

Q1) Drive an expression for calculating the overall heat transfer coefficient for an evaporator or condenser.

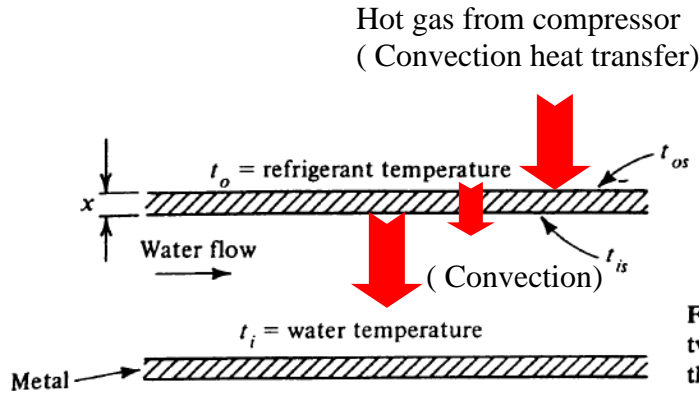


Figure 12-3 Heat transfer between refrigerant and water through a tube.

$$q = h_o A_o (t_o - t_{os}) \quad (12-1)$$

$$q = \frac{k}{x} A_m (t_{os} - t_{is}) \quad (12-2)$$

$$q = h_i A_i (t_{is} - t_i) \quad (12-3)$$

where q = rate of heat transfer, W

h_o = heat-transfer coefficient on outside of tube, $W/m^2 \cdot K$

A_o = outside area of tube, m^2

t_o = refrigerant temperature, $^{\circ}C$

t_{os} = temperature of outside surface of tube, $^{\circ}C$

k = conductivity of tube metal, $W/m \cdot K$

x = thickness of tube, m

t_{is} = temperature of inside surface of tube, $^{\circ}C$

A_m = mean circumferential area of tube, m^2

h_i = heat-transfer coefficient on inside of tube, $W/m^2 \cdot K$

A_i = inside area of tube, m^2

t_i = water temperature, $^{\circ}C$

To express the overall heat-transfer coefficient the area on which the coefficient is based must be specified. Two acceptable expressions for the overall heat-transfer coefficient are

$$q = U_o A_o (t_o - t_i) \quad (12-4)$$

and

$$q = U_i A_i (t_o - t_i) \quad (12-5)$$

where U_o = overall heat-transfer coefficient based on outside area, $W/m^2 \cdot K$

U_i = overall heat-transfer coefficient based on inside area, $W/m^2 \cdot K$

From Eqs. (12-4) and (12-5) it is clear that $U_o A_o = U_i A_i$. The U value is always associated with an area. Knowledge of U_o or U_i facilitates computation of the rate of heat transfer q .

To compute the U value from knowledge of the individual heat-transfer coefficients, first divide Eq. (12-1) by $h_o A_o$, Eq. (12-2) by $k A_m / x$, and Eq. (12-3) by

$h_i A_i$, leaving only the temperature differences on the right sides of the equations. Next add the three equations, giving

$$\frac{q}{h_o A_o} + \frac{qx}{kA_m} + \frac{q}{h_i A_i} = (t_o - t_{os}) + (t_{os} - t_{is}) + (t_{is} - t_i) = t_o - t_i \quad (12-6)$$

Alternate expressions for $t_o - t_i$ are available from Eqs. (12-4) and (12-5)

$$t_o - t_i = \frac{q}{U_o A_o} = \frac{q}{U_i A_i} \quad (12-7)$$

Equating Eqs. (12-6) and (12-7) and canceling q provides an expression for computing the U values

$$\frac{1}{U_o A_o} = \frac{1}{U_i A_i} = \frac{1}{h_o A_o} + \frac{x}{kA_m} + \frac{1}{h_i A_i} \quad (12-8)$$

The physical interpretation of the terms in Eq. (12-8) is that $1/U_o A_o$ and $1/U_i A_i$ are the total resistances to heat transfer between the refrigerant and water. This total resistance is the sum of the individual resistances

1. From the refrigerant to the outside surface of the tube $1/h_o A_o$
2. Through the tube $x/(kA_m)$
3. From the inside surface of the tube to the water $1/h_i A_i$

Q2) What is the expression for heat transfer coefficient for fluid flowing inside tubes.

