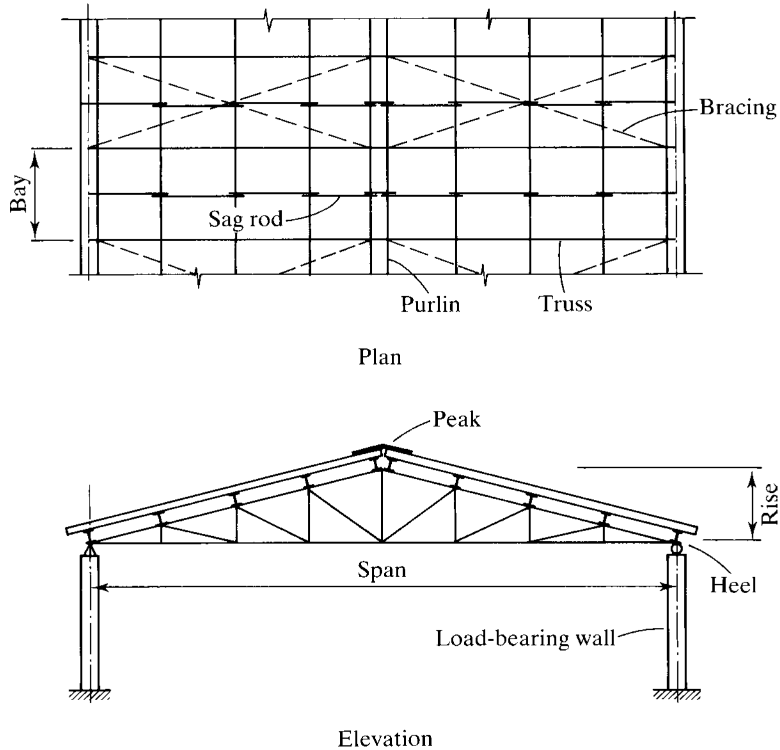
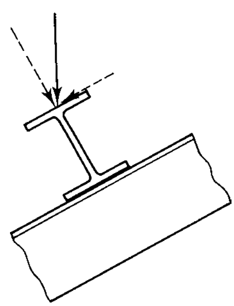


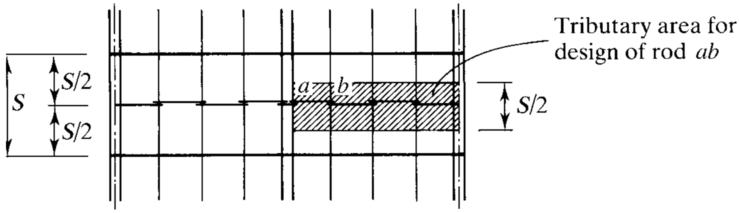
FIGURE 3.29



to this type of bending (most shapes used for purlins are very weak in this respect), and the amount of support furnished by the roofing. If a metal deck is used, it will usually be rigidly attached to the purlins, and sag rods may not be needed. Sometimes, however, the weight of the purlin itself is enough to cause problems, and sag rods may be needed to provide support during construction before the deck is in place.

If sag rods are used, they are designed to support the component of roof loads parallel to the roof. Each segment between purlins is assumed to support everything below it; thus the top rod is designed for the load on the roof area tributary to the rod, from the heel of the truss to the peak, as shown in Figure 3.30. Although the force will be different in each segment of rod, the usual practice is to use one size throughout.

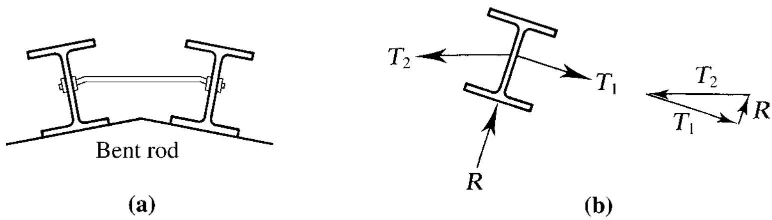
FIGURE 3.30



The extra amount of material in question is insignificant, and the use of the same size for each segment eliminates the possibility of a mix-up during construction.

A possible treatment at the peak or ridge is shown in Figure 3.31a. The tie rod between ridge purlins must resist the load from all of the sag rods on either side. The tensile force in this horizontal member has as one of its components the force in the upper sag-rod segment. A free-body diagram of one ridge purlin illustrates this effect, as shown in Figure 3.31b.

FIGURE 3.31



EXAMPLE 3.15

Fink trusses spaced at 20 feet on centers support W6 × 12 purlins, as shown in Figure 3.32a. The purlins are supported at their midpoints by sag rods. Use A36 steel and design the sag rods and the tie rod at the ridge for the following service loads.

Metal deck:	2 psf
Built-up roof:	5 psf
Snow:	18 psf of horizontal projection of the roof surface
Purlin weight:	12 pounds per foot (lb/ft) of length

SOLUTION Calculate loads.

Tributary width for each sag rod = 20/2 = 10 ft

Tributary area for deck and built-up roof = 10(46.6) = 466 ft^2

Dead load (deck and roof) = (2 + 5)(466) = 3262 lb

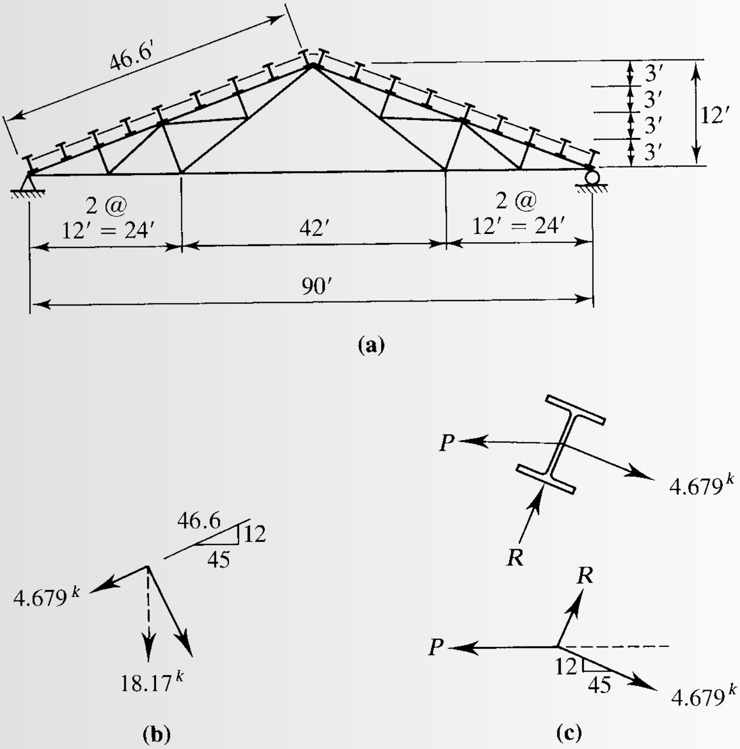
Total purlin weight = 12(10)(9) = 1080 lb

Total dead load = 3262 + 1080 = 4342 lb

Tributary area for snow load = 10(45) = 450 ft^2

Total snow load = 18(450) = 8100 lb

FIGURE 3.32



LRFD SOLUTION

Check load combinations.

Combination 2: $1.2D + 0.5S = 1.2(4342) + 0.5(8100) = 9260$ lb

Combination 3: $1.2D + 1.6S = 1.2(4342) + 1.6(8100) = 18{,}170$ lb

Combination 3 controls. (By inspection, the remaining combinations will not govern.)

For the component parallel to the roof (Figure 3.32b),

$$T = (18.17)\frac{12}{46.6} = 4.679 \text{ kips}$$

$$\text{Required } A_b = \frac{T}{\phi_t(0.75F_u)} = \frac{4.679}{0.75(0.75)(58)} = 0.1434 \text{ in.}^2$$

ANSWER

Use a ⅝-inch-diameter threaded rod ($A_b = 0.3068$ in.^{2}).

Tie rod at the ridge (Figure 3.32c):

$$P = (4.679)\frac{46.6}{45} = 4.845 \text{ kips}$$

$$\text{Required } A_b = \frac{4.845}{0.75(0.75)(58)} = 0.1485 \text{ in.}^2$$

ANSWER Use a ⅝-inch-diameter threaded rod ($A_b = 0.3068$ in.^{2}).

