Chapter Five

Dynamics of Fluid Flow

Dr. Abdulkareem A. Wahab

5.1 / Introduction:

This chapter includes the study of forces causing fluid flow. Thus dynamics of fluid flow is the study of fluid motion with the forces causing flow.

5.2 / Equation of motion:

According to Newton's second law of motion, the net force F_x acting on a fluid element in the direction of x is equal to mass (m) of the fluid element multiplied by the acceleration (a) in the x – direction. Thus mathematically:

$$F_x = m\, a_x \qquad (5.1)$$

5.3 / Euler's equation of motion:

This equation of motion in which the forces due to gravity and pressure are taken into consideration. This is derived by considering the motion of the fluid element along a stream – lines as:

Consider a stream – line in which flow is taking place in S – direction as shown in Fig. (5.1). Consider a cylindrical element of cross – section dA and:

1. **Pressure force (pdA) in the direction of flow.**
2. **Pressure force (p + $\frac{\partial p}{\partial s}$ds) dA opposite to the direction of flow .**
3. **Weight of element (W = γ V = ρ g V = ρ g dA dS). $a_s = \frac{dv}{dt} = \frac{v\partial v}{\partial s}$**

[(Note : (where v is a function of s & t , so , and (cos Θ = $\frac{dz}{ds}$)].

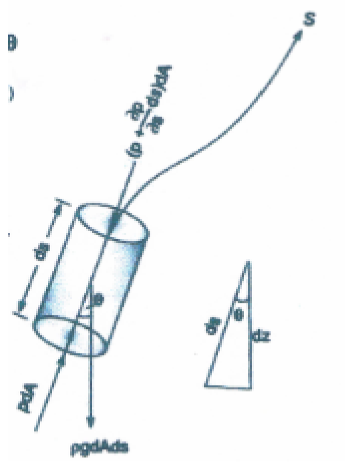

Fig. (5.1)

F = m a

$$\sum F = m\,a$$

$$pdA - \left(p + \frac{\partial p}{\partial s} ds \right) dA - \rho\, g\, dA\, dS \cos \Theta = \rho\, dA\, ds \,.\, \frac{v \partial v}{\partial s}$$

$$\frac{dp}{\rho} + g\,dz + v\,dv = 0 \qquad (5.2)$$

***Equation*(5.2) is known as Euler's equation of motion.**

In which, p – pressure, ρ – mass density, g – gravity, z – head, v – velocity.

5.4 / Bernoulli's equation from Euler's equation:

Bernoulli's equation is obtained by integrating the Euler's equation of motion (5.2):

$$\int \frac{dp}{\rho} + \int g\,dz + \int v dv = \text{constant}$$

If flow is compressible, ρ is constant and:

$$\frac{p}{\rho} + g\,z + \frac{v^2}{2} = \text{constant}$$

$$\frac{p}{\rho g} + z + \frac{v^2}{2g} = \text{constant} \qquad (5.3)$$

Equation (5.3) is a Bernoulli's equation, in which,

$\frac{p}{\rho\, g}$ - **pressure energy per unit weight of fluid (pressure head)**

$\frac{v^2}{2g}$ - **kinetic energy per unit weight of fluid (kinetic head).**

Z - potential energy per unit weight of fluid (potential head).

Assumptions:

The following are theassumptions made in the derivation of Bernoulli's equation:

1 – The fluid is ideal (viscosity is zero).

2 – The flow is steady. ($\frac{\partial v}{\partial t} = 0$)

3 - The flow is incompressible. (ρ = constant).

4 – The flow is irrotational.

5.5 / Bernoulli′s equation for real fluid:

In real fluid, there are some losses, these losses have to be taken into consideration. Thus the Bernoulli′sequation for real fluids between points 1 and 2 is given as:

$$\frac{p_1}{\rho\, g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho\, g} + \frac{v_2^2}{2g} + z_2 + H_L \qquad (5.\ 4)$$

In which h_l is loss of energy (head loss) between points 1 & 2.

5.6 / Instruments for measure the rate of flow:

1 – Venture meter.

2 – Orifice meter

3 – Pitot - tube .

