Chapter Four

Kinematics of Flow and Ideal Flow

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4.1 / Introduction:

Kinematic is defined as that branch of science which deals with motion of particles without considering the forces causing the motion. Once the velocity is known, then the pressure distribution and hence forces acting on the fluid can be determined.

4.2 Types of fluid flow:

The fluid flow is classified as:

1 – Steady & unsteady flows. 2 – Uniform & non-uniform flows.

3 – Laminar & turbulent flows. 4 – Compressible & incompressible flows.

5 – Rotational & irrational flows. 6 – One, two, three – dimensional flows

1. **Steady & unsteady flows :**

Steady flow is defined as that type of flow in which the fluid characteristics like velocity, pressure, density, etc. At a point do not change with time. Thus for steady flow, mathematically, we have:

$$\left(\frac{\partial V}{\partial t}\right) = 0 \quad , \quad \left(\frac{\partial p}{\partial t}\right) = 0 \quad , \quad \left(\frac{\partial \rho}{\partial t}\right) = 0$$

Unsteady flow is that type of flow, in which the velocity, pressure or density at a point changes with respect to time. Thus, mathematically, for unsteady flow:

$$\left(\frac{\partial V}{\partial t}\right) \neq 0 \quad , \quad \left(\frac{\partial p}{\partial t}\right) \neq 0 \quad , \left(\frac{\partial \rho}{\partial t}\right) \neq 0$$

2 – Uniform and nonuniform flows:

Uniform flow is defined as that type of flow in which the velocity at any given time does not change with respect to space (i.e., length of direction of the flow). Mathematically, for uniform flow:

$$\left(\frac{\partial V}{\partial s}\right) = 0$$

Non uniform flow is that type of flow in which the velocity at any point time changes with respect to space. Thus, mathematically, for nonuniform flow:

$$\left(\frac{\partial V}{\partial s}\right) \neq 0$$

3 – Laminar and Turbulent flows:

Laminar flow is defined as that type of flow in which the fluid particles move along stream line and all the stream lines are straight and parallel. Thus the particles moves laminar or layers gliding smoothly over the adjacent layer (this type of flow is called also viscous flow).

Turbulent flow is that type of flow in which the fluid particles move in zig- zag way. Due to the movement of fluid particles in a zig – zag way, the eddies formation takes place which are responsible for high energy loss. For pipe flow, the type of flow is determined by a Reynolds number (Re):

$$Re = \frac{VD\rho}{\mu} = \frac{VD}{\nu}$$

(V– Velocity, D – diameter of pipe, ρ – mass density, μ – dynamic viscosity, ν – kinematic viscosity)

If the value of Reynolds number is less than 2000, the flow is called laminar. (In the pipe flow).

If the value of Reynolds number is more than 4000, it is called turbulent flow. (In the pipe flow)

If the value of Reynolds number lies between 2000 & 4000, the flow is called Transition flow. (In the pipe flow).

4 – Compressible & incompressible flows:

Compressible flow is defined as that type of flow in which mass density of the fluid changes from point to point or in the other words the mass density (ρ) is not constant for the fluid. Thus, mathematically, for compressible flow:

$$\rho \neq \text{Constant}$$

Incompressible flow is that type of flow in which the density is constant for the fluid flow. Liquids are generally incompressible while gases are compressible. Mathematically, for compressible flow:

$$\rho = \text{constant}$$

5 – Rotational & Irrotational flows:

Rotational flow is that type of flow in which the fluid particles while flowing along the stream lines, also rotate about their own axis , and if the fluid particles while flowing along stream lines , do not rotate about their own axis then that type of flow is called irrotational flow.

6 – One, Two, and Three – Dimensional flows:

One – dimensional flow is that type of flow in which the flow parameter, such as velocity is a function of time and one space co – ordinate only, say x , hence mathematically , for one – dimensional flow :

$$u = f(x), \quad v = 0 \quad , \quad w = 0$$

Where u, v and w are velocity components in x, y and z directions respectively.

Two – dimensional flow is that type of flow in which the velocity is a function of time and two rectangular space co – ordinates say x and y. Thus, mathematically for two – dimensional flow:

$$u = f_1(x, y), \; v = f_2(x, y) \text{ and } w = 0$$

Three – dimensional flow is that type of flow in which the velocity is a function of time and three mutually perpendicular direction. Thus, mathematically, for three – dimensional flow.

$$u = f_1(x, y, z), \; v = f_2(x, y, z) \text{ and } w = f_3(x, y, z)$$

4.3 / Rate of volume flow (Discharge) (Q):

It is defined as the quantity of a fluid flowing per second through a section of a pipe or a channel. For an incompressible fluid (liquid) the rate of flow or discharge is expressed as the volume of fluid flowing across the section per second. For compressible fluids, the rate of flow is usually expressed as the weight of fluid flowing across the section.

$$Q = A\,V \qquad (4.1)$$

Where,

Q – Discharge. (m^3/s)

A – Cross – sectional area ($A = \pi r^2 = \frac{\pi d^2}{4}$ m^2)

(r – radius, d – diameter)

V – Velocity. (m/s)

$$\dot{m} = \rho\, Q = \rho\, A\, V \qquad (4.2)$$

$\dot{m}$ - Mass flow rate. (Kg / s), ρ - mass density. (Kg / m^3)

4.4 / Continuity equation:

The equation based on the principle of conservation of mass is called continuity equation. Thus for a fluid flowing through the pipe at all the cross – section, the quantity of fluid per second is constant. Consider two cross – section of a pipe as shown in Fig. (4.1).