ASD SOLUTION By inspection, load combination 3 will control.

$$D + S = 4342 + 8100 = 12{,}440 \text{ lb}$$

The component parallel to the roof is

$$T = 12.44\left(\frac{12}{46.6}\right) = 3.203 \text{ kips}$$

The allowable tensile stress is $F_t = 0.375F_u = 0.375(58) = 21.75$ ksi.

$$\text{Required } A_b = \frac{T}{F_t} = \frac{3.203}{21.75} = 0.1473 \text{ in.}^2$$

ANSWER Use a ⅝-inch-diameter threaded rod ($A_b = 0.3068$ in.^{2}) for the sag rods.
Tie rod at the ridge:

$$P = 3.203\left(\frac{46.6}{45}\right) = 3.317 \text{ kips}$$

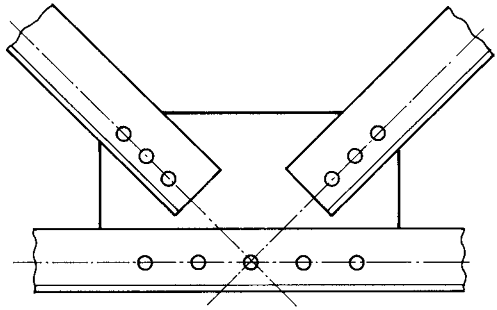
$$\text{Required } A_b = \frac{3.317}{21.75} = 0.1525 \text{ in.}^2$$

ANSWER Use a ⅝-inch-diameter threaded rod ($A_b = 0.3068$ in.^{2}) for the tie rod at the ridge.

For the usual truss geometry and loading, the bottom chord will be in tension and the top chord will be in compression. Some web members will be in tension and others will be in compression. When wind effects are included and consideration is given to different wind directions, the force in some web members may alternate between tension and compression. In this case, the affected member must be designed to function as both a tension member and a compression member.

In bolted trusses, double-angle sections are frequently used for both chord and web members. This design facilitates the connection of members meeting at a joint by permitting the use of a single gusset plate, as illustrated in Figure 3.33. When structural tee-shapes are used as chord members in welded trusses, the web angles can usually be welded to the stem of the tee. If the force in a web member is small, single angles can be used, although doing so eliminates the plane of symmetry from the truss and causes the web member to be eccentrically loaded. Chord members are usually fabricated as continuous pieces and spliced if necessary.

FIGURE 3.33



The fact that chord members are continuous and joints are bolted or welded would seem to invalidate the usual assumption that the truss is pin-connected. Joint rigidity does introduce some bending moment into the members, but it is usually small and considered to be a secondary effect. The usual practice is to ignore it. Bending caused by loads directly applied to members between the joints, however, must be taken into account. We consider this condition in Chapter 6, "Beam–Columns."

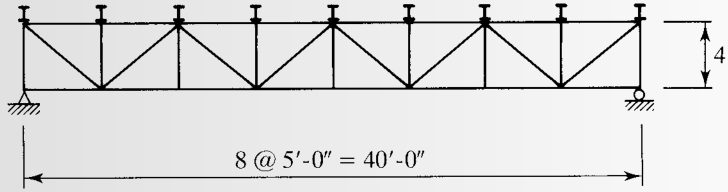
The *working lines* of the members in a properly detailed truss intersect at the *working point* at each joint. For a bolted truss, the bolt lines are the working lines, and in welded trusses the centroidal axes of the welds are the working lines. For truss analysis, member lengths are measured from working point to working point.

EXAMPLE 3.16

Select a structural tee for the bottom chord of the Warren roof truss shown in Figure 3.34. The trusses are welded and spaced at 20 feet. Assume that the bottom chord connection is made with 9-inch-long longitudinal welds at the flange. Use A992 steel and the following load data (wind is not considered in this example):

Purlins:	M8 × 6.5
Snow:	20 psf of horizontal projection
Metal deck:	2 psf
Roofing:	4 psf
Insulation:	3 psf

FIGURE 3.34



SOLUTION Calculate loads:

Snow = 20(40)(20) = 16,000 lb

Dead load (exclusive of purlins) =	Deck	2 psf
	Roof	4
	Insulation	3
	Total	9 psf

Total dead load = 9(40)(20) = 7200 lb

Total purlin weight = 6.5(20)(9) = 1170 lb

Estimate the truss weight as 10% of the other loads:

0.10(16,000 + 7200 + 1170) = 2437 lb

Loads at an interior joint are

$$D = \frac{7200}{8} + \frac{2437}{8} + 6.5(20) = 1335 \text{ lb}$$

$$S = \frac{16{,}000}{8} = 2000 \text{ lb}$$

At an exterior joint, the tributary roof area is half of that at an interior joint. The corresponding loads are

$$D = \frac{7200}{2(8)} + \frac{2437}{2(8)} + 6.5(20) = 732.3 \text{ lb}$$

$$S = \frac{16{,}000}{2(8)} = 1000 \text{ lb}$$

LRFD SOLUTION Load combination 3 will control:

$$P_u = 1.2D + 1.6S$$

At an interior joint,

$$P_u = 1.2(1.335) + 1.6(2.0) = 4.802 \text{ kips}$$

At an exterior joint,

$$P_u = 1.2(0.7323) + 1.6(1.0) = 2.479 \text{ kips}$$

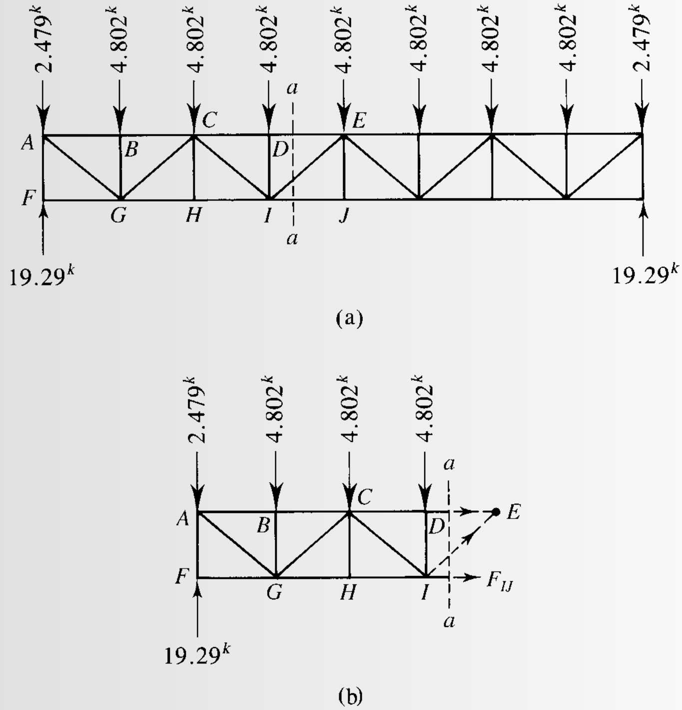
The loaded truss is shown in Figure 3.35a.

The bottom chord is designed by determining the force in each member of the bottom chord and selecting a cross section to resist the largest force. In this example, the force in member *IJ* will control. For the free body left of section *a–a* shown in Figure 3.35b,

$$\sum M_E = 19.29(20) - 2.479(20) - 4.802(15 + 10 + 5) - 4F_{IJ} = 0$$

$$F_{IJ} = 48.04 \text{ kips}$$

FIGURE 3.35



For the gross section,

$$\text{Required } A_g = \frac{F_{IJ}}{0.90F_y} = \frac{48.04}{0.90(50)} = 1.07 \text{ in.}^2$$

For the net section,

$$\text{Required } A_e = \frac{F_{IJ}}{0.75F_u} = \frac{48.04}{0.75(65)} = 0.985 \text{ in.}^2$$

Try an MT5 × 3.75:

$$A_g = 1.11 \text{ in.}^2 > 1.07 \text{ in.}^2 \qquad \text{(OK)}$$

Compute the shear lag factor U from Equation 3.1.

$$U = 1 - \left(\frac{\bar{x}}{\ell}\right) = 1 - \left(\frac{1.51}{9}\right) = 0.8322$$

$$A_e = A_gU = 1.11(0.8322) = 0.924 \text{ in.}^2 < 0.985 \text{ in.}^2 \quad \text{(N.G.)}$$

Try an MT6 × 5:

$$A_g = 1.48 \text{ in.}^2 > 1.07 \text{ in.}^2 \quad \text{(OK)}$$

$$U = 1 - \left(\frac{\bar{x}}{\ell}\right) = 1 - \left(\frac{1.86}{9}\right) = 0.7933$$

$$A_e = A_gU = 1.48(0.7933) = 1.17 \text{ in.}^2 > 0.985 \text{ in.}^2 \qquad \text{(OK)}$$

If we assume that the bottom chord is braced at the panel points,

$$\frac{L}{r} = \frac{5(12)}{0.594} = 101 < 300 \qquad \text{(OK)}$$

ANSWER Use an MT6 × 5.

