Solved problems of f(t)/t

Division by 't' property -

If $L[f(t)] = f(s)$ then

$$L\left[\frac{f(t)}{t}\right] = \int_s^{\infty} f(s)\, ds$$

Que find laplace transform of

1> $\dfrac{e^{4t}}{t}$

$\rightarrow$ $L[e^{4t}] = \dfrac{1}{s-4} = f(s)$

$$L\left[\frac{e^{4t}}{t}\right] = \int_s^{\infty} f(s)\, ds = \int_s^{\infty} \frac{1}{s-4}\, ds$$

$\because \int \frac{1}{x}\, dx = \log x + c$

$$\therefore L\left[\frac{e^{4t}}{t}\right] = \int_s^{\infty} \frac{1}{s-4}\, ds$$

$$= \Big[\log(s-4)\Big]_s^{\infty}$$

$$= 0 - \log(s-4)$$

$$= -\log(s-4)$$

$$= \text{Log}\left(\frac{1}{s-4}\right)$$

$$\frac{\sin 3t}{t}$$

$$L[\sin 3t] = \frac{3}{s^2+3^2} = f(s)$$

$$L\left[\frac{\sin 3t}{t}\right] = \int_s^{\infty} f(s)\,ds$$

$$= \int_s^{\infty} \frac{3}{s^2+3^2}\,ds$$

$$= 3\int_s^{\infty} \frac{1}{s^2+3^2}\,ds$$

$$\int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + c$$

$$L\left[\frac{\sin 3t}{t}\right] = \left[\cancel{3} \times \frac{1}{\cancel{3}} \tan^{-1}\left(\frac{s}{3}\right)\right]_s^{\infty}$$

$$= \left[\tan^{-1}\left(\frac{s}{3}\right)\right]_s^{\infty}$$

$$= \left[\tan^{-1}\left(\frac{s}{3}\right)\right]_s^{\infty}$$

$$= 0 - \tan^{-1}\left(\frac{s}{3}\right)$$

$= -\tan^{-1}(s/a)$

3> $$\frac{\sinh t - \cos 2t}{t} = \frac{\sinh t}{t} - \frac{\cos 2t}{t}$$

$$\because \; L[\sinh t] = \frac{1}{s^2 - 1^2}$$

$$L[\cos 2t] = \frac{s}{s^2 + 2^2}$$

$$\therefore \; L\left[\frac{\sinh t}{t}\right] - L\left[\frac{\cos 2t}{t}\right]$$

$$= \int_s^{\infty} \frac{1}{s^2 - 1^2}\, ds - \int_s^{\infty} \frac{s}{s^2 + 2^2}\, ds$$

$$\int \frac{1}{x^2 - a^2}\, dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + c$$

$$= \int_s^{\infty} \frac{1}{s^2 - 1^2}\, ds - \int_s^{\infty} \frac{s}{s^2 + 2^2}\, ds$$

$$\int \frac{1}{x^2 - a^2}\, dx = \frac{1}{2a} \log \left| \frac{x-a}{x+a} \right| + c$$

$$\int \frac{f'(x)}{f(x)}\, dx = \log f(x) + c$$

So, $f(s) = s^2 + 2^2$

And $f'(s) = 2s$

$$= \int_s^{\infty} \frac{1}{s^2 - 1^2}\, ds - \frac{1}{2}\int_s^{\infty} \frac{2s}{s^2 + 2^2}\, ds$$

$$= \left[\frac{1}{2\times 1} \log \left|\frac{s-1}{s+1}\right| - \frac{1}{2} \log |s^2 + 4|\right]_s^{\infty}$$

$$= 0 - \left[\frac{1}{2} \log \left|\frac{s-1}{s+1}\right] - \frac{1}{2} \log |s^2 + 4|\right]$$

$$= -\frac{1}{2}\left[\log \left|\frac{s-1}{s+1}\right| + \log |s^2 + 4|\right]$$

$$= -\frac{1}{2}\left[\log \left|\frac{s-1}{s+1}\right| + \log |s^2 + 4|\right]$$

$$= -\frac{1}{2}\left[\log \left|\frac{(s-1)(s^2+4)}{(s+1)}\right]$$

As

$$\log_{10}\left(\frac{a}{b}\right) + \log_{10}(c)$$

Equal

$$\log_{10}\left(\frac{ac}{b}\right)$$

4> $$\frac{e^{-t} + \cos t}{t} = \frac{e^{-t}}{t} + \frac{\cos t}{t}$$

$$L[e^{-t}] = \frac{1}{s+1}, \quad L[\cos t] = \frac{s}{s^2 + 1^2}$$

$$\therefore \quad L\left[\frac{e^{-t}}{t}\right] + L\left[\frac{\cos t}{t}\right]$$

$$\therefore \quad L\left[\frac{e-t}{t}\right] + L\left[\frac{\cos t}{t}\right]$$

$$= \int_s^{\infty} \frac{1}{s+1}\, ds + \frac{1}{2}\int_s^{\infty} \frac{2s}{s^2+1}\, ds$$

$$= \left[\log(s+1) + \frac{1}{2}\log(s^2+1)\right]_s^{\infty}$$

$$= 0 - \left[\log(s+1) + \frac{1}{2}\log(s^2+1)\right]$$

$$= 0 - \left[\log(s+1) + \frac{1}{2}\log(s^2+1)\right]$$

$$= -\left[\log(s+1) + \log(s^2+1)^{1/2}\right]$$