Lecture 7

 Sets and Subsets

 **Set** - A collection of objects. The specific objects within the set are called the elements or member of the set . Capital letters are commonly used to name sets.

Examples:

 𝑆𝑒𝑡 𝐴 = {𝑎, 𝑏, 𝑐, 𝑑} 𝑜𝑟 𝑆𝑒𝑡 𝐵 = {1, 2, 3, 4}

**Set Notation** - Braces { } can be used to list the members of a set, with each

member separated by a comma. This is called the “Roster Method.” A

description can also be used in the braces. This is called “Set-builder”

notation.

Example:

**Roster Method**

 Set *A*: The natural numbers from 1 to 10.

 Members of *A*: 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

 Set Notation: 𝐴 = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

 Set Builder Not.: {𝑥 𝑥 𝑖𝑠 𝑎 𝑛𝑎𝑡𝑢𝑟𝑎𝑙 𝑛𝑢𝑚𝑏𝑒𝑟 𝑓𝑟𝑜𝑚 1 𝑡𝑜 10}

**Ellipsis** - Three dots (…) used within the braces to indicate that the list continues

 in the established pattern. This is helpful notation to use for **long lists or**

 **infinite lists**. If the dots come at the end of the list, they indicate that the

 list goes on indefinitely (i.e. an infinite set).

Examples: Set *A*: Lowercase letters of the English alphabet

 Set Notation: {𝑎, 𝑏, 𝑐, … , 𝑧}

**Cardinality of a Set** – The number of distinct elements in a set.

 Example: Set 𝐴: The days of the week

 Members of Set *A*: Monday, Tuesday, Wednesday,

 Thursday, Friday, Saturday, Sunday

 Cardinality of Set 𝐴 = 𝒏(𝑨) = 7

**Equal Sets** – Two sets that contain exactly the same elements, regardless of the

 order listed or possible repetition of elements.

 Example: 𝐴 = {1, 1, 2, 3, 4} , 𝐵 = {4, 3, 2, 1, 2, 3, 4, } .

 Sets 𝐴 𝑎𝑛𝑑 𝐵 are equal because they contain exactly the same

 elements (i.e. 1, 2, 3, & 4). This can be written as 𝑨 = 𝑩.

**Equivalent Sets** – Two sets that contain the same number of distinct elements.

 Example: 𝐴 = {𝐹𝑜𝑜𝑡𝑏𝑎𝑙𝑙, 𝐵𝑎𝑠𝑘𝑒𝑡𝑏𝑎𝑙𝑙, 𝐵𝑎𝑠𝑒𝑏𝑎𝑙𝑙, 𝑆𝑜𝑐𝑐𝑒𝑟}

**Both Sets have 4 elements elements**

 𝐵 = {𝑝𝑒𝑛𝑛𝑦, 𝑛𝑖𝑐𝑘𝑒𝑙, 𝑑𝑖𝑚𝑒, 𝑞𝑢𝑎𝑟𝑡𝑒𝑟}

 𝑛(𝐴) = 4 𝑎𝑛𝑑 𝑛(𝐵) = 4

 𝐴 𝑎𝑛𝑑 𝐵 𝑎𝑟𝑒 𝐸𝑞𝑢𝑖𝑣𝑎𝑙𝑒𝑛𝑡 𝑆𝑒𝑡𝑠, 𝑚𝑒𝑎𝑛𝑖𝑛𝑔 𝑛(𝐴) = 𝑛(𝐵).

**Note: If two sets are E qual, they are a lso Equivalent**

 Example: 𝑆𝑒𝑡 𝐴 = {𝑎, 𝑏, 𝑐, 𝑑} 𝑆𝑒𝑡 𝐵 = {𝑑, 𝑑, 𝑐, 𝑐, 𝑏, 𝑏, 𝑎, 𝑎}

Sets *A* and *B* have the exact same element {a, b, c, d}

 Are Sets *A* and *B* Equal? Yes

Sets *A* and *B* have the exact same number of distinct elements

n(*A*) =b(*B*)=4

 Are Sets *A* and *B*

 Equivalent? Yes

**The Empty Set (or Null Set)** – The set that contains no elements.

 It can be represented by either { } 𝑜𝑟 ∅.