ASD SOLUTION Load combination 3 will control. At an interior joint,

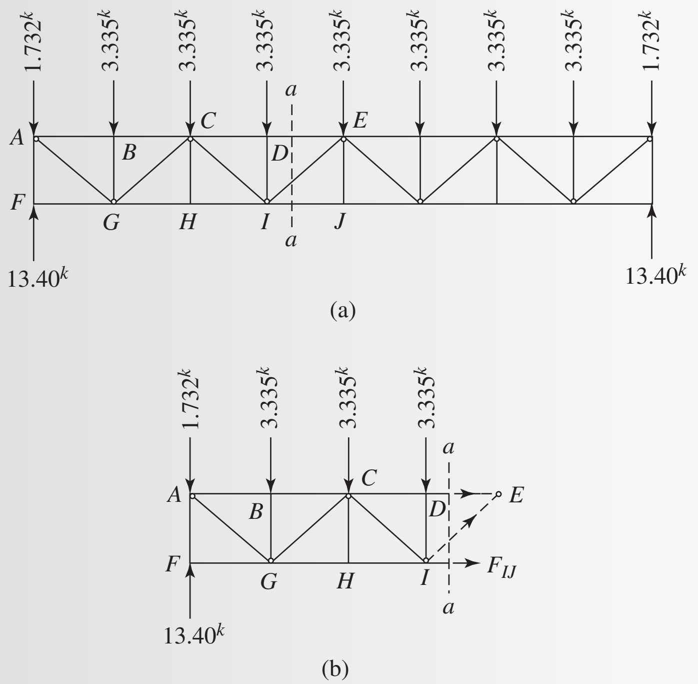
$$P_a = D + S = 1.335 + 2.0 = 3.335 \text{ kips}$$

At an exterior joint,

$$P_a = 0.7323 + 1.0 = 1.732 \text{ kips}$$

The loaded truss is shown in Figure 3.36a.

FIGURE 3.36



Member *IJ* is the bottom chord member with the largest force. For the free body shown in Figure 3.36b,

$$\sum M_E = 13.40(20) - 1.732(20) - 3.335(15 + 10 + 5) - 4F_{IJ} = 0$$

$$F_{IJ} = 33.33 \text{ kips}$$

For the gross section, $F_t = 0.6F_y = 0.6(36) = 21.6$ ksi

$$\text{Required } A_g = \frac{F_{IJ}}{F_t} = \frac{33.33}{21.6} = 1.54 \text{ in.}^2$$

For the net section, $F_t = 0.5F_u = 0.5(58) = 29.0$ ksi

$$\text{Required } A_e = \frac{F_{IJ}}{F_t} = \frac{33.33}{29.0} = 1.15 \text{ in.}^2$$

Try an MT6 × 5.4:

$$A_g = 1.59 \text{ in.}^2 > 1.54 \text{ in.}^2 \qquad \text{(OK)}$$

$$U = 1 - \frac{\bar{x}}{\ell} = 1 - \frac{1.86}{9} = 0.7933$$

$$A_e = A_g U = 1.59(0.7933) = 1.26 \text{ in.}^2 > 1.15 \text{ in.}^2 \qquad \text{(OK)}$$

Assuming that the bottom chord is braced at the panel points, we get

$$\frac{L}{r} = \frac{5(12)}{0.566} = 106 < 300 \qquad \text{(OK)}$$

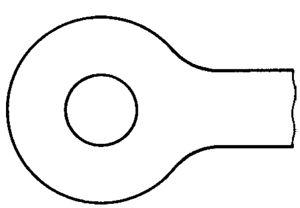
ANSWER Use an MT6 × 5.4.

3.9 PIN-CONNECTED MEMBERS

When a member is to be pin-connected, a hole is made in both the member and the parts to which it is connected and a pin is placed through the holes. This provides a connection that is as moment-free as can be fabricated. Tension members connected in this manner are subject to several types of failure, which are covered in AISC D5 and D6 and discussed in the following paragraphs.

The eyebar is a special type of pin-connected member in which the end containing the pin hole is enlarged, as shown in Figure 3.37. The design strength is based on yielding of the gross section. Detailed rules for proportioning eyebars are given in AISC D6 and are not repeated here. These requirements are based on experience and test programs for forged eyebars, but they are conservative when applied to eyebars thermally cut from plates (the present fabrication method). Eyebars were widely used in the past as single tension members in bridge trusses or were linked in chainlike fashion in suspension bridges. They are rarely used today.

FIGURE 3.37



Pin-connected members should be designed for the following limit states (see Figure 3.38).

1. **Tension** on the net effective area (Figure 3.38a):

   $\phi_t = 0.75,\ \Omega_t = 2.00, \qquad P_n = F_u(2tb_e)$ (AISC Equation D5-1)

2. **Shear** on the effective area (Figure 3.38b):

   $\phi_{sf} = 0.75,\ \Omega_{sf} = 2.00, \qquad P_n = 0.6F_uA_{sf}$ (AISC Equation D5-2)

3. **Bearing.** This requirement is given in Chapter J ("Connections, Joints, and Fasteners"), Section J7 (Figure 3.38c):

   $\phi = 0.75,\ \Omega = 2.00, \qquad P_n = 1.8F_yA_{pb}$ (AISC Equation J7-1)

4. **Tension** on the gross section:

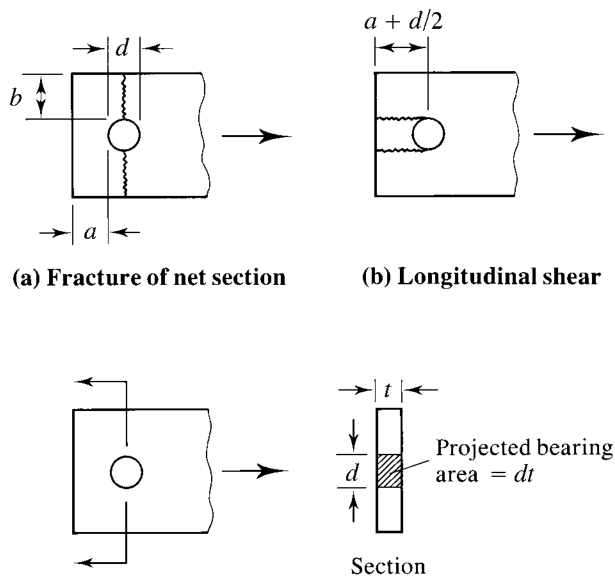
   $\phi_t = 0.90,\ \Omega_t = 1.67, \qquad P_n = F_yA_g$ (AISC Equation D2-1)

where

t = thickness of connected part
b_e = $2t + 0.63 \le b$
b = distance from edge of pin hole to edge of member, perpendicular to direction of force
A_{sf} = $2t(a + d/2)$
a = distance from edge of pin hole to edge of member, parallel to direction of force
d = pin diameter
A_{pb} = projected bearing area = dt

Additional requirements for the relative proportions of the pin and the member are covered in AISC D5.2

FIGURE 3.38



(a) Fracture of net section

(b) Longitudinal shear

(c) Bearing

Problems

Tensile Strength

3.2-1 A PL $\frac{3}{8} \times 7$ tension member is connected with three 1-inch-diameter bolts, as shown in Figure P3.2-1. The steel is A36. Assume that $A_e = A_n$ and compute the following.

a. The design strength for LRFD.

b. The allowable strength for ASD.

