Example-9

Consider the following specifications for Digital Image:
Image frame resolution or dimension = 1200x800 pixels/frame
Colored (RGB) information for each pixel = 24 Bits/ pixel.
The pixels are equal probable to have any color value.
Find:

a- the amount of information carried by one frame (in bits/frame).
b- the amount of information produced by 1000 frames.
c- the rate of information (in bps), if the above 1000 frames are sent within 100 sec.
d- the required channel bandwidth if the signal to noise power ratio is 45 dB.

Solution: First, we need to know the detail of any digital image. It consists of a number of picture elements also called pixel or pel or dot. One can notice this small element when get very close to TV screen. The single image also known as frame or just picture is 2-dimentional representation of large number of pixels. This number is determined by the height (H) and the width (W) of the picture or frame. In above example WxH=1200x800 (also called the resolution). The actual resolution is the number of bits in each pixel. Higher resolution produces better quality picture.

a- Given WxH=1200x800 and 24 bits/pixel we need to find the total information of one frame:

$$I_{frame} = 1200x800\frac{pixel}{frame}x24\frac{Bits}{pixel} = 2304x10^4\ \frac{Bits}{Frame}$$

b- $I_T = 2304x10^4\ \frac{Bits}{Frame}x\ 1000\ Frames = 2304x10^7\ Bits$

c- $R_b = 2304x10^7\ \frac{Bits}{100\ sec} = 2304x10^5\ bps$

d- Here $C_r = R_b = 2304x10^5\ bps$

and $\left.\frac{S}{N}\right|_{ratio} = 10^{[S/N]_{dB}\div 10} = 10^{45\div 10} = 10^{4.5} = 31622.8\ (ratio)$

Using the channel capacity theorem: $C_r = B\ Log_2(1+\frac{S}{N})$, then

$$B = \frac{C_r}{Log_2(1+\frac{S}{N})} = \frac{2304x10^5}{Log_2(1+31622.8)} = \frac{2304x10^5}{14.95} = 15.4x\ 10^6 Hz = 15.4\ MHZ$$

Example-10

Repeat the requirements for Example-9 if the image is a Gray scale image with 8 bits/pixel.

Solution: The image here is gray scale image (also called Black and White or B/W picture). So instead of colored 24 bits per pixel we have 8 bits per pixel.

a- Given WxH=1200x800 as before and 8 bits/pixel :

$$I_{frame} = 1200x800 \frac{pixel}{frame} x8 \frac{Bits}{pixel} = 768x10^4 \ \frac{Bits}{Frame}$$

b- $I_T = 768x10^4 \ \frac{Bits}{Frame} x\ 1000\ Frames = 768x10^7 \ \ Bits$

c- $R_b = 768x10^7 \ \frac{Bits}{100\ sec} = 768x10^5\ bps$

d- Here $C_r = R_b = 768x10^5\ bps$

and $\left.\frac{S}{N}\right|_{ratio} = 10^{[S/N]_{dB} \div 10} = 10^{45 \div 10} = 10^{4.5} = 31622.8\ (ratio)$

Using the channel capacity theorem: $C_r = B\ Log_2(1+\frac{S}{N})$, then

$$B = \frac{C_r}{Log_2(1+\frac{S}{N})} = \frac{768x10^5}{Log_2(1+31622.8)} = \frac{768x10^5}{14.95} = 52.42x10^5 Hz = 5.242\ MHZ$$

Example-11

Which of the following has more information (in bits)

a- Computer file storage for 10 Minutes recording of source having entropy of 4 Bits/symbol and average symbol time of 0.01 msec/symbol.

b- 100 sec of audio record files with 44 k samples/sec and 8 bits/sample.