12-3 Liquid in tubes; heat transfer and pressure drop The expression for the heat-transfer coefficient for fluids flowing inside tubes, as was first shown in Fig. 2-6, is of the form

$$Nu = C Re^n Pr^m$$

where n and m are exponents. The constant C and exponents in the equation are

$$\frac{hD}{k} = 0.023 \left(\frac{VD\rho}{\mu} \right)^{0.8} \left(\frac{c_p \mu}{k} \right)^{0.4} \quad (12-9)$$

where h = convection coefficient, $W/m^2 \cdot K$

D = ID of tube, m

k = thermal conductivity of fluid, $W/m \cdot K$

V = mean velocity of fluid, m/s

ρ = density of fluid, kg/m^3

μ = viscosity of fluid, $Pa \cdot s$

c_p = specific heat of fluid, $J/kg \cdot K$

Eq. (12-9) is applicable to turbulent flow, which typically prevails with the

Example 12-1 Compute the heat-transfer coefficient for water flow inside the tubes (8 mm ID) of an evaporator if the water temperature is 10°C and its velocity is 2 m/s.

Solution The properties of water at 10°C are

$$\mu = 0.00131 \text{ Pa} \cdot \text{s} \quad \rho = 1000 \text{ kg/m}^3 \quad k = 0.573 \text{ W/m} \cdot \text{K} \quad c_p = 4190 \text{ J/kg} \cdot \text{K}$$

The Reynolds number is

$$\text{Re} = \frac{(2 \text{ m/s}) (0.008 \text{ m}) (1000 \text{ kg/m}^3)}{0.00131 \text{ Pa} \cdot \text{s}} = 12,214$$

This value of the Reynolds number indicates that the flow is turbulent, so Eq. (12-9) applies. The Prandtl number is

$$\text{Pr} = \frac{(4190 \text{ J/kg} \cdot \text{K}) (0.00131 \text{ Pa} \cdot \text{s})}{0.573 \text{ W/m} \cdot \text{K}} = 9.6$$

The Nusselt number can now be computed from Eq. (12-9)

$$\text{Nu} = 0.023(12,214^{0.8}) (9.6^{0.4}) = 106$$

from which the heat-transfer coefficient can be computed as

$$h = \frac{0.573 \text{ W/m} \cdot \text{K}}{0.008 \text{ m}} (106) = 7592 \text{ W/m}^2 \cdot \text{K}$$

Q3) What is the expression for pressure drop of fluid flowing in straight tubes.

As the fluid flows inside the tubes through a condenser or evaporator, a pressure drop occurs both in the straight tubes and in the U-bends or heads of the heat exchanger. Some drop in pressure is also attributable to entrance and exit losses. The expression for pressure drop of fluid flowing in straight tubes from Chap 7 is

$$\Delta p = f \frac{L}{D} \frac{V^2}{2} \rho \quad (12-10)$$

where Δp = pressure drop, Pa

f = friction factor, dimensionless

L = length of tube, m

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The other contributors to pressure drop resulting from changes in flow area and direction are also almost exactly proportional to the square of the flow rate, so if the pressure drop and flow rate Δp_1 and w_1 are known, the pressure drop Δp_2 at a different flow rate w_2 can be predicted:

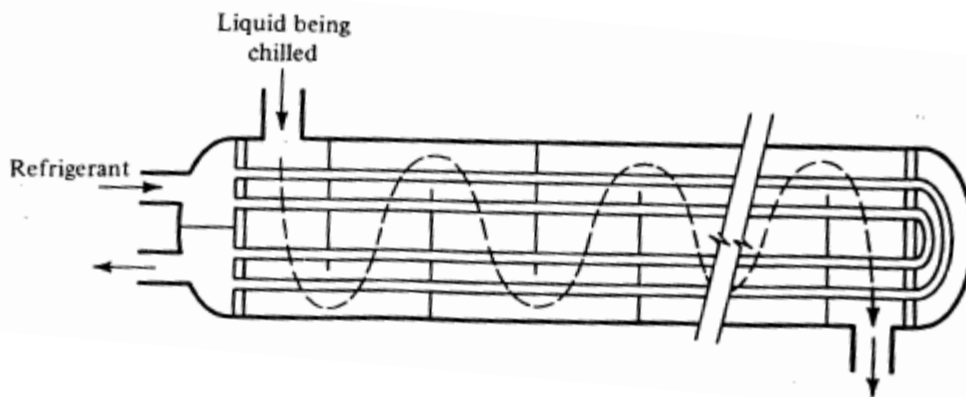
$$\Delta p_2 = \Delta p_1 \left(\frac{w_2}{w_1} \right)^2 \quad (12-11)$$

