Al-Mustaqbal University

College of Engineering and Technology

Department of Biomedical Engineering

Stage: three

Signal Processing

2023-2024

Lecture (7): Discrete Fourier Transform (DFT)

Discrete Fourier Transform

DFT: is the tool that separates the frequency components. Viewing the signal in terms of its frequency components gives us a better understanding of its characteristics.

The Discrete Fourier Transform (DFT) of a signal x may be defined by:
Time-domain signal representation: Signals occur, mostly, in a form that is called the time-domain representation. In this form, the signal amplitude, x(t), is represented against time (t). If the signal is denned at all instants of time, it is referred as a continuous-time or analog signal. If the signal defined only at discrete instants of time, then the signal is referred as a discrete signal.

Frequency-domain signal representation: In this representation, the variation of a signal in terms of frequency is used to characterize the signal. At each frequency, the amplitude and phase or, equivalently, the amplitudes of the cosine and sine components of the sinusoid are required for representing a signal.

The DFT and the IDFT

The DFT is defined by

$$x(n) = \sum_{k=0}^{k=N-1} x(k) e^{-j\left(\frac{2\pi}{N}\right)nk}$$

$k = 0,1,2, \ldots, N-1$

and its inverse (the IDFT) is given by:

$$x(k) = 1/N \sum_{n=0}^{n=N-1} x(n) e^{-j\left(\frac{2\pi}{N}\right)nk}$$

$n = 0,1,2, \ldots, N-1$

x(n) denotes the input signal at time (sample) n, and *x(k)* denotes the k^{th} spectral sample.

N: number of samples

Example: Compute the DFT of the 4-point sequence {2, 0, -1, 3}.

Solution:

$x[n] = \{2, 0, -1, 3\}, N = 4$

$$x(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi}{N}kn}$$

$$x(k) = \sum_{n=0}^{3} x(n) e^{-j\frac{2\pi}{4}kn}$$

$x(k) = x(0) + x(1)\, e^{-j\frac{\pi}{2}k} + x(2) e^{-j\pi k} + x(3) e^{-j\frac{3\pi}{2}k}$

$$x(k) = 2 - 1e^{-j\pi k} + 3e^{-j\frac{3\pi}{2}k}$$

for k=0

$$x(0) = 2 - 1 + 3 = 4$$

for k=1

$$x(1) = 2 - 1(cos\pi - j\ sin\pi) + 3(cos\frac{3\pi}{2} - j\ sin\frac{3\pi}{2})$$

$$x(1) = 2 - 1(-1 - j0) + 3\big(0 - j(-1)\big)$$

$$= 2 + 1 + 3j = 3 + 3j$$

for k=2

$$x(2) = 2 - 1e^{-j2\pi} + 3e^{-j3\pi}$$

$$= 2 - 1 - 3 = -2$$

for k=3

$$x(3) = 2 - 1e^{-j3\pi} + 3e^{-j\frac{9\pi}{2}}$$

$$x(3) = 2 + 1 - j3 = 3 - 3j$$

So, DFT of x(k) =[4, 3+3j, -2, 3-3j]

Example:

Given a sequence x[n] for $0 \le n \le 3$, where x[0] = 1, x[1] = 2, x[2] = 3, and x[3] = 4.

Evaluate its DFT X[k].

Thus, for $k = 0$

$$
\begin{aligned}
X[0] &= \sum_{n=0}^{3} x[n]e^{-j0} = x[0]e^{-j0} + x[1]e^{-j0} + x[2]e^{-j0} + x[3]e^{-j0} \\
&= x[0] + x[1] + x[2] + x[3] \\
&= 1 + 2 + 3 + 4 = 10
\end{aligned}
$$

for $k = 1$

$$
\begin{aligned}
X[1] &= \sum_{n=0}^{3} x(n)e^{-j\frac{\pi n}{2}} = x[0]e^{-j0} + x[1]e^{-j\frac{\pi}{2}} + x[2]e^{-j\pi} + x[3]e^{-j\frac{3\pi}{2}} \\
&= x[0] - jx[1] - x[2] + jx[3] \\
&= 1 - j2 - 3 + j4 = -2 + j2
\end{aligned}
$$

for $k = 2$

$$
\begin{aligned}
X[2] &= \sum_{n=0}^{3} x(n)e^{-j\pi n} = x[0]e^{-j0} + x[1]e^{-j\pi} + x[2]e^{-j2\pi} + x[3]e^{-j3\pi} \\
&= x[0] - x[1] - x[2] - x[3] \\
&= 1 - 2 + 3 - 4 = -2
\end{aligned}
$$

for $k = 3$

$$
\begin{aligned}
X[3] &= \sum_{n=0}^{3} x[n]e^{-j\frac{3\pi}{2}} = x[0]e^{-j0} + x[1]e^{-j\frac{3\pi}{2}} + x[2]e^{-j3\pi} + x[3]e^{-j\frac{\pi}{2}} \\
&= x[0] + jx[1] - x[2] - jx[3] \\
&= 1 + j2 - 3 - j4 = -2 - j
\end{aligned}
$$

X(k)={10, -2+2j, -2, -2-j}