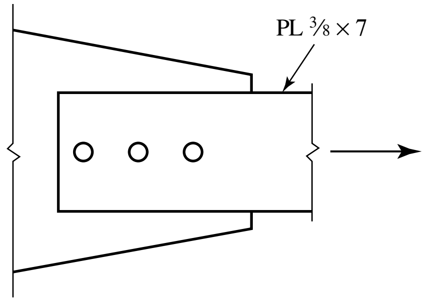


FIGURE P3.2-1

3.2-2 A PL $\frac{1}{2} \times 8$ tension member is connected with six 1-inch-diameter bolts, as shown in Figure P3.2-2. The steel is ASTM A242. Assume that $A_e = A_n$ and compute the following.

a. The design strength for LRFD.

b. The allowable strength for ASD.

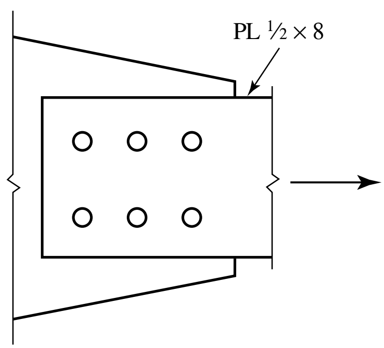


FIGURE P3.2-2

3.2-3 A C12 × 30 is connected with 1-in. diameter bolts in each flange, as shown in Figure P3.2-3. If $F_y = 50$ ksi, $F_u = 65$ ksi, and $A_e = 0.90A_n$, compute the following.

a. The design strength for LRFD.

b. The allowable strength for ASD.

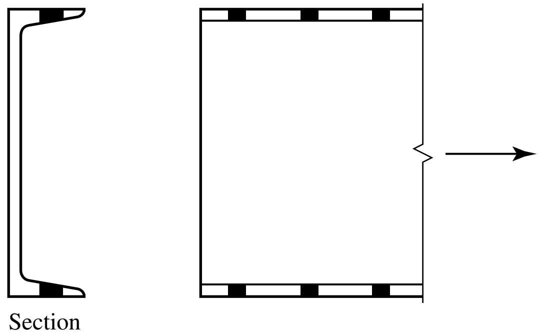


FIGURE P3.2-3

3.2-4 A PL $\frac{3}{8} \times 6$ tension member is welded to a gusset plate as shown in Figure P3.2-4. The steel is A36. Assume that $A_e = A_g$ and compute the following.

a. The design strength for LRFD.

b. The allowable strength for ASD.

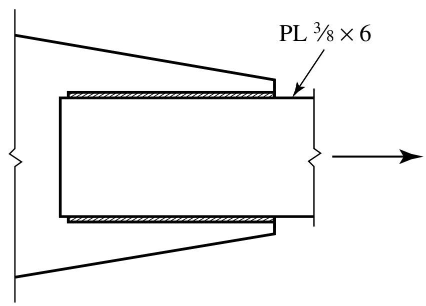


FIGURE P3.2-4

3.2-5 The tension member shown in Figure P3.2-5 is a PL $\frac{1}{2} \times 8$ of A36 steel. The member is connected to a gusset plate with-$1\frac{1}{8}$ inch-diameter bolts. It is subjected to the dead and live loads shown. Does this member have enough strength? Assume that $A_e = A_n$.

a. Use LRFD.

b. Use ASD.

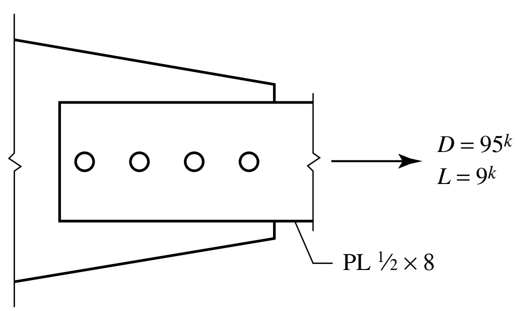


FIGURE P3.2-5

3.2-6 A double-angle tension member, 2L 3 × 2 × ¼ LLBB, of A36 steel is subjected to a dead load of 12 kips and a live load of 36 kips. It is connected to a gusset plate with ¾-inch-diameter bolts through the long legs. Does this member have enough strength? Assume that $A_e = 0.85A_n$.

a. Use LRFD.

b. Use ASD.

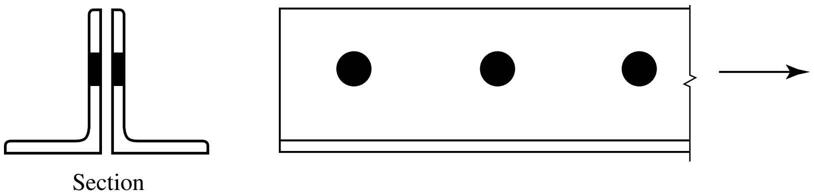


FIGURE P3.2-6

3.2-7 A C8 × 11.5 is connected to a gusset plate with ⅞-inch-diameter bolts as shown in Figure P3.2-7. The steel is A572 Grade 50. If the member is subjected to dead load and live load only, what is the total service load capacity if the live-to-dead load ratio is 3? Assume that $A_e = 0.85A_n$.

a. Use LRFD.

b. Use ASD.

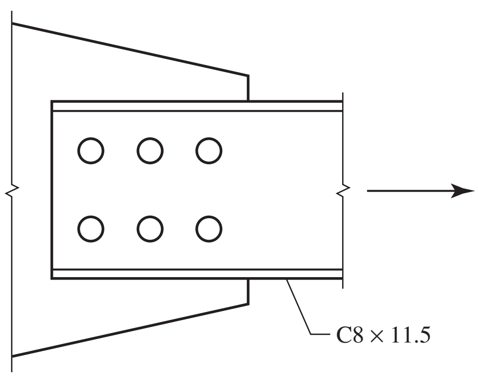


FIGURE P3.2-7

Effective area

3.3-1 Determine the effective area A_e for each case shown in Figure P3.3-1.

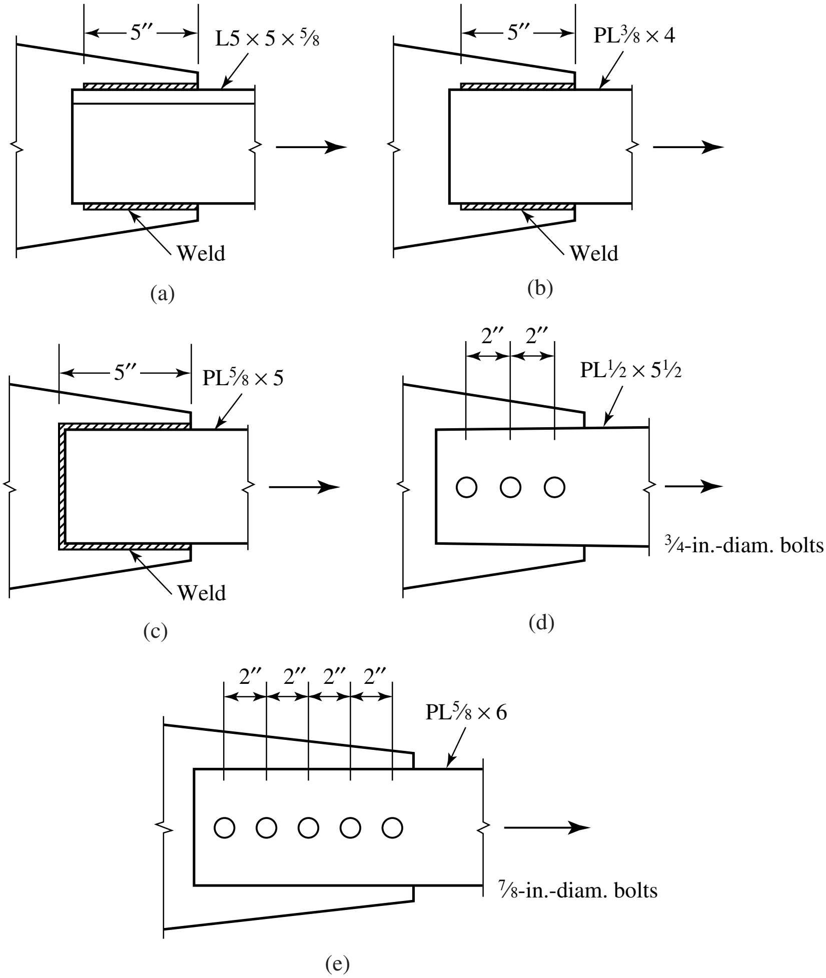


FIGURE P3.3-1

3.3-2 For the tension member shown, compute the following.

a. The tensile design strength.

b. The allowable tensile strength.