Q4) What is the expression for heat transfer coefficient for liquid being cooled inside the shell.

$$\frac{hD}{k} = (\text{terms controlled by geometry}) (\text{Re})^{0.6} (\text{Pr})^{0.3} \left(\frac{\mu}{\mu_w} \right)^{0.14} \quad (12-12)$$

where μ = viscosity of fluid at bulk temperature, $\text{Pa} \cdot \text{s}$

μ_w = viscosity of fluid at tube-wall temperature, $\text{Pa} \cdot \text{s}$



Q5) Draw a diagram for a pressure drop of water flow in the shell of an evaporator.

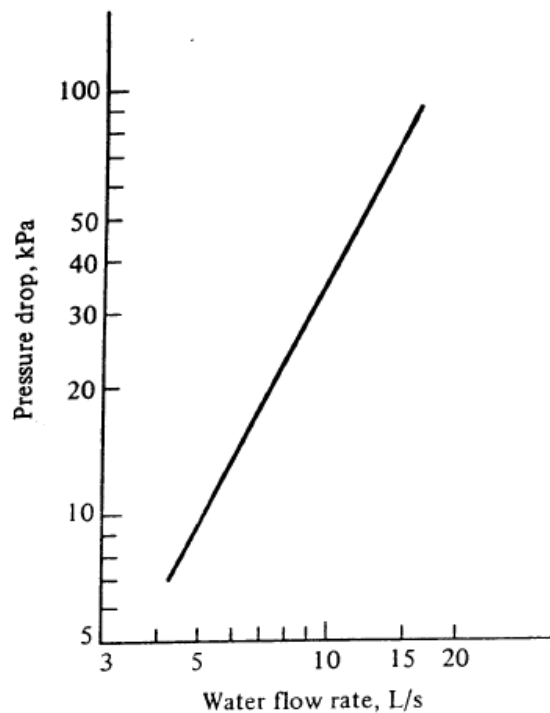


Figure 12-5 Pressure drop of water flowing in the shell of an evaporator. (York Division of Borg Warner.)

The figure above shows that an increase in water flow rate will be increasing the pressure drop of water while flowing in the shell.

Q6) Explain the expressions for calculating the temperature distribution and effectiveness for a bar fin.

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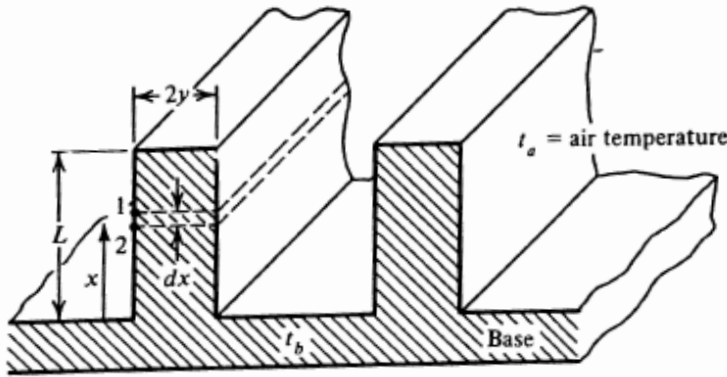


Figure 12-6 Bar fin.

from the end of the fin plus that transferred to the element from the air equals the rate of heat transferred out of the element at position 2 toward the base. For one-half a fin width and a fin depth of Z m, the heat balance in symbols is

$$kyZ \left(\frac{dt}{dx} \right)_1 + Z dx h_f (t_a - t) = kyZ \left(\frac{dt}{dx} \right)_2 \quad (12-14)$$

where t_a = temperature of air

t = temperature of fin

Canceling Z and factoring gives

$$ky \left[\left(\frac{dt}{dx} \right)_2 - \left(\frac{dt}{dx} \right)_1 \right] = dx h_f (t_a - t) \quad (12-15)$$

