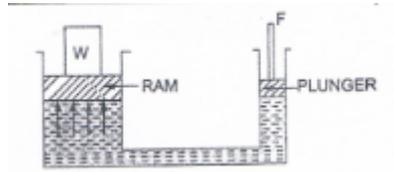


**Problems of Chapter Two**

Dr. Abdulkareem Abdulwahab

**Problem 2.1 /**

A hydraulic press has a ram of 30 cm diameter and a plunger of 4.5 cm diameter. Find the weight ( W ) lifted by the hydraulic press , when the force ( F ) applied at the plunger is 500 N .

**Solution :**

$$\text{Area of ram ( A )} = \frac{\pi}{4} D^2 = \frac{\pi}{4} \times ( 0.3 )^2 = 0.07 \text{ m}^2$$

$$\text{Area of plunger ( a )} = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times ( 0.045 )^2 = 0.0016 \text{ m}^2$$

$$\text{Pressure due to plunger} = \frac{F}{a} = \frac{500}{0.0016} = 314465 \text{ N / m}^2$$

Due to **Pascal Law** , the pressure will be equally transmitted in all direction.

$$\text{i . e the pressure at ram} = 314465 \text{ N/ m}^2$$

$$\text{But pressure at ram} = \frac{W}{a} = \frac{W}{0.0016}$$

$$314465 = \frac{W}{0.0016} \quad , \quad W = 22222 \text{ N} = 22. 222 \text{ kN} .$$

**Problem 2.2 /**

The pressure at a point in a fluid is given 3.924 N/cm<sup>2</sup>. Find the corresponding height of fluid when the fluid is : ( 1 ) water , ( 2 ) oil of sp. gr. 0.9 .

**Solution :**

$$P = 3.924 \text{ N/cm}^2 = 3.924 \times 10^4 \text{ N /m}^2$$

$$S_{\text{oil}} = \frac{\rho_{\text{oil}}}{\rho_{\text{water}}} \quad , \quad \rho_{\text{oil}} = S_{\text{oil}} \rho_{\text{water}} = 0.9 \times 1000 = 900 \text{ kg / m}^3$$

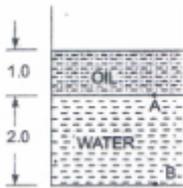
$$(1) P = \rho_w g h_w, \quad h_w = \frac{P}{\rho_w g} = \frac{3.924 \times 10^4}{1000 \times 9.81} = 4 \text{ m water}$$

$$(2) P = \rho_{oil} g h_{oil}, \quad h_{oil} = \frac{P}{\rho_{oil} g} = \frac{3.924 \times 10^4}{900 \times 9.81} = 4.44 \text{ m oil}$$

### Problem 2.3 /

An open tank contains water up to a depth of 2 m and above it an oil of sp. gr. 0.9 for a depth 1 m . Find the pressure ( 1 ) at the interface of the two liquids and ( 2 ) at the bottom of the tank.

Solution :



( 1 ) At interface :

$$S_{oil} = \frac{\rho_{oil}}{\rho_{water}}, \quad \rho_{oil} = S_{oil} \rho_w = 0.9 \times 1000 = 900 \text{ kg/ m}^3$$

$$P = \rho_{oil} g h_{oil} = 900 \times 9.81 \times 1 = 8829 \text{ N/ m}^2$$

( 2 ) At the bottom ( point B ) :

$$P = \rho_w g h_w + 8829$$

$$P = 1000 \times 9.81 \times 2 + 8829 = 19620 + 8829 = 28449 \text{ N/ m}^2$$

### Problem 2.4 /

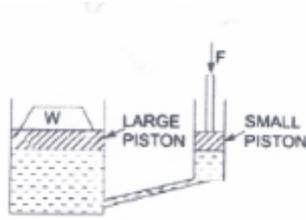
The diameter of a small piston and a large piston of a hydraulic jack are 3 cm and 10 cm respectively. A force of 80 N is applied on a small piston. Find the load lifted by the large piston , when ( 1 ) the piston are at the same level , ( 2 ) small piston is 40 cm above the large piston. ( the density of the liquid in the jack is  $1000 \text{ kg/ m}^3$  ).