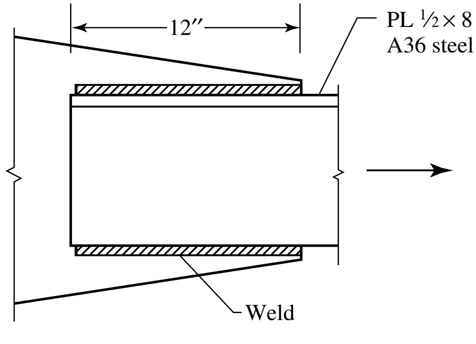


FIGURE P3.3-2

3.3-3 Determine the nominal tensile strength *based on the effective net area*.

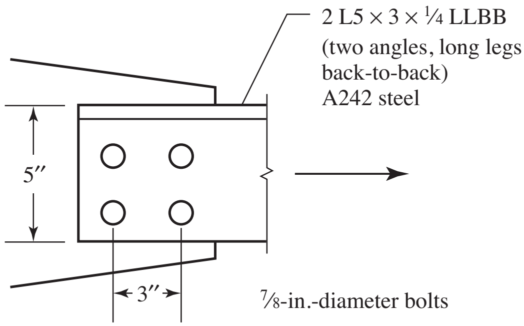


FIGURE P3.3-3

3.3-4 For the tension member shown, compute the following.

a. The tensile design strength.

b. The allowable tensile strength.

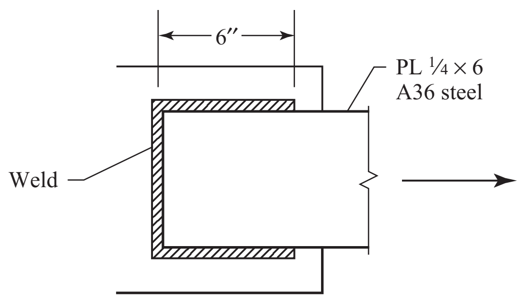


FIGURE P3.3-4

3.3-5 A W16 × 45 of A992 steel is connected to a plate at each flange as shown in Figure P3.3-5. Determine the nominal strength *based on the net section* as follows:

a. Use Equation 3.1 for the shear lag factor, U.

b. Use the alternative value of U from AISC Table D3.1.

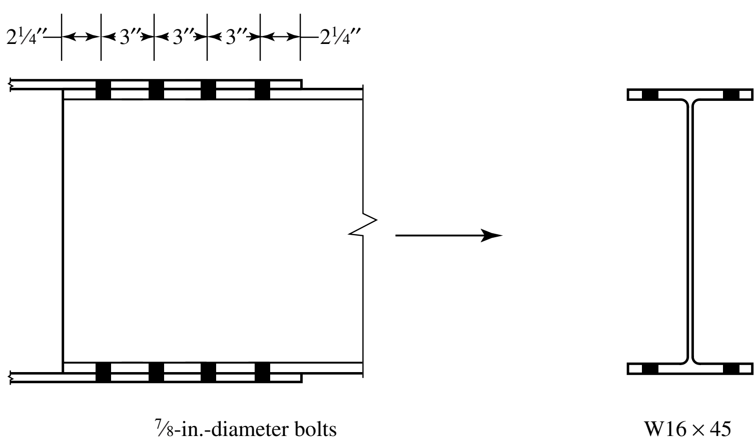


FIGURE P3.3-5

3.3-6 The tension member shown in Figure P3.3-6 is a C12 × 20.7 of A572 Grade 50 steel. Will it safely support a service dead load of 60 kips and a service live load of 125 kips? Use Equation 3.1 for U.

a. Use LRFD.

b. Use ASD.

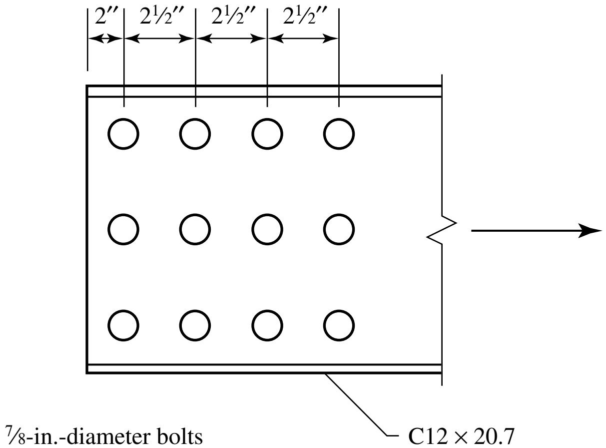


FIGURE P3.3-6

3.3-7 A double-angle tension member, 2L4 × 3 × ¼ LLBB, is connected with welds as shown in Figure P3.3-7. A36 steel is used.

a. Compute the available strength for LRFD.

b. Compute the available strength for ASD.

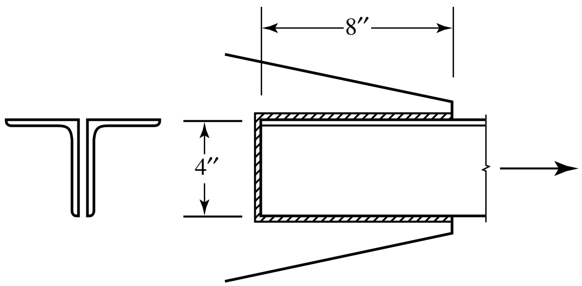


FIGURE P3.3-7

3.3-8 An L5 × 5 × ½ tension member of A242 steel is connected to a gusset plate with six ¾-inch-diameter bolts as shown in Figure P3.3-8. If the member is subject to dead load and live load only, what is the maximum total service load that can be applied if the ratio of live load to dead load is 2.0? Use the alternative value of U from AISC Table D3.1.

a. Use LRFD.

b. Use ASD.

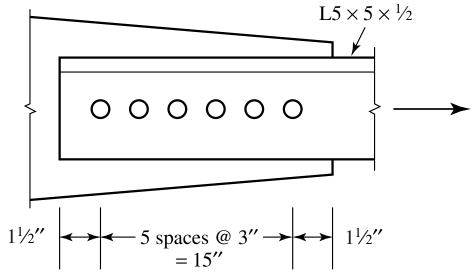


FIGURE P3.3-8

Staggered Fasteners

3.4-1 A36 steel is used for the tension member shown in Figure P3.4-1.

a. Determine the nominal strength based on the gross area.

b. Determine the nominal strength based on the net area.

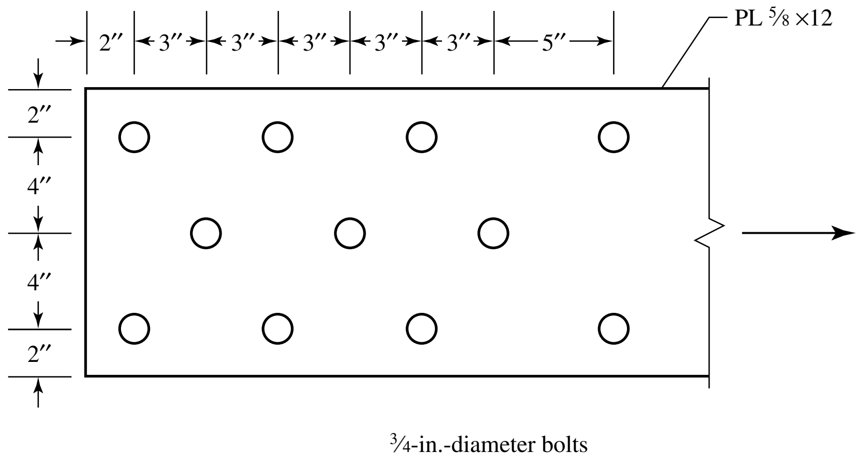


FIGURE P3.4-1

3.4-2 The tension member shown in Figure 3.4-2 is a PL ⅝ × 10, and the steel is A36. The bolts are ⅞-inch in diameter.

a. Determine the design strength for LRFD.

b. Determine the allowable strength for ASD.