 Note: Writing the empty set as {∅} is not correct!

**Symbols commonly used with Sets –**

 ∈ → 𝑖𝑛𝑑𝑖𝑐𝑎𝑡𝑒𝑠 𝑎𝑛 𝑜𝑏𝑗𝑒𝑐𝑡 𝑖𝑠 𝑎𝑛 𝒆𝒍𝒆𝒎𝒆𝒏𝑡 𝑜𝑓 𝑎 𝑠𝑒𝑡.

 ∈ → 𝑖𝑛𝑑𝑖𝑐𝑎𝑡𝑒𝑠 𝑎𝑛 𝑜𝑏𝑗𝑒𝑐𝑡 𝑖𝑠 𝒏𝒐𝒕 𝑎𝑛 𝑒𝑙𝑒𝑚𝑒𝑛𝑡 𝑜𝑓 𝑎 𝑠𝑒𝑡.

 ⊆ → 𝑖𝑛𝑑𝑖𝑐𝑎𝑡𝑒𝑠 𝑎 𝑠𝑒𝑡 𝑖𝑠 𝑎 𝒔𝒖𝒃𝒔𝒆𝒕 𝑜𝑓 𝑎𝑛𝑜𝑡ℎ𝑒𝑟 𝑠𝑒𝑡.

 ⊂ → 𝑖𝑛𝑑𝑖𝑐𝑎𝑡𝑒𝑠 𝑎 𝑠𝑒𝑡 𝑖𝑠 𝑎 𝒑𝒓𝒐𝒑𝒆𝒓 𝒔𝒖𝒃𝒔𝒆𝒕 𝑜𝑓 𝑎𝑛𝑜𝑡ℎ𝑒𝑟 𝑠𝑒𝑡.

 ∩ → 𝑖𝑛𝑑𝑖𝑐𝑎𝑡𝑒𝑠 𝑡ℎ𝑒 𝒊𝒏𝒕𝒆𝒓𝒔𝒆𝒄𝒕𝒊𝒐𝒏 𝑜𝑓 𝑠𝑒𝑡𝑠.

 ∪ → 𝑖𝑛𝑑𝑖𝑐𝑎𝑡𝑒𝑠 𝑡ℎ𝑒 𝒖𝒏𝒊𝒐𝒏 𝑜𝑓 𝑠𝑒𝑡𝑠.

**Subsets** - For Sets *A* and *B*, Set *A* is **a Subset** of Set *B* if every element in Set *A* is

 also in Set *B*. It is written as *𝑨* ⊆ *𝑩*.

**Proper Subsets** - For Sets *A* and *B*, Set *A* is a Proper Subset of Set *B* if every

 element in Set *A* is also in Set *B*, but Set *A* does not equal Set *B*. (𝑨 ≠ 𝑩)

 It is written as 𝑨 ⊂ 𝑩.

Example : 𝑆𝑒𝑡 𝐴 = {2, 4, 6} 𝑆𝑒𝑡 𝐵 = {0, 2, 4, 6, 8}

 {2, 4, 6} ⊆ {0, 2, 4, 6, 8} and {2, 4, 6} ⊂ {0, 2, 4, 6, 8}

Set *A* is a **Proper Subset** of Set *B* because every element in *A* is also in B, but A ≠ 𝐵. 𝑨 ⊂ 𝑩

Set *A* is a **Subset** of Set *B* because every element in *A* is also in *B*. 𝑨 ⊆ 𝑩

**Note**: The Empty Set is a Subset of every Set.

 The Empty Set is also a Proper Subset of every Set except the Empty Set.