<u>1 – Venture meter:</u>

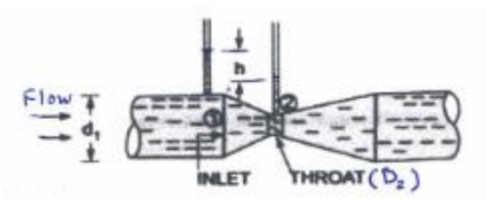

Fig. (5.2)

A venture meter is a device used for measuring the rate of a flow of a fluid flowing through a pipe. It consists of three parts:

(1) short converging part , (2) Throat , (3) Diverging part .

It is based on the principle of Bernoulli′s equation.

Applying Bernoulli equation at section 1 & section 2 (throat) , we get :

$$\frac{P_1}{\rho\, g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho\, g} + \frac{V_2^2}{2g} + Z_2$$

As pipe is horizontal, hence $Z_1 = Z_2$

$$\frac{P_1}{\rho\, g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho\, g} + \frac{V_2^2}{2g} \;, \; \frac{P_1 - P_2}{\rho\, g} = \frac{V_2^2}{2\, g} - \frac{V_1^2}{2\, g}$$

$\frac{P_1 - P_2}{\rho\, g}$ **is the difference of pressure heads at section 1 & section 2**

And it is equal to (h):

$$h = \frac{V_2^2}{2g} - \frac{V_1^2}{2g} \qquad (5.5)$$

Now, applying continuity equation at section 1 & 2:

$A_1 V_1 = A_2 V_2$, $V_1 = \frac{A_2 V_2}{A_1}$

Substituting the of V_1in the equation (5 .5),

$$h = \frac{V_2^2}{2g}\left[\frac{A_1^2 - A_2^2}{A_1^2}\right]$$

$$V_2^2 = 2g\,h\,\frac{A_1^2}{A_1^2 - A_2^2}$$

$$V_2 = \frac{A_1}{\sqrt{A_1^2 - A_2^2}}\sqrt{2gh}$$

But, $Q = A_2 V_2$

$$Q = \frac{A_2 A_1}{\sqrt{A_1^2 - A_2^2}}\sqrt{2gh} \qquad (5.6)$$

Where,

X - reading of differential manometer.

S_h – Sp. gravity of the liquid manometer.

S_o – Sp. gravity of the liquid flowing through pipe.

2. Orifice meter:

It is a device used for measuring the rate of flow of a fluid through a pipe. It also works on the same principle as that of venture meter. It consists of a flat circular which has a circular sharp edges hole called orifice, which is concentric with the pipe. The orifice diameter is kept generally 0.5 times the diameter of the pipe (or 0.4 to 0.8 times the pipe diameter).

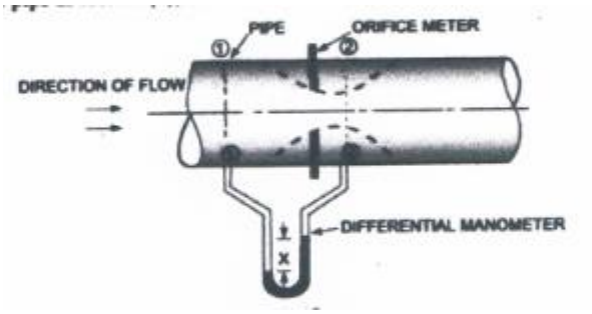

Fig.(5.3)

Applying Bernoulli equation between section 1 & section 2, we get:

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + Z_2$$

As the pipe is horizontal, $Z_1 = Z_2$

$$\frac{P_1}{\rho g} - \frac{P_2}{\rho g} = \frac{V_2^2}{2g} - \frac{V_1^2}{2g}$$

$$h = \frac{V_2^2 - V_1^2}{2g}$$

$$2g\,h = V_2^2 - V_1^2$$

$$V_2 = \sqrt{2gh + V_1^2} \qquad (5.8)$$

Now, section 2 is at the (vena – contracta) and A_2 represents the area at the (vena – contracta). If the Ao is the area of orifice, then we have:

$$C_c = \frac{A_2}{A_o}$$

Where, Cc - Coefficient of contraction.