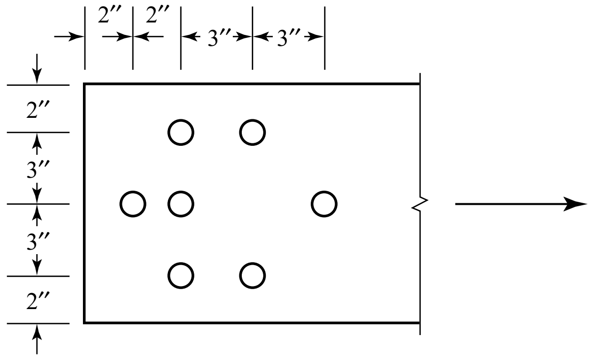


FIGURE P3.4-2

3.4-3 An MC 9 × 23.9 is connected with ¾-inch-diameter bolts as shown in Figure P3.4-3. A572 Grade 50 steel is used.

a. Determine the design strength.

b. Determine the allowable strength.

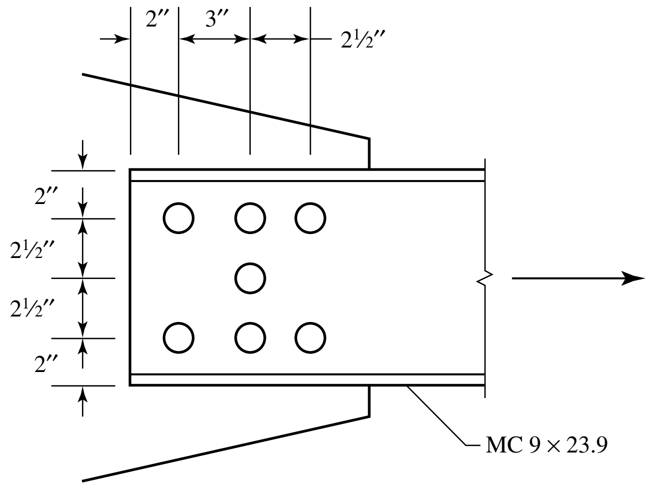


FIGURE P3.4-3

3.4-4 A992 steel is used for the tension member shown in Figure P3.4-4. The bolts are ¾ inch in diameter. The connection is to a ⅜-in.-thick gusset plate.

a. Determine the nominal strength based on the gross area.

b. Determine the nominal strength based on the effective net area.

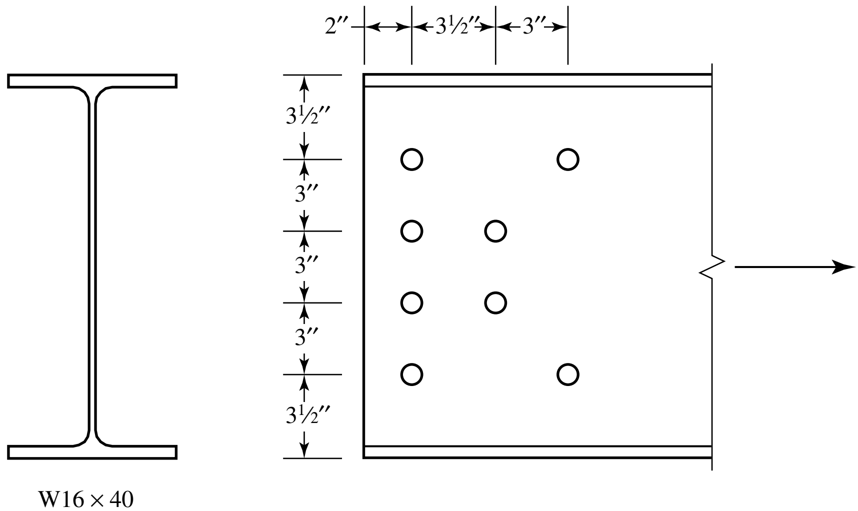


FIGURE P3.4-4

3.4-5 The tension member shown in Figure P3.4-5 is an L6 × 3½ × 5⁄16. The bolts are ¾ inch in diameter. If A36 steel is used, is the member adequate for a service dead load of 31 kips and a service live load of 31 kips?

a. Use LRFD.

b. Use ASD.

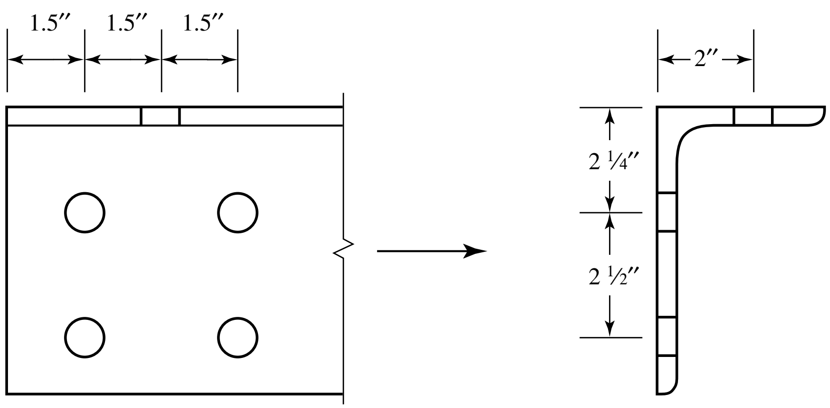


FIGURE P3.4-5

3.4-6 A double-channel shape, 2C10 × 20, of A572 Grade 50 steel is used for a built-up tension member as shown in Figure P3.4-6. The holes are for ½-inch-diameter bolts. Determine the total service load capacity if the live load is three times the dead load.

a. Use LRFD.

b. Use ASD.

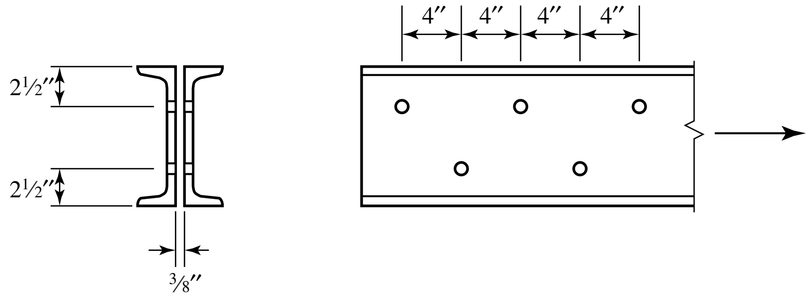


FIGURE P3.4-6

Block Shear

3.5-1 The tension member is a PL⅜ × 5½ of A242 steel. It is connected to a ⅜-in. thick gusset plate, also of A242 steel, with ¾-inch diameter bolts as shown in Figure P3.5-1. Determine the nominal block shear strength of the tension member.

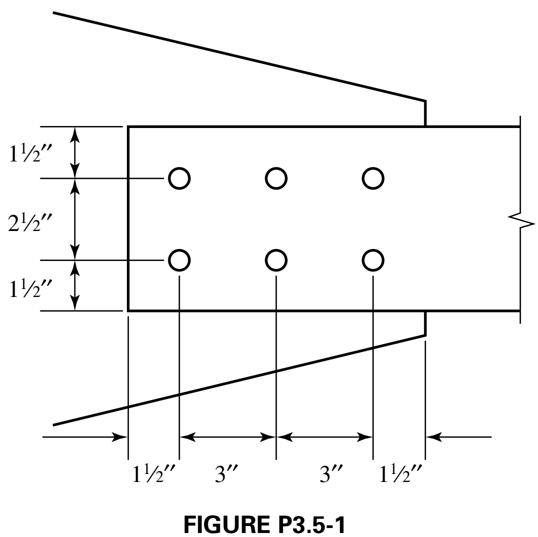


FIGURE P3.5-1

3.5-2 A square hollow structural section (HSS) is used as a tension member and is welded to a gusset plate of A36 steel as shown in Figure P3.5-2. Compute the nominal block shear strength of the gusset plate.

