Higher order partial derivative :

- 2^{nd} Order partial derivative for functions with two variable

$F(x,y)$ has partial derivative $F_x = \frac{\partial f}{\partial x}$ and $F_y = \frac{\partial f}{\partial y}$.

- The 2^{nd} order partial derivative are denoted by:

$$F_{xx} = \frac{\partial^2 F}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial F_x}{\partial x}$$

$$F_{yy} = \frac{\partial^2 F}{\partial y^2} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial F_y}{\partial y}$$

$$F_{xy} = \frac{\partial^2 F}{\partial y\, \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial f_x}{\partial y}$$

$$F_{yx} = \frac{\partial^2 F}{\partial x\, \partial y} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial F_y}{\partial x}$$

Ex : let $F(x,y) = x\cos y + y e^x$ find F_{xx}, F_{yy}, F_{xy}, F_{yx} ?

Sol:

1- $F_{xx} = \cos y * 1 + y e^x * 1$

$= \cos y + y e^x$

$= 0 + y e^x * 1 = y e^x$

2- $F_{yy} = X(-\sin y) + e^X * 1$

$= -X \sin y + e^X$

$= -X \cos y + 0$

$= -X \cos y$

3- $F_{xy} = \cos y + y e^X$

$= -\sin y + e^X * 1$

$= -\sin y + e^X$

4- $F_{yx} = -X \sin y + e^X$

$= -\sin y (1) + e^X (1)$

$= -\sin y + e^X$

- Consider the 3rd order partial derivative of the function $Z = f(x,y)$.

$$\frac{\partial^3 f}{\partial x^3} = \frac{\partial}{\partial x}\left(\frac{\partial^2 f_x}{\partial x^2}\right) = \frac{\partial f_{xx}}{\partial x} = f_{xxx} \quad \text{Similarity}$$

$$\frac{\partial^3 f}{\partial y^3} = f_{yyy}$$

$$\frac{\partial^3 f}{\partial y \partial x^2} = \frac{\partial}{\partial y}\left(\frac{\partial^2 f}{\partial x^2}\right) = \frac{\partial f_{xx}}{\partial y} = f_{xxy}$$

Ex: $f(x,y) = x\cos y + y e^x$ find f_{xxx}, f_{yyy}, f_{xxy}, f_{yyx} ?

Sol:

$$f_{xxx} = \cos y * 1 + y\, e^x * 1$$

$$= \cos y + y e^x$$

$$= 0 + y * e^x * 1 \quad = y e^x$$

$$= y e^x * 1 = y e^x$$

$$f_{yyy} = x\,(-\sin y) + e^x * 1$$

$$= -x \sin y + e^x \quad = -x \cos y + 0$$

$$= -x \cos y \quad = -x\,(-\sin y) = x \sin y$$

H.W $\Rightarrow$ f_{xxy}, f_{yyx} ?