Lecture 6: Torque and rotational motion collision

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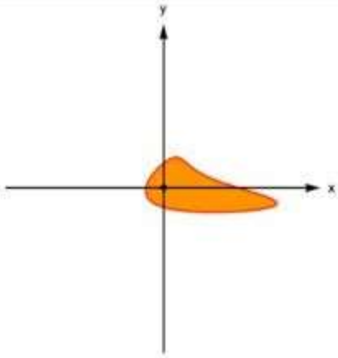
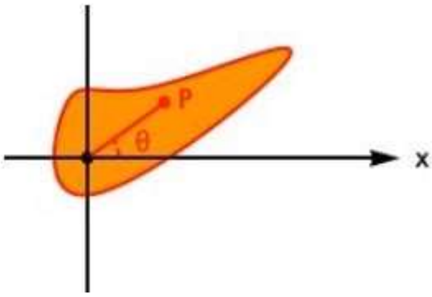
rotations of a rigid body

➜ suppose we have a body which rotates about some axis



➜ we can define its orientation at any moment by an angle, θ

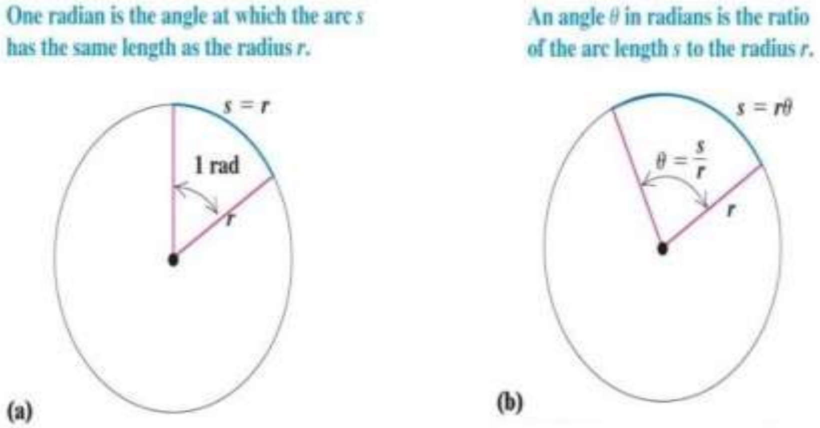
(any point P will do)



radians

➜ measuring θ in degrees turns out to be a poor choice

➜ **radians** are a more natural choice of angular unit



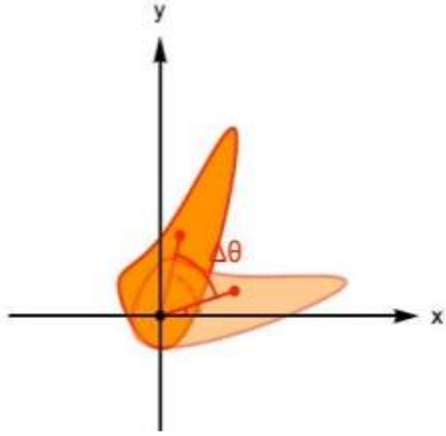
$$1\,\text{rad} = \frac{360^\circ}{2\pi} = 57.3^\circ$$

angular velocity

➜ describe the rate of rotation by the change in angle in a given time

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{\theta_2 - \theta_1}{t_2 - t_1}$$

(notice, just like linear motion but with x→θ)



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Q/ ---------- describe the rate rotation by the change in angle in a given time.

a) Acceleration velocity

b) **Angular velocity**

c) Radians

d) Collision

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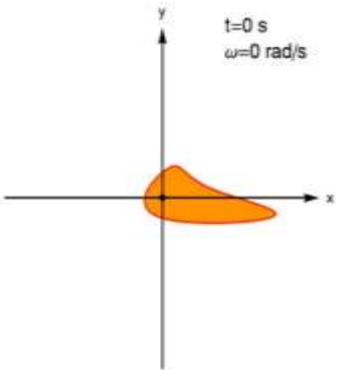
angular acceleration

➔ suppose the rate of rotation changes - we need angular acceleration

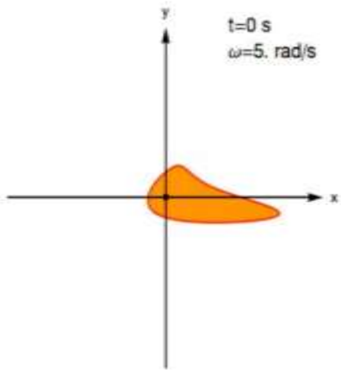
$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega_2 - \omega_1}{t_2 - t_1}$$