**Number of Subsets(power of set)** – The number of distinct subsets of a set containing n elements is given by $2^{n}$ .

**Number of Proper Subsets** – The number of distinct proper subsets of a set

 containing n elements is given by $2^{n} $**− 𝟏**.

 **Example**: How many Subsets and Proper Subsets does Set *A* have?

 𝑆𝑒𝑡 𝐴 = {𝑏𝑎𝑛𝑎𝑛𝑎𝑠, 𝑜𝑟𝑎𝑛𝑔𝑒𝑠, 𝑠𝑡𝑟𝑎𝑤𝑏𝑒𝑟𝑟𝑖𝑒𝑠}

 𝑛 = 3

 **Subsets** = $2^{n}$ = $2^{3}$ = 8 **Proper Subsets** = $2^{n}$ − 1 = 7

 **Example**: List the Proper Subsets for the Example above.

 1. {𝑏𝑎𝑛𝑎𝑛𝑎𝑠} 5. {𝑏𝑎𝑛𝑎𝑛𝑎𝑠, 𝑠𝑡𝑟𝑎𝑤𝑏𝑒𝑟𝑟𝑖𝑒𝑠}

 2. {𝑜𝑟𝑎𝑛𝑔𝑒𝑠} 6. {𝑜𝑟𝑎𝑛𝑔𝑒𝑠, 𝑠𝑡𝑟𝑎𝑤𝑏𝑒𝑟𝑟𝑖𝑒𝑠}

 3. {𝑠𝑡𝑟𝑎𝑤𝑏𝑒𝑟𝑟𝑖𝑒𝑠} 7. ∅

 4. {𝑏𝑎𝑛𝑎𝑛𝑎𝑠, 𝑜𝑟𝑎𝑛𝑔𝑒𝑠}

**Intersection of Sets** – The Intersection of Sets *A* and *B* is the set of elements that

 are in both *A* and *B*, i.e. what they have in common. It can be written as 𝑨 ∩ 𝑩

**Union of Sets** – The Union of Sets *A* and *B* is the set of elements that are

 members of Set *A*, Set *B*, or both Sets. It can be written as 𝑨 ∪ 𝑩.

 **Example**: Find the Intersection and the Union for the Sets *A* and *B*.

 𝑆𝑒𝑡 𝐴 = {𝑅𝑒𝑑, 𝐵𝑙𝑢𝑒, 𝐺𝑟𝑒𝑒𝑛}

Set *A* and *B* only have 2 elements in common

 𝑆𝑒𝑡 𝐵 = {𝑌𝑒𝑙𝑙𝑜𝑤, 𝑂𝑟𝑎𝑛𝑔𝑒, 𝑅𝑒𝑑, 𝑃𝑢𝑟𝑝𝑙𝑒, 𝐺𝑟𝑒𝑒𝑛}

 Intersection: 𝑨 ∩ 𝑩 = {𝑅𝑒𝑑, 𝐺𝑟𝑒𝑒𝑛}

 Union: 𝑨 ∪ 𝑩 = {𝑅𝑒𝑑, 𝐵𝑙𝑢𝑒, 𝐺𝑟𝑒𝑒𝑛, 𝑌𝑒𝑙𝑙𝑜𝑤, 𝑂𝑟𝑎𝑛𝑔𝑒, 𝑃𝑢𝑟𝑝𝑙𝑒}

List each distinct element only once, even if it appears in both Set *A* and Set *B*.

**Complement of a Set** - The Complement of Set *A*, written as *A’* , is the set of all elements in the given Universal Set (U) that are not in Set *A*.

 **Example**: Let 𝑈 = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} and 𝐴 = {1,3, 5, 7, 9}

 Find 𝐴′ .

Cross off everything in U that is also in A. What is left over will be the elements that are in A’

 𝑈 = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

 So, 𝐴′ = {2, 4, 6, 8, 10}