For the differential length dx the change in the temperature gradient is

$$\left(\frac{dt}{dx} \right)_1 - \left(\frac{dt}{dx} \right)_2 = \frac{d}{dx} \left(\frac{dt}{dx} \right) dx = \frac{d^2 t}{dx^2} dx \quad (12-16)$$

Substituting into Eq. 12-15, we get

$$\frac{d^2 t}{dx^2} = \frac{h_f (t_a - t)}{ky} \quad (12-17)$$

By solving the second-order differential equation (12-17) the temperature distribution throughout the fin can be shown to be

$$\frac{t - t_b}{t_a - t_b} = \frac{\cosh M(L - x)}{\cosh ML} \quad (12-18)$$

where t_b = temperature of base of fin, °C

$$M = \sqrt{\frac{h_f}{ky}}$$

When a finned coil cools air, points in the fin farther away from the base are higher

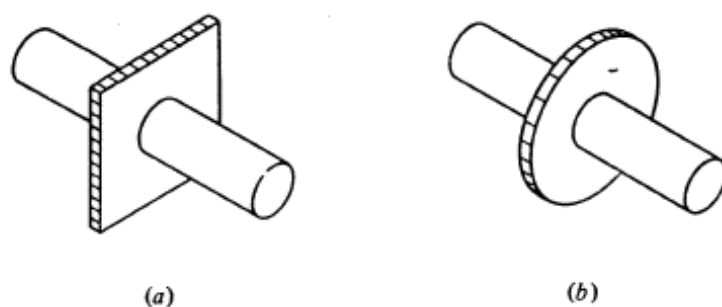


Figure 12-7 Determining fin effectiveness of a rectangular plate fin (a) by treating it as an (b) annular fin of the same area.

in temperature than points close to the base. The net result of the higher temperature of most of the fin is that less heat is transferred than if the entire fin were at temperature t_b . The ratio of the actual rate of heat transfer to that which would be transferred if the fin were at temperature t_b is called the *fin effectiveness*

$$\text{Fin effectiveness} = \eta = \frac{\text{actual } q}{q \text{ if fin were at base temperature}} \quad (12-19)$$

Harper and Brown⁴ found that the fin effectiveness for the bar fin at Fig. 12-6 can be represented by

$$\eta = \frac{\tanh ML}{ML}$$

The bar fin is not a common shape but the dominant type of finned surface is the rectangular plate fin mounted on cylindrical tubes. The net result is a rectangular or square fin mounted on a circular base, one section of which is shown in Fig. 12-7a. The fin effectiveness of the rectangular plate fin is often calculated by using properties of the corresponding annular fin (Fig. 12-7b), for which a graph of the fin effectiveness is available, as in Fig. 12-8. The corresponding annular fin has the same area and thickness as the plate fin it represents.

Example 12-2 What is the fin effectiveness of a rectangular plate fin made of aluminum 0.3 mm thick mounted on a 16-mm-OD tube if the vertical tube spacing is 50 mm and the horizontal spacing is 40 mm? The air-side heat-transfer coefficient is $65 \text{ W/m}^2 \cdot \text{K}$, and the conductivity of aluminum is $202 \text{ W/m} \cdot \text{K}$.

Solution The annular fin having the same area as the plate fin (Fig. 12-9) has an external radius of 25.2 mm. The half-thickness of the fin $y = 0.15 \text{ mm}$

$$M = \sqrt{\frac{65}{202(0.00015)}} = 46.3 \text{ m}^{-1}$$

$$(r_e - r_i)M = (0.0252 - 0.008)(46.3) = 0.8$$

From Fig. 12-8 for $(r_e - r_i)M = 0.8$ and $r_e/r_i = 25.2/8 = 3.15$ the fin effectiveness η is 0.72.