(notice, just like linear motion but with v➔ω)

positive constant α



negative constant α
begins with positive ω



angular motion vs. linear motion

➜ the analogy between angular motion & linear motion is strong

➜ for constant acceleration we have

$$v = v_0 + at$$
$$x = x_0 + v_0 t + \frac{1}{2}at^2$$
$$v^2 = v_0^2 + 2a(x - x_0)$$

➜ for constant angular acceleration we have

$$\omega = \omega_0 + \alpha t$$
$$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$$
$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$

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Q/ ($\omega = \omega_\circ + \propto t$) for

a) **Constant angular acceleration**
b) t = 0
c) $v = 0$
d) a = 0

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Q/ ($v = v_0 + at$) for

a) **Constant acceleration**
b) $x = 0$
c) $v = x$
d) a = v

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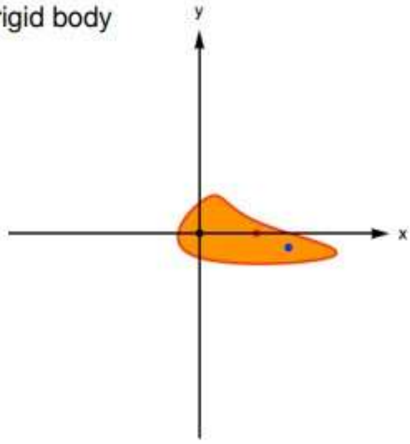
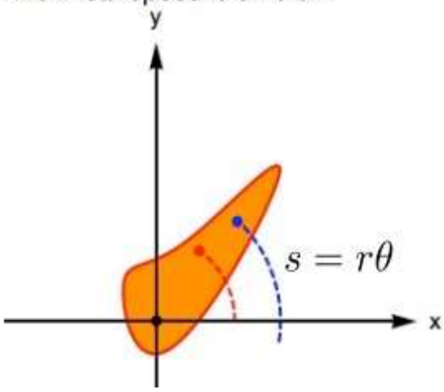
motion of points in a rigid body

→ consider the motion of a couple of points within the rigid body



the blue point at a large radius travels further in the same time than the red point

so although the angular speed is the same, the linear speed is different



$$v = \frac{\Delta s}{\Delta t} = r\frac{\Delta \theta}{\Delta t} = r\omega$$

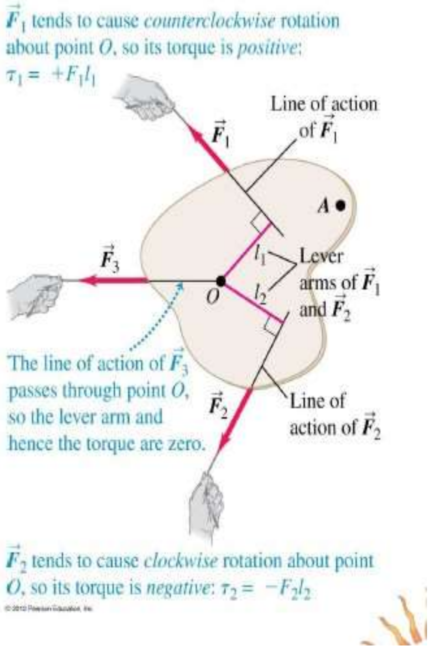
Force vs. Torque

- Forces cause accelerations
- What cause angular accelerations?
- There are three factors that determine the effectiveness of the force:
  - The *magnitude* of the force
  - The *position* of the application of the force
  - The *angle* at which the force is applied

Torque Definition

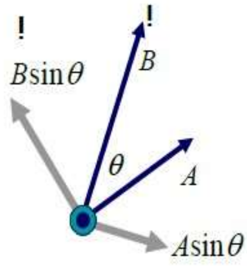
- Torque, τ, is the tendency of a force to rotate an object about some axis
- Let **F** be a force acting on an object, and let **r** be a position vector from a rotational center to the point of application of the force, with **F** perpendicular to **r**. The magnitude of the torque is given by

$$\tau = rF$$

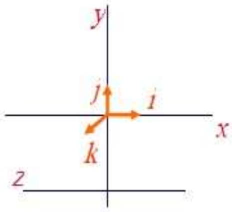


Cross Product

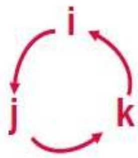
$$\vec{C} = \vec{A} \times \vec{B}$$



- The cross product of two vectors says something about how perpendicular they are.
- Magnitude: $\left|\vec{C}\right| = \left|\vec{A}\times\vec{B}\right| = AB\sin\theta$
  - θ is smaller angle between the vectors
  - Cross product of any parallel vectors = zero
  - Cross product is maximum for perpendicular vectors
  - Cross products of Cartesian unit vectors:

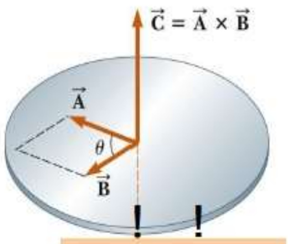


$$\hat{i}\times\hat{j}=\hat{k};\ \hat{i}\times\hat{k}=-\hat{j};\ \hat{j}\times\hat{k}=\hat{i}$$
$$\hat{i}\times\hat{i}=0;\ \hat{j}\times\hat{j}=0;\ \hat{k}\times\hat{k}=0$$



Cross Product

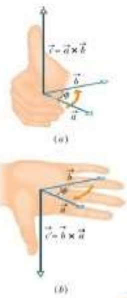
- Direction: C perpendicular to both A and B (right-hand rule)
  - Place A and B tail to tail
  - Right hand, not left hand
  - Four fingers are pointed along the first vector A
  - "sweep" from first vector A into second vector B through the smaller angle between them
  - Your outstretched thumb points the direction of C
- First practice $\vec{A}\times\vec{B}=\vec{B}\times\vec{A}?$



$$\vec{A}\times\vec{B}=\vec{B}\times\vec{A}?$$

Right-hand rule

$$\vec{A}\times\vec{B}=-\vec{B}\times\vec{A}$$



Torque Units and Direction

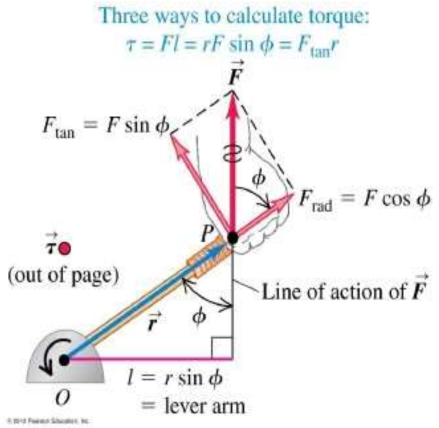
- The SI units of torque are N·m
- Torque is a vector quantity

$$\vec{\tau} = \vec{r} \times \vec{F}$$

- Torque **magnitude** is given by

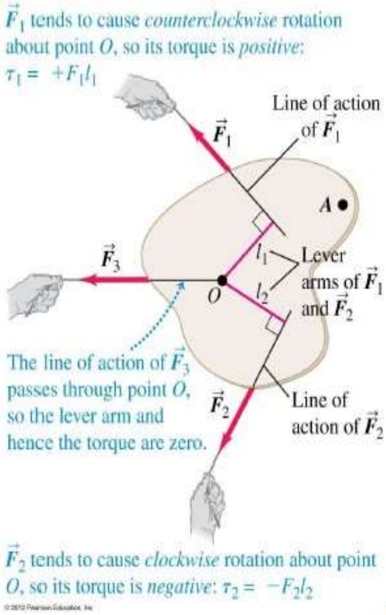
$$\tau = rF\sin\phi = Fl$$

- Torque will have **direction**
  - If the turning tendency of the force is counterclockwise, the torque will be positive
  - If the turning tendency is clockwise, the torque will be negative



Net Torque

- The force $\mathbf{F}_1$ will tend to cause a counterclockwise rotation about O
- The force $\mathbf{F}_2$ will tend to cause a clockwise rotation about O
- $\Sigma\tau = \tau_1 + \tau_2 + \tau_3 = F_1l_1 - F_2l_2$
- If $\Sigma\tau \neq 0$, starts rotating
- If $\Sigma\tau = 0$, rotation rate does not change



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Q/ If --------- start rotating.

a) $\sum \tau = F1$

b) $\boldsymbol{\sum \tau \neq 0}$

c) $\sum \tau = 0$

d) b and c

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Q/ If --------- rotation rate does not change.

a) $\sum \tau = -1$

b) $\sum \tau \neq 0$

c) $\boldsymbol{\sum \tau = 0}$

d) $\sum \tau = 1$

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Q/ The SI unites of torque are --------.

a) N.m

b) N

c) m

d) W

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