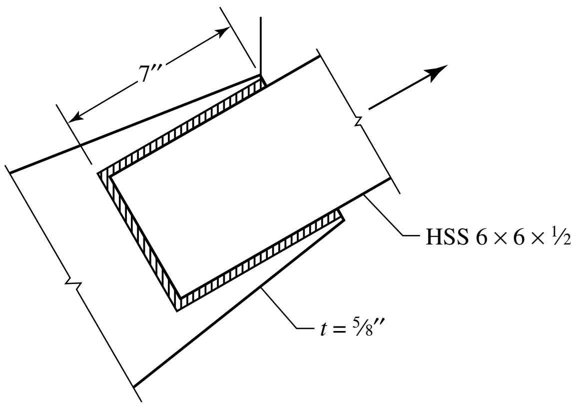


FIGURE P3.5-2

3.5-3 A WT8 × 13 of A992 steel is used as a tension member. The connection is with ⅞-in. diameter bolts as shown in Figure P3.5-3. Compute the nominal block shear strength.

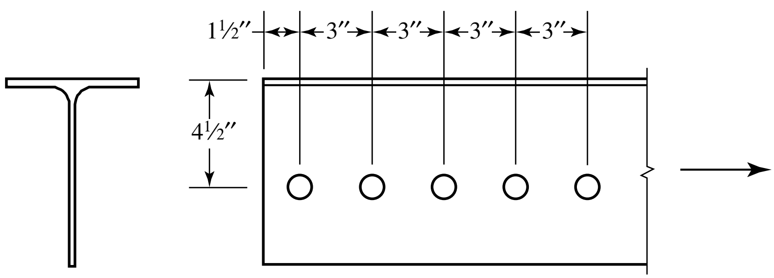


FIGURE P3.5-3

3.5-4 Compute the available block shear strength of the gusset plate.

a. Use LRFD.

b. Use ASD.

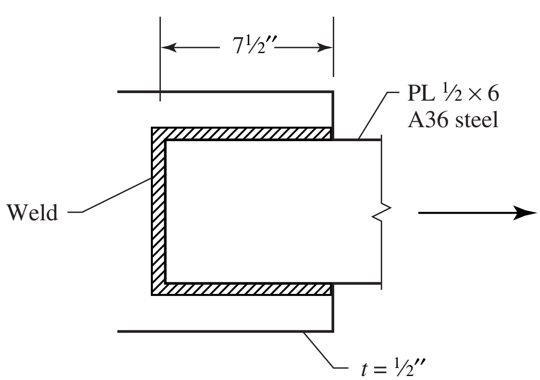


FIGURE P3.5-4

3.5-5 A C7 × 9.8 tension member is connected to a ⅜-in.-thick gusset plate as shown in Figure P3.5-5. Both the member and the gusset plate are A36 steel.

a. Compute the available block shear strength of the tension member for both LRFD and ASD.

b. Compute the available block shear strength of the gusset plate for both LRFD and ASD.

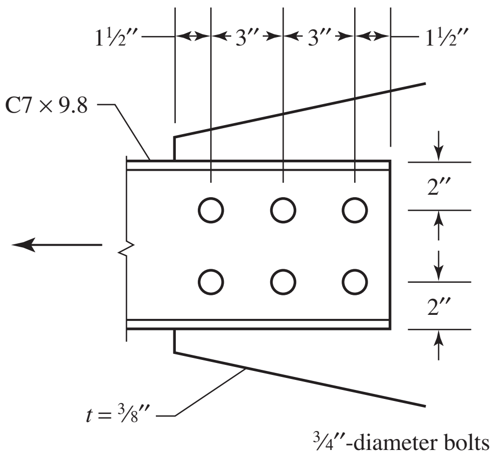


FIGURE P3.5-5

3.5-6 A double-channel shape, 2C8 × 18.75, is used as a tension member. The channels are bolted to a ⅜-inch gusset plate with ⅞-inch diameter bolts. The tension member is A572 Grade 50 steel and the gusset plate is A36. If LRFD is used, how much factored tensile load can be applied? Consider *all* limit states.

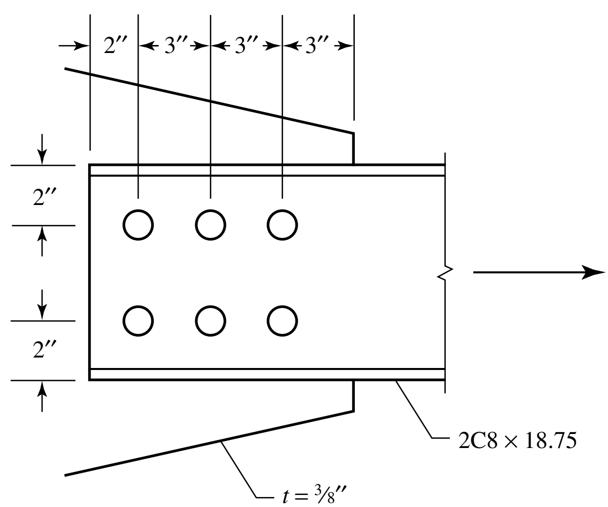


FIGURE P3.5-6

Design of Tension Members

3.6-1 Select a single-angle tension member of A36 steel to resist the following service loads: dead load = 50 kips, live load = 100 kips, and wind load = 45 kips. The member will be connected through one leg with 1-inch diameter bolts in two lines. There will be four bolts in each line. The member length is 20 feet.

a. Use LRFD.

b. Use ASD.

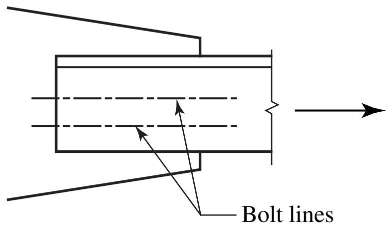


FIGURE P3.6-1

3.6-2 Use A36 steel and select a double-angle tension member to resist a service dead load of 20 kips and a service live load of 60 kips. Assume that the member will be connected to a 3⁄8-inch-thick gusset plate with a single line of five 7⁄8-inch diameter bolts. The member is 15 feet long.

a. Use LRFD.

b. Use ASD.

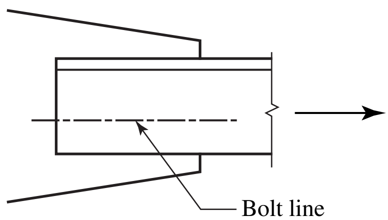


FIGURE P3.6-2

3.6-3 Select an ST shape to be used as a 20-ft-long tension member to resist the following service loads: dead load = 38 kips, live load = 115 kips, and snow load = 75 kips. The connection is through the flange with three 3⁄4-inch diameter bolts in each line. Use A572 Grade 50 steel.

a. Use LRFD.

b. Use ASD.

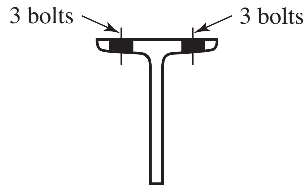


FIGURE P3.6-3

3.6-4 Select an S shape for the tension member shown in Figure P3.6-4. The member shown will be connected between two plates with eight ⅞-in. diameter bolts. The service dead load is 216 kips, the service live load is 25 kips, and the length is 22 ft. Use A36 steel.

a. Use LRFD.

b. Use ASD.

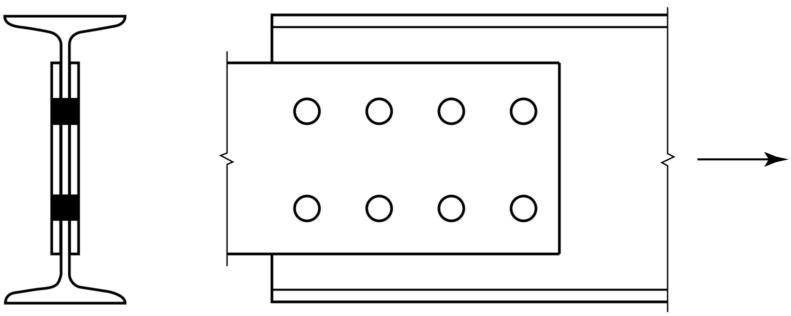


FIGURE P3.6-4

3.6-5 Choose a pipe to be used as a tension member to resist a service dead load of 10 kips and a service live load of 25 kips. The ends will be connected by welding completely around the circumference of the pipe. The length is 8 feet.

a. Use LRFD.

b. Use ASD.

