Al-Mustaqbal University

College of Engineering and Technology

Department of Biomedical Engineering

Stage: three

Signal Processing

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Lecture (8): Non-recursive digital filter (FIR)

Transfer Function of Finite Impulse Response(FIR)

An Finite Impulse Response (FIR) filter is completely specified by the following input-output relationship

$y(n) = \sum_{k=0}^{N} b(k)x(n-k)$ (1) (Difference Equation)

y(n)=$b_0 x(n) + b_1 x(n-1) + b_2 x(n-2) + \cdots + b_k x(n-k)$

OR

$H(z) = \frac{Y(z)}{X(z)} = \sum_{k=0}^{N} b(k)z^{-k}$ (2) (Transfer Function)

H(z)=$b_0 + b_1 z^{-1} + b_2 z^{-2} + \cdots + b_k z^{-k}$

The length of filter =k+1

The filter coefficients b_0, b_1, b_2, b_K are the impulse response h(n) of the filter .

Properties of FIR filter

- FIR filters have a finite-duration impulse response.
- FIR filters are non-recursive filters.
- No feedback
- Always stable
- Simple to implement

Example\\Given the following FIR filter

y(n)=0.1x(n) + 0.25x(n-1) + 0.2x(n-2)

Determine the **transfer function**, **filter length**, **nonzero coefficients**, and **impulse response**.

Sol\\ Applying z-transform on both sides of the difference equation yields

Y(z)=0.1X(z)+0.25z^{-1}X(z) +0.2z^{-2}X(z)

1)Then the transfer function is found to be

$$H(z) = \frac{Y(z)}{X(z)} = 0.1 + 0.25z^{-1} + 0.2z^{-2}$$

2)filter length is k+1=3

3)nonzero coefficients b_0=0.1 b_1=0.25 b_2=0.2

4)impulse response h(n)=0.1 $\delta[n]$+0.25 $\delta[n-1]$+0.2 $\delta[n-2]$

Example\\ A FIR filter has a set of filter coefficients $\{b_k\} = \{3, -1, 2, 1\}$. Determine the **difference equation** for the filter.

Sol// The length of the filter is 4.

$y[n] = 3x[n] - x[n-1] + 2x[n-2] + x[n-3]$

Example\\ Determine the first four samples in the impulse response for the FIR filter.

$y[n] = 0.5(x[n] + x[n-1] + x[n-2])$

Sol\\ Substituting δ [n] for $x[n]$ and h[n] for $y[n]$.

$h[n] = 0.5(\delta[n] + \delta[n-1] + \delta[n-2])$

$h[0] = 0.5(\delta[0] + \delta[-1] + \delta[-2]) = 0.5(1.0 + 0.0 + 0.0) = 0.5$

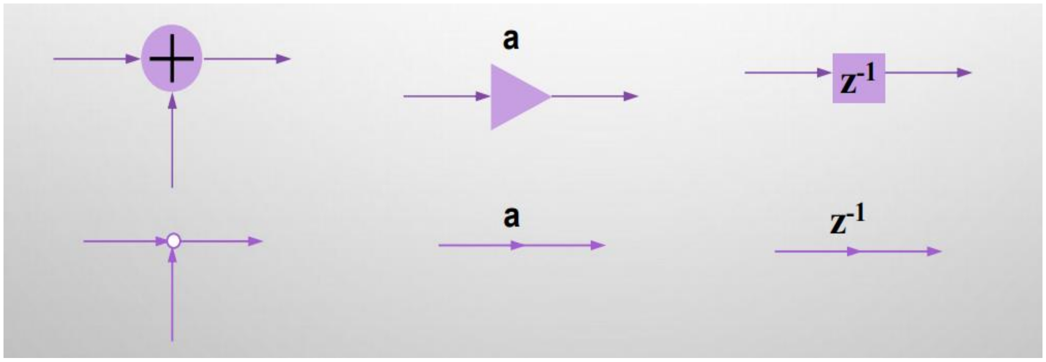
$h[1] = 0.5(\delta[1] + \delta[0] + \delta[-1]) = 0.5(0.0 + 1.0 + 0.0) = 0.5$

$h[2] = 0.5(\delta[2] + \delta[1] + \delta[0]) = 0.5(0.0 + 0.0 + 1.0) = 0.5$

$h[3] = 0.5(\delta[3] + \delta[2] + \delta[1]) = 0.5(0.0 + 0.0 + 0.0) = 0$

Basic elements of digital filter structures

- Adder has two inputs and one output.
- Multiplier (gain) has single-input, single-output.
- Delay element delays the signal passing through it by one sample. It is implemented by using a shift register.

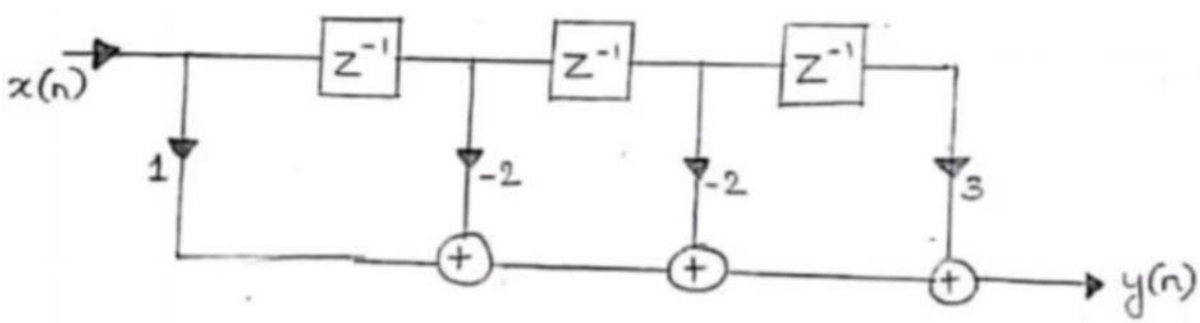


FIR Filter structures

Direct form: a direct implementation of the convolution operation . the number of delay equal to the order of the filter.

Example\\Draw the direct form structure for the FIR filter represented by the following difference equation

$$y[n]= x[n] - 2x[n-1] - 2x[n-2] + 3x[n-3]$$



Sol:

Example//Based on the transfer function, realize the digital filter using the direct form.

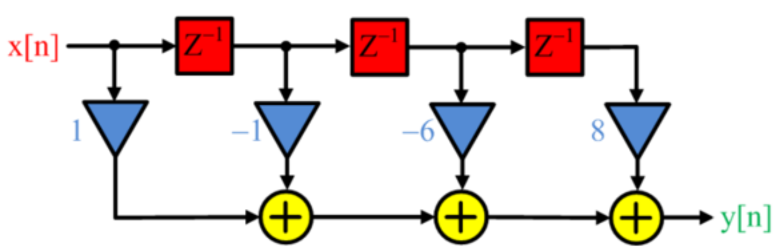
$$H(z) = (1-2z^{-1})\,(1+ z^{-1}-4\,z^{-2})$$

Sol// Since the transfer function has only the numerator part or zeroes, therefore this is an FIR filter.

$$H(z) = \frac{y(z)}{x(z)} = (1-2z^{-1})(1+z^{-1}-4z^{-2})$$

$$y(z) = x(z) - z^{-1}x(z) - 6z^{-2}x(z) + 8z^{-3}x(z)$$

$$y(n) = x(n) - x(n-1) - 6x(n-2) + 8x(n-3)$$



Example// for this difference equation find the direct-form

$$y(n) = 3x(n) + 3x(n-1) + 2x(n-2) - 2x(n-3)$$