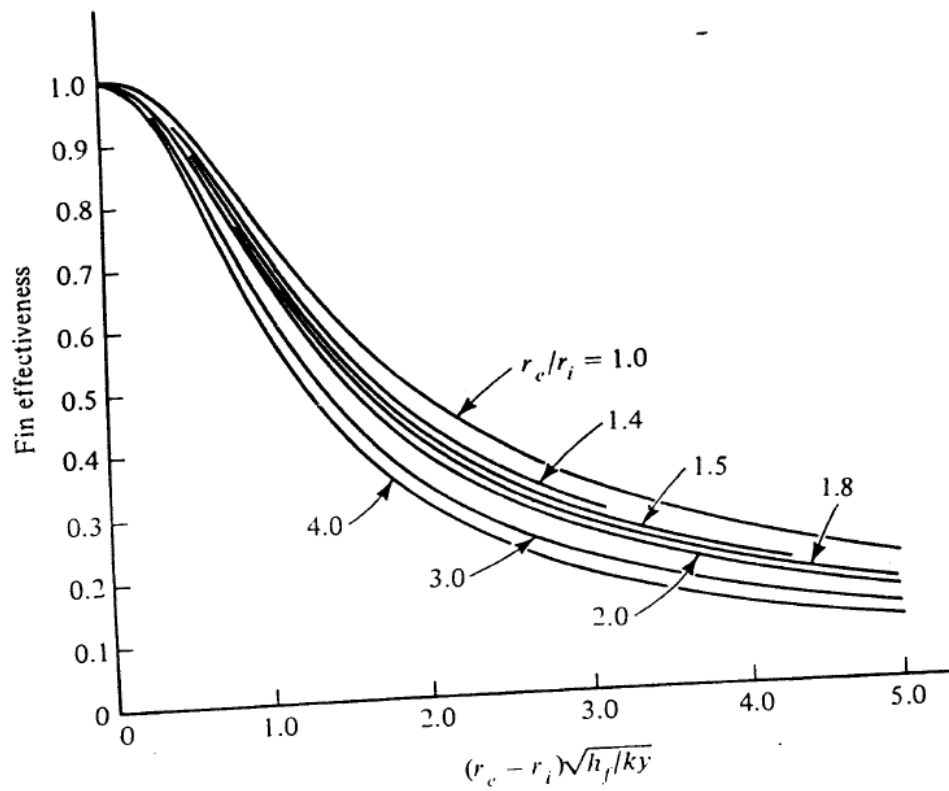


Figure 12-8 Fin effectiveness of an annular fin.⁵ The external radius of the fin is r_e m and the internal radius is r_i m.

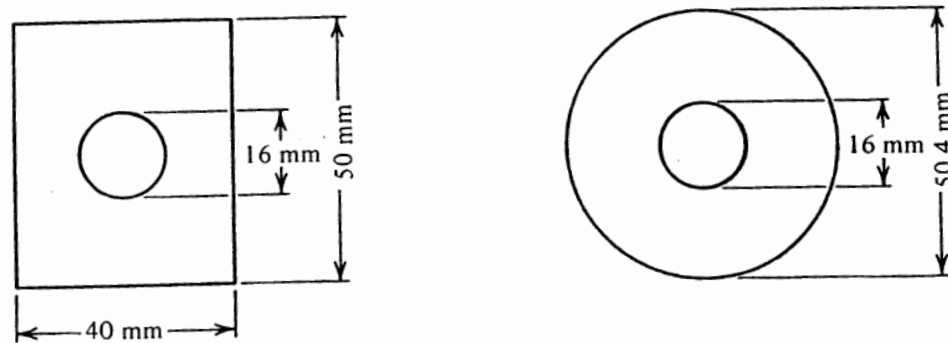


Figure 12-9 Annular fin of same area as rectangular plate fin.