Solution: In each case we need to calculate the amount of information in bits and then compare the results:

a- We have T_x =0.01 m sec./symbol = 10^{-5} sec./Symbol, then $R_x = 1/T_x = 10^5$ Symbols/sec
Also we have H(x) = 4 Bits/symbols and total time of record T = 10 Min.= 10x60=600 sec.

$$I_{Ta} = R_x.H(x).T = 10^5 \frac{symbol}{sec} x4 \frac{Bits}{Symbol} x600\ sec = 240x10^6\ Bits = 240MBits$$

b- We have R_s =44 k Samples/sec = $44x10^3$ samples/sec.
Also, we have similar to source entropy 8 Bits/samples and total time of record T =100 sec.

$$I_{Tb} = 44x10^3 \frac{samples}{sec} x8 \frac{Bits}{Sample} x100\ sec = 352x10^5 \ \ Bits = 35.2\ MBits$$

Comparing I_{Ta} with I_{Tb} reveals that $I_{Ta} > I_{Tb}$

Example-12

Consider the transmission of binary information over public switched telephone network (PSTN) characterized by bandwidth B= 4 kHz at rate of 20 kbps and signal power of S=5 Watt, then:

a- Find the corresponding S/N in dB, and the noise power.
b- Find the AWGN noise power spectral density N_o.
c- Find the new S/N required if the information rate is halved

Solution: Given B= 4 kHz , C_r =20 kbps and S=5 Watt, then:

a- Find S/N (dB), and N

Using $C_r = B\, Log_2(1+\frac{S}{N})$, gives $\frac{C_r}{B} = Log_2(1+\frac{S}{N})$, then $\left(1+\frac{S}{N}\right) = 2^{\left(\frac{C_r}{B}\right)}$

$\frac{S}{N} = 2^{\left(\frac{C_r}{B}\right)}-1 = 2^{\left(\frac{20000}{4000}\right)}-1 = 31$ (ratio) or $\frac{S}{N} = 10\,Log_{10}(31) = 14.914\ dB$

$\frac{S}{N} = 31$ then N=S/31=5/31 = 0.1613 Watts

b- From the relation $N=B.N_o$ then, $N_o = N/B = 0.04$ Watts/Hz.

c- As in (a) above: $\frac{S}{N} = 2^{\left(\frac{C_r}{B}\right)}-1$ but now with C_r half the given rate i.e. C_r =10 kbps.

$\frac{S}{N} = 2^{\left(\frac{C_r}{B}\right)}-1 = 2^{\left(\frac{10000}{4000}\right)}-1 = 4.657$ ratio or $\frac{S}{N} = 10\,Log_{10}(4.657) = 6.681\ dB$

Example-13 Answer TRUE or FALSE and correct the FALSE statements

No.	The given statement	Ans. T/F	Correction of FALSE Statement	Additional Remarks
1	Self-information is always +ve	T	------	since I_x= -LogP(x) then it is +ve
2	H(x) for certain binary source is 1.5 Bit/Symbol	F	There is no binary source that have H(x)>1 Bit/Symbol	For Binary source $H_{max}(x)$= Log_2 2 =1 Bit/symbol
3	Shannon-Hartley Equation specifies that R_{max}=2B symbols/sec (B is channel Bandwidth)	F	Nyquist's theorem specifies that R_{max}=2B symbols/sec (B is channel Bandwidth)	
4	The capacity of noiseless ternary channel is $Log_2 3$ = 1.585 Bits/symbol	T	------	C=Log_2M-H(y|x) general for noiseless H(y|x)=0 and for ternary M=3.
5	The source entropy H(x) for continues source depends on the power (or variance) of x	T	------	Large magnitude of x shows large entropy for continues source

6	Mutual information may be +ve, 0, or -ve	T	------	$I_{x:y} = \log[\frac{P(y\|x)}{P(y)}]$ If the ratio [.] >1 then +ve. If [.]=1 then 0, and if [.]<1 then -ve
7	The presence of DC level (or average) affects the information for CRV	F	The DC level has no effect on information of CRV	
8	C_r is increased as the noise power (N) is increased.	F	According to $C_r = B\, Log_2(1+\frac{S}{N})$, C_r is inversely proportional with N	N is denominator
9	If S/N = 50 dB, then the corresponding ratio is 500.	F	If S/N = 50 dB, then the corresponding ratio is 10^5.	$\frac{S}{N}\Big]_{ratio} = 10^{[\frac{S}{N}]_{dB} \div 10}$ $=10^{50/10}=10^5$
10	dBm is measure for signal power when measured in mWatt.	T	------	dBm is not S/N it is $10Log_{10}S$ when S measured in mWatt

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