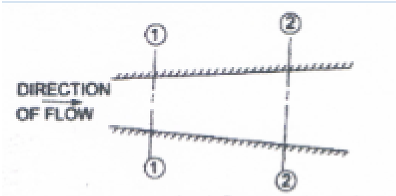

Fig.4.1 *Fluid flowing through a pipe.*

Let, V_1 , ρ_1 , A_1 at cross – section 1 - 1

Let, V_2 , ρ_2 , A_2 at cross - section 2 - 2

Mass flow rate at cross – section 1 - 1 = $\rho_1 A_1 V_1$

Mass flow rate at cross – section 2 - 2 = $\rho_2 A_2 V_2$

According to law of conservation of mass:

$$\rho_1 A_1 V_1 = \rho_2 A_2 V_2 \qquad (4.3)$$

Equation (4.3) is applicable to the compressible as well as incompressible fluids and is called (Continuity equation). If the fluid is incompressible, then the equation (4.3) becomes:

$$A_1 V_1 = A_2 V_2 \qquad (4.4)$$

(Because the density is constant, $\rho_1 = \rho_2$)

4.5 / Continuity equation in three dimensional:

Consider a fluid element of lengths dx, dy, and dz in the direction of x, y, z. Let u, v and w are the inlet velocity components in x, y and z directions respectively .Mass of fluid entering the face ABCD per second. Fig. (4.2).

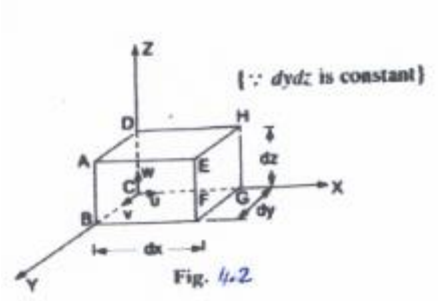

Fig. 4.2

Mass of fluid entering the face ABCD = $\rho V_x A = \rho u (dy \times dz)$

Mass of fluid leaving the face EFGH = $\rho u\, dydz + \frac{\partial}{\partial x}(\rho u\, dy\, dz)\, dx$

Gain of mass in x – direction = Mass through ABCD – Mass through EFGH

$$= \rho u\, dydz - \left(\rho u dydz + \frac{\partial}{\partial x}(\rho u\, dy\, dz) dx\right)$$

$$= \rho u\, dx\, dz - \rho u\, dydz - \frac{\partial}{\partial x}(\rho u\, dydz)\, dx$$

$$= -\frac{\partial}{\partial x}(\rho\, u\, dy\, dz) dx \qquad (1)$$

Gain of mass in Z – direction = $-\frac{\partial}{\partial z}(\rho w\, dx\, dy) dz$ (2)

Gain of mass in y – direction = $-\frac{\partial}{\partial y}(\rho\, v\, dx\, dz) dy$ (3)

Net gain of mass = $-\left[\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho\, v) + \frac{\partial}{\partial z}(\rho\, w)\right] dx\, dy\, dz$ (4.5)

But , mass of fluid in element is = $\rho \times$ volume = $\rho\, dx\, dy\, dz$

Rate of mass flow = $\frac{\partial}{\partial t}(\rho\, dx\, dy\, dz)$ (4.6)

Equating the equation (4.5) with equation (4.6):

$$-\left[\frac{\partial}{\partial x}(\rho\, u) + \frac{\partial}{\partial y}(\rho\, v) + \frac{\partial}{\partial z}(\rho\, w)\right] dx\, dy\, dz = \frac{\partial \rho}{\partial t}\, dx\, dy\, dz$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho\, w) = 0 \qquad (4.7)$$

Equation (4.7) is the continuity equation in Cartesian co – ordinates in its most general form, this equation is applicable to:

1. **Steady and unsteady flow.**

2. Uniform and non – uniform flow.

3. Compressible and incompressible flow.

For steady flow, $\frac{\partial \rho}{\partial t} = 0$, hence the equation (4.7) becomes:

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0 \qquad (4.8)$$

If the fluid is incompressible, then ρ is constant an the equation (4.8) becomes as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \qquad (4.9)$$

Equation (4.9) is the continuity equation in three – dimensional. For a two – dimensional, the components velocity w = 0, and hence continuity equation becomes as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (4.\ 10)$$