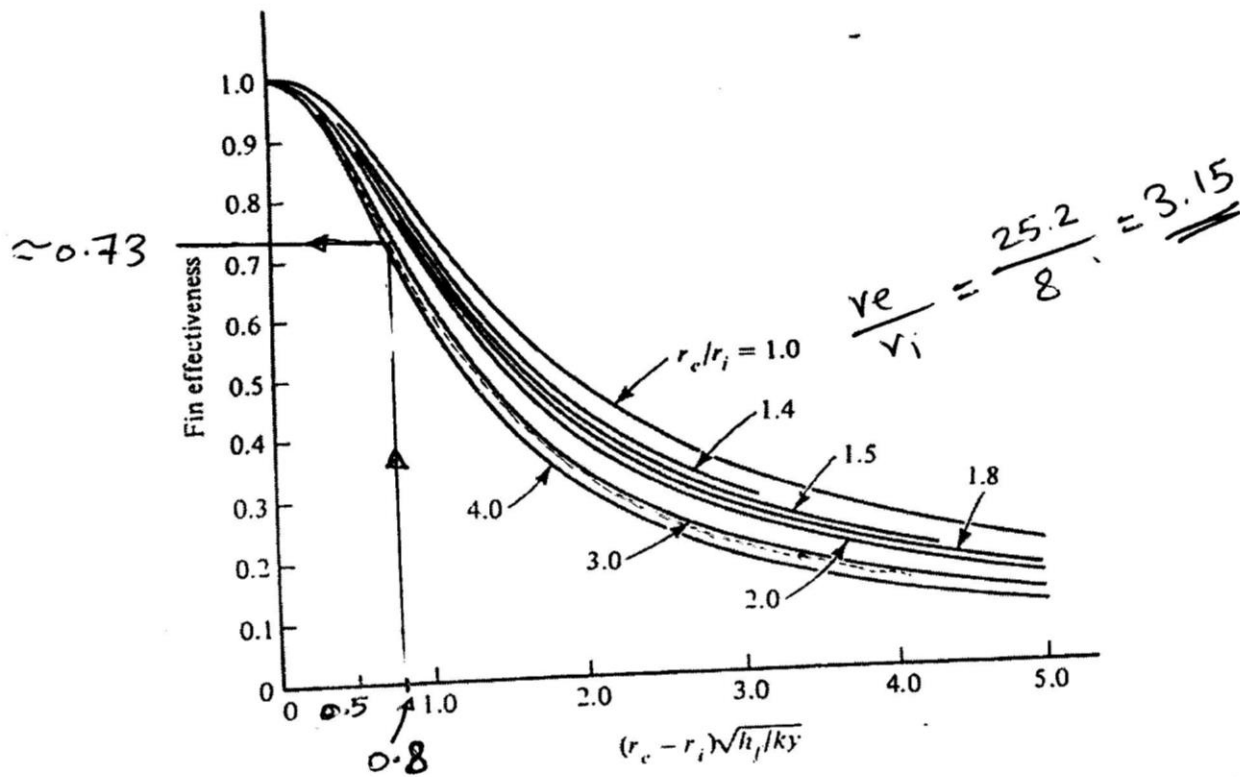


Figure 12-8 Fin effectiveness of an annular fin.⁵ The external radius of the fin is r_e m and the internal radius is r_i m.

$$(r_e - r_i) M = (0.0252 - 0.0008) \times (46.3) = \underline{\underline{0.8}}$$

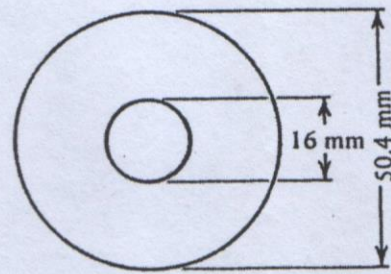
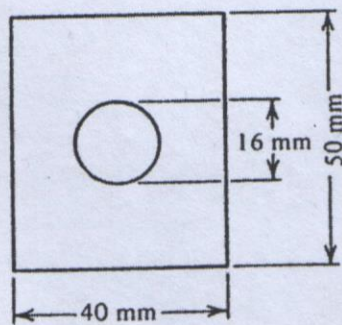


Figure 12-9 Annular fin of same area as rectangular plate fin.

$$A_{\text{rectangular}} = A_{\text{annular}}$$

$$\frac{40}{1000} \times \frac{50}{1000} = \frac{\pi (D_e)^2}{4}$$

$$\Rightarrow (D_e)^2 = \left[\frac{4 \times 40 \times 50}{10^6} \right] / \pi = 2.546 \times 10^{-3}$$

$$D_e = \sqrt{2.546 \times 10^{-3}} = 0.0504 \text{ m} \\ = 50.4 \text{ mm}$$

$$r_e = \frac{D_e}{2} = \underline{0.0252 \text{ m}}$$

$$r_i = \frac{D_i}{2} = \frac{16}{2} = 8 \text{ mm} = \underline{0.008 \text{ m}}$$

Q7) Write an expression for calculating the over all heat transfer coefficient for finned coil

The air-side area of a finned condenser or evaporator is composed of two portions, the prime area and the extended area. The *prime* area A_p is that of the tube between the fins, and the *extended* area A_e is that of the fin. Since the prime area is at the base temperature, it has a fin effectiveness of 1.0. It is to the extended surface that the fin effectiveness less than 1.0 applies. Equation (12-8) for the overall heat-transfer coefficient can be revised to read

$$\frac{1}{U_o A_o} = \frac{1}{U_i A_i} = \frac{1}{h_f (A_p + \eta A_e)} + \frac{x}{k A_m} + \frac{1}{h_i A_i} \quad (12-20)$$