Then, $A_2 = C_c A_o$

From continuity equation, we have, $A_1 V_1 = A_2 V_2$

$$V_1 = \frac{A_2}{A_1} V_2 = \frac{A_o C_c}{A_1} V_2$$

Substituting the value of V_1 in equation (5. 8) :

$$V_2 = \sqrt{2gh + \frac{A_o^2 C_c^2 V_2^2}{A_1^2}}$$

$$V_2 = \frac{\sqrt{2gh}}{\sqrt{1-(\frac{A_o}{A_1})^2\, C_c^2}}$$, **After simplified** , $$V_2 = \frac{A_1\sqrt{2g\,h}}{C_c\sqrt{A_1^2 - A_0^2}}$$

The discharge $Q = C_d\, V_2\, A_2 = C_d V_2\, A_o\, C_c = \dfrac{C_d A_o C_c A_1\sqrt{2\,g\,h}}{C_c\sqrt{A_1^2 - A_o^2}}$

$$Q = \frac{C_d A_o A_1\sqrt{2gh}}{\sqrt{A_1^2 - A_o^2}} \qquad (5.9)$$

Where,

C_d **- Coefficient of discharge for orifice meter.**

3.Pitot – tube:

It is a device used for measuring the velocity of flow at any point in a pipe or a channel. It is based on the principle that if the velocity of flow at a point becomes zero, the pressure there is increased due to the conversion of the kinetic energy into pressure energy.

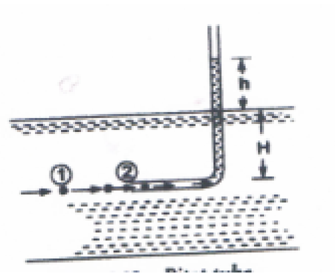

Fig. (5.4)

Applying Bernoulli's between points 1 & 2, we get:

$$\frac{P_1}{\rho\, g} + \frac{V_1^2}{2g} + Z_1 = \frac{P_2}{\rho\, g} + \frac{V_2^2}{2g} + Z_2$$

$Z_1 = Z_2$**, because the points 1 & 2 are on the same line, and** $V_2 = 0$**.**

$$\frac{P_1}{\rho\, g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho\, g}$$

$$H + \frac{V_1^2}{2g} = (\,h + H\,)$$

$$\frac{V_1^2}{2g} = h \quad , \quad V_1 = \sqrt{2gh}$$

In this equation, the velocity V_1 is theoretical velocity, but the actual velocity is :

$$(V_1)_{actual} = C_v\sqrt{2gh} \qquad (5.10)$$

<u>There are many arrangements with Pitot – tube (as shown in Figures) :</u>

1 – Pitot- tube along with a vertical piezometer tube, as shown in Fig. (5.5)

2 – Pitot –tube connected with piezometer tube as shown in Fig. (5. 6).

3 – Pitot – tube and vertical piezometer tube connected with a differential U – tube manometer as shown in Fig. (5. 7).

4 – Pitot – tube, which consists of two circular concentric tubes one inside, the other with some annular space in between as shown in Fig. (5.8). The outlet of these two tubes are connected to the differential manometer where the differential of pressure head (h) is measured by knowing the difference of the levels of the manometer liquid, say x, then:

$$h = x\left[\frac{S_m}{S_o} - 1\right]$$

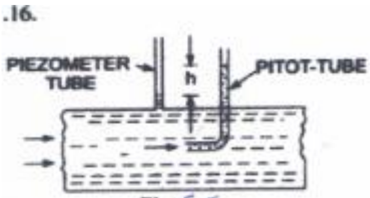

Fig. 5.6

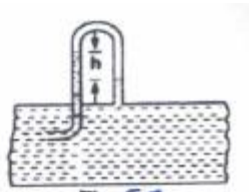

Fig. 5.5

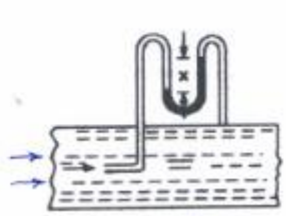

Fig. 5.7

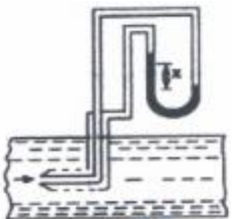

Fig. 5.8