3.6-6 Use LRFD and select an American Standard Channel shape for the following tensile loads: dead load = 54 kips, live load = 80 kips, and wind load = 75 kips. The connection will be with two 9-in.-long longitudinal welds. Use an estimated shear lag factor of $U = 0.85$. Once the member has been selected, compute the value of U with Equation 3.1 and revise the design if necessary. The length is 17.5 ft. Use $F_y = 50$ ksi and $F_u = 65$ ksi.

Threaded Rods and Cables

3.7-1 Select a threaded rod to resist a service dead load of 43 kips and a service live load of 4 kips. Use A36 steel.

a. Use LRFD.

b. Use ASD.

3.7-2 A W16 × 36 is supported by two tension rods *AB* and *CD,* as shown in Figure P3.7-2. The 30-kip load is a service live load. Use load and resistance factor design and select threaded rods of A36 steel for the following load cases.

a. The 30-kip load cannot move from the location shown.

b. The 30-kip load can be located anywhere between the two rods.

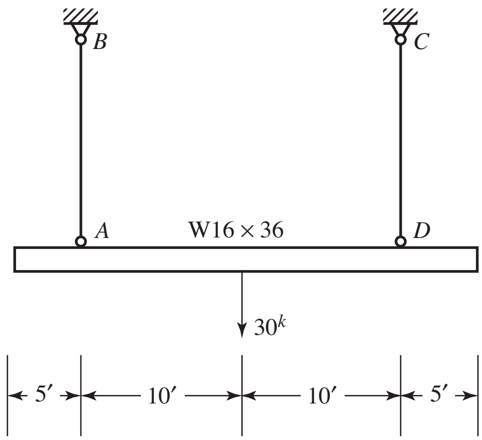


FIGURE P3.7-2

3.7-3 Same as problem 3.7-2, but use allowable *stress* design.

3.7-4 As shown in Figure P3.7-4, members *AC* and *BD* are used to brace the pin-connected structure against a horizontal wind load of 10 kips. Both of these members are assumed to be tension members and not resist any compression. For the load direction shown, member *AC* will resist the load in tension, and member *BD* will be unloaded. Select threaded rods of A36 steel for these members. Use load and resistance factor design.

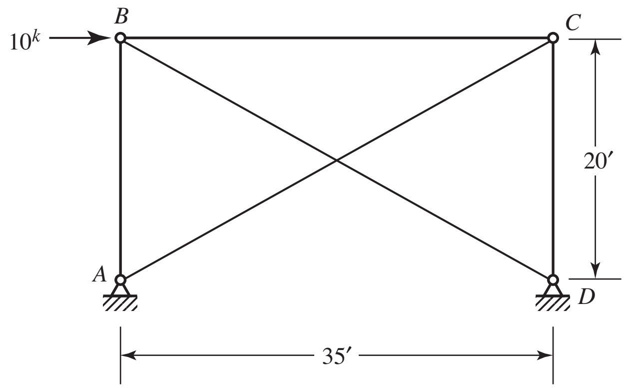


FIGURE P3.7-4

3.7-5 What size A36 threaded rod is required for member *AB*, as shown in Figure P3.7-5? The load is a service live load. (Neglect the weight of member *CB*.)

a. Use LRFD.

b. Use ASD.

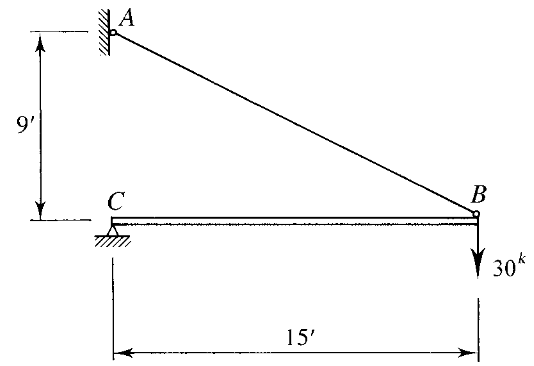


FIGURE P3.7-5

3.7-6 A pipe is supported at 12-foot intervals by a bent, threaded rod, as shown in Figure P3.7-6. If an 8-inch-diameter standard weight steel pipe full of water is used, what size A36 steel rod is required?

a. Use LRFD.

b. Use ASD.

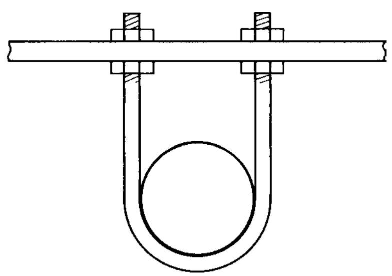


FIGURE P3.7-6

Tension Members in Roof Trusses

3.8-1 Use A992 steel and select a structural tee for the top chord of the welded roof truss shown in Figure P3.8-1. All connections are made with longitudinal plus transverse welds. Assume a connection length of 12 inches. The spacing of trusses in the roof system is 15 feet. Design for the following loads.

Snow: 20 psf of horizontal projection

Roofing: 12 psf

MC8 × 8.5 purlins

Truss weight: 1000 lb (estimated)

a. Use LRFD.
b. Use ASD.

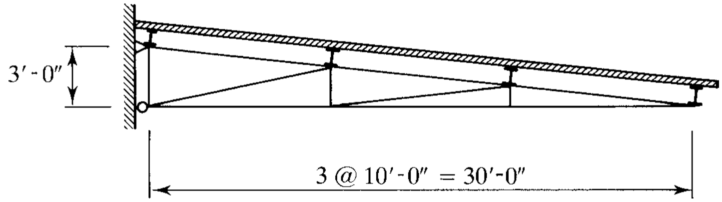


FIGURE P3.8-1

3.8-2 Use ASD and select single-angle shapes for the web tension members of the truss loaded as shown in Figure P3.8-2. The loads are service loads. All connections are with longitudinal welds. Use A36 steel and an estimated shear lag factor, *U*, of 0.85.

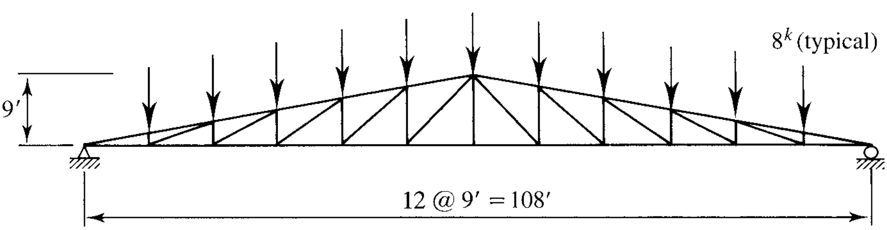


FIGURE P3.8-2

3.8-3 Compute the *factored* joint loads for the truss of Problem 3.8-2 for the following conditions.

Trusses spaced at 18 feet

Weight of roofing = 8 psf

Snow load = 20 psf of horizontal projection

W10 × 33 purlins located only at the joints

Total estimated truss weight = 5000 lb

3.8-4 Use LRFD and design the tension members of the roof truss shown in Figure P3.8-4. Use double-angle shapes throughout and assume ⅜-inch-thick gusset plates and welded connections. Assume a shear lag factor of $U = 0.80$. The trusses are spaced at 30 feet. Use A36 steel and design for the following loads.

Metal deck:	4 psf of roof surface
Built-up roof:	12 psf of roof surface
Purlins:	3 psf of roof surface (estimated)
Snow:	20 psf of horizontal projection
Truss weight:	5 psf of horizontal projection (estimated)

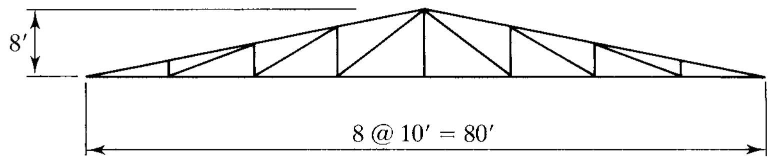


FIGURE P3.8-4

3.8-5 Use A36 steel and design sag rods for the truss of Problem 3.8-4. Assume that, once attached, the metal deck will provide lateral support for the purlins; therefore, the sag rods need to be designed for the purlin weight only.

a. Use LRFD.

b. Use ASD.