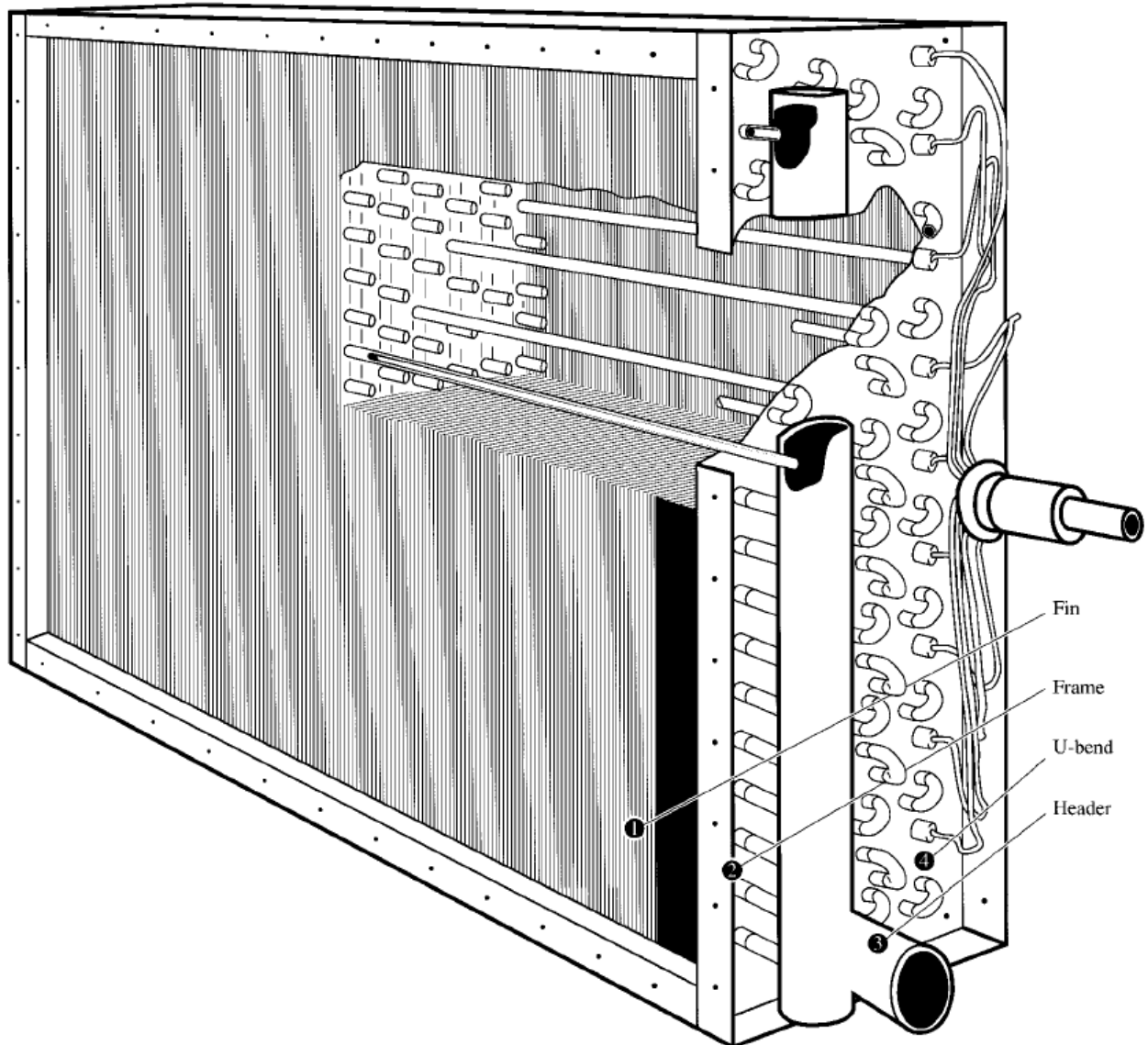


FIGURE 15.26b Structure of a DX coil. (Source: York International Corporation. Reprinted with permission.)