Al-Mustaqbal University

College of Engineering and Technology

Department of Biomedical Engineering

Stage: three

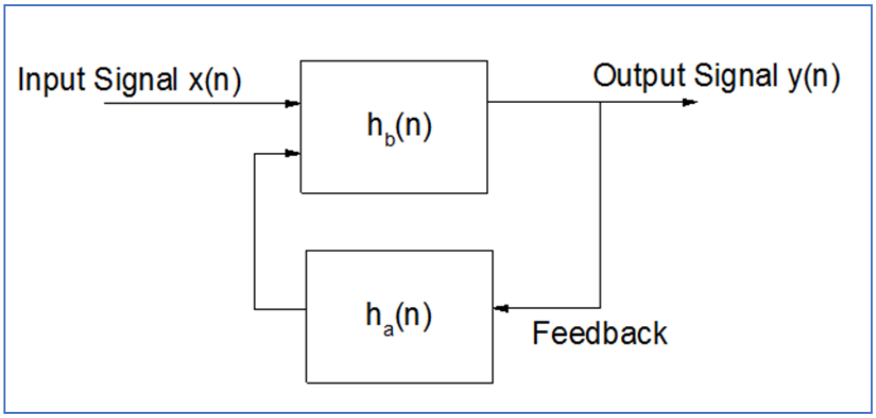
Signal Processing

2023-2024

Lecture (9): Recursive digital filter (IIR)

Transfer Function of Infinite Impulse Response (IIR)

The infinite impulse response (IIR) filter is a recursive filter in that the output from the filter is computed by using the current and previous inputs and previous outputs. Because the filter uses previous values of the output, there is feedback of the output in the filter structure.



$$y[n] = \sum_{k=0}^{M} b_k x[n-k] - \sum_{k=1}^{N} a_k y[n-k]$$ (1) (Difference Equation)

$$y[n] = b_0 x(n) + b_1 x(n-1) + \cdots + b_M X(n-M) - a_1 y(n-1) \ldots\ldots - a_N y(n-N)$$

OR

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1+\sum_{k=1}^{N} a_k z^{-k}}$$ (2) (Transfer Function)

where a_k and b_k are filter coefficients

Properties of IIR filter

- IIR filters stands infinite impulse response filter.
- IIR filters are recursive filters.
- Feedback system is available in IIR filter.
- IIR filter sometimes unstable
- The transfer function contains both poles and zeros.

Example//Determine the first three samples in the impulse response for the IIR filter.

$y[n] - 0.2\, y[n-1] = x[n] + x[n-1]$

Substituting δ[n] for $x[n]$ and h[n] for $y[n]$.

$h[n] - 0.2\, h[n-1] = \delta[n] + \delta[n-1]$

$h[n] = 0.2\, h[n-1] + \delta[n] + \delta[n-1]$

h[0] = 0.2 h [0 – 1] +δ [0] + δ [0 – 1]

h[0] = 0.2 h [– 1] +δ [0] + δ [– 1]

= 0.0 + 1.0 + 0.0 = 1.0

h[1] = 0.2 h [1 – 1] +δ [1] + δ [1 – 1]

= 0.2 (1) + 0.0 + 1.0 = 1.2

h[2] = 0.2 h [2 – 1] +δ [2] + δ [2 – 1]

= 0.2 (1.2) + 0.0 + 0.0 = 0.24

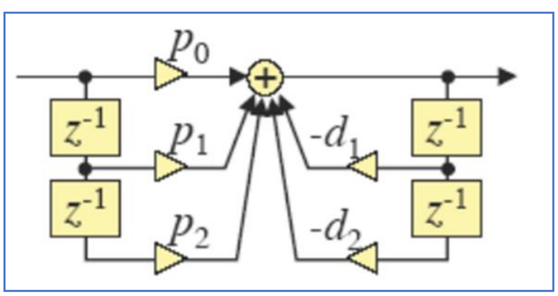
h[3] = 0.2 h [3 – 1] +δ [3] + δ [3 – 1]

= 0.2 (0.24) + 0.0 + 0.0 = 0.048

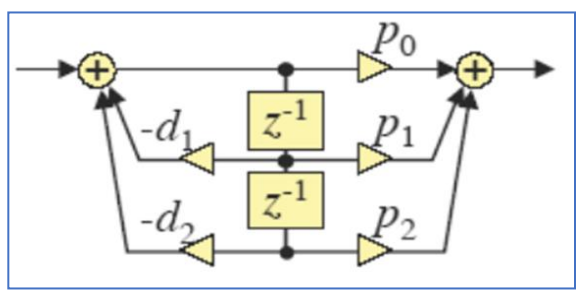
IIR Filter structures

IIR system/filter can be realized in several structures:

1. Direct Form I
2. Direct Form II



Direct Form I



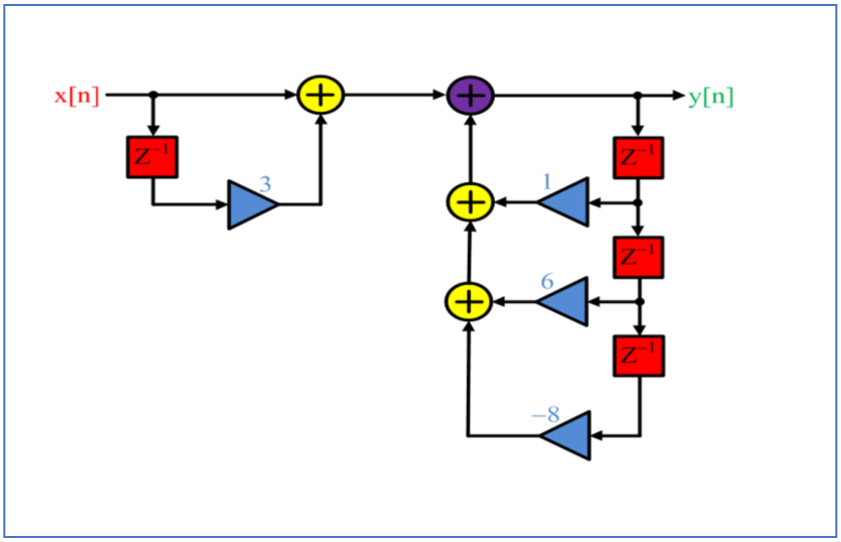
Direct Form II

Example// Realize the infinite impulse response (IIR) filter using the direct form-I from the transfer function:

$$H(z) = \frac{1 + 3z^{-1}}{(1 - 2z^{-1})(1 + z^{-1} - 4z^{-2})}$$

$$H(z) = \frac{y(z)}{x(z)} = \frac{1 + 3z^{-1}}{1 - z^{-1} - 6z^{-2} + 8z^{-3}}$$

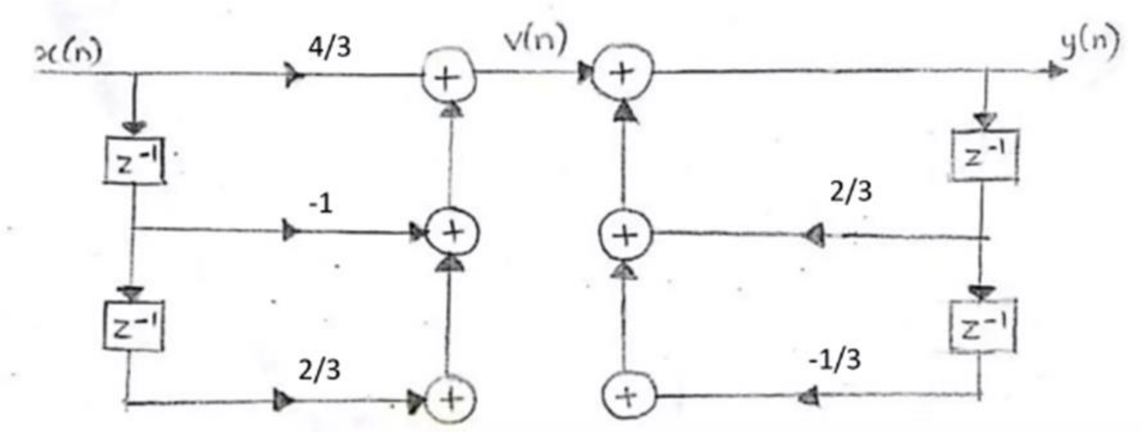
$$y(z) = z^{-1}y(z) + 6z^{-2}y(z) - 8z^{-3}y(z) + x(z) + 3z^{-1}x(z)$$



Example // implement the filter represented by following difference equation

$$y(n) = \frac{4}{3}x(n) - x(n-1) + \frac{2}{3}x(n-2) + \frac{2}{3}y(n-1) - \frac{1}{3}y(n-2)$$

Sol// by using Direct-form I



Direct-form II