Solution :

$$\text{Area of small piston ( a )} = \frac{\pi}{4} d^2 = \frac{\pi}{4} \times 3^2 = 7.068 \text{ cm}^2$$

$$\text{Area of large piston ( A )} = \frac{\pi}{4} D^2 = \frac{\pi}{4} \times ( 10 )^2 = 78.54 \text{ cm}^2$$

( 1 ) when the pistons are at the same level :



$$\text{Pressure on small piston} = \frac{F}{a} = \frac{80}{7.068} \quad ( 1 )$$

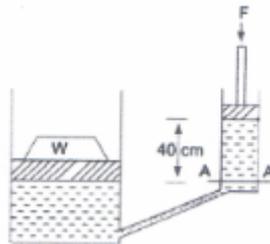
$$\text{Pressure on large piston} = \frac{W}{A} = \frac{W}{78.54} \quad ( 2 )$$

pressure on large piston = pressure on small piston

$$\frac{W}{78.54} = \frac{80}{7.068}$$

$$W = 888.96 \text{ N}$$

( 2 ) when small piston is 40 cm above the large piston :



$$\text{Pressure on small piston} = \frac{80}{7.068} = 11.32 \text{ N/ cm}^2$$

$$\begin{aligned} \text{Pressure due to 40 cm of liquid} &= \rho g h = 1000 \times 9.81 \times 0.4 \text{ N/m}^2 \\ &= \frac{1000 \times 9.81 \times 0.4}{10^4} = 0.3924 \text{ N/ cm}^2 \end{aligned}$$

$$\text{Pressure at section A - A} = 11.32 + 0.3924 = 11.71 \text{ N/ cm}^2$$

Pressure on large piston = pressure at section A - A

$$\frac{W}{78.54} = 11.71$$

$$W = 11.71 \times 78.54 = 919.7 \text{ N}$$

### Problem 2.5 /

What are the gauge pressure and absolute pressure at a point 3 m below the free surface of a liquid having a density of  $1.53 \times 10^3 \text{ kg/m}^3$ , if the atmospheric pressure is equivalent to 750 mm of mercury ? The specific gravity ( S ) of mercury( Hg ) is 13.6 .

#### Solution:

$$P_{\text{atm.}} = \rho_m g h_m = 13.6 \times 1000 \times 9.81 \times 0.75 = 100062 \text{ N/m}^2$$

$$P_{\text{gag.}} = \rho_L g h_L = 1.53 \times 10^3 \times 9.81 \times 3 = 45028 \text{ N/m}^2$$

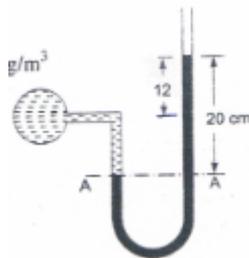
$$P_{\text{abs.}} = P_{\text{atm.}} + P_{\text{gag.}} = 100962 + 45028 = 145090 \text{ N/m}^2$$

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### Problem 2.6 /

The right limb ( leg ) of a simple U – tube manometer containing mercury is open to the atmospheric , while the left limb is connected to a pipe in which a fluid of sp.gr.0.9 is flowing. The center of the pipe is 12 cm below the level of mercury in the right limb. Find the pressure of fluid in the pipe , if the difference of mercury level in the two limbs is 20 cm.

#### Solution :



Equating the pressure above A – A in two limbs , we get

$$P + \rho_f g h_f = \rho_m g h_m$$

$$P + 0.9 \times 1000 \times 9.81 \times 0.08 = 13.6 \times 1000 \times 9.81 \times 0.2$$

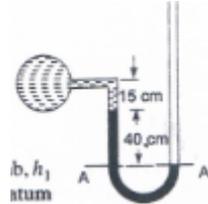
$$P = 25977 \text{ N / m}^2$$

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Problem 2.7 / A simple U – tube manometer containing mercury is connected to a pipe in which a fluid of sp. gr. 0.8 and having vacuum pressure is flowing .

The other end of manometer is open to atmosphere . Find the vacuum pressure in pipe , if the difference of mercury level in the two limbs is 40 cm and the height of fluid in the left from the center of pipe is 15 cm below.

**Solution :**



Equating pressures above datum line A – A in two limbs , we get

$$P + \rho_f g h_f + \rho_m g h_m = 0$$

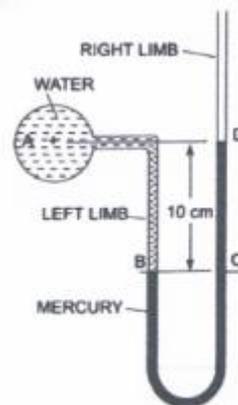
$$P + 0.8 \times 1000 \times 9.81 \times 0.15 + 13.6 \times 1000 \times 9.81 \times 0.4 = 0$$

$$P = - 54543.6 \text{ N / m}^2$$

**Problem 2.8 /**

A U - tube manometer is used to measure the pressure of water in a pipe line, which is in excess of atmospheric pressure. The limb of manometer contains mercury and is open to atmosphere . The contact between water and mercury is in the left limb. Determine the pressure of water in the main line , if the difference in level of mercury in the limbs of U- tube is 10 cm and the free surface of mercury is in level with the center of the pipe . If the pressure of water in pipe line is reduced to 9810 N/m<sup>2</sup> , calculate the new difference in the level of mercury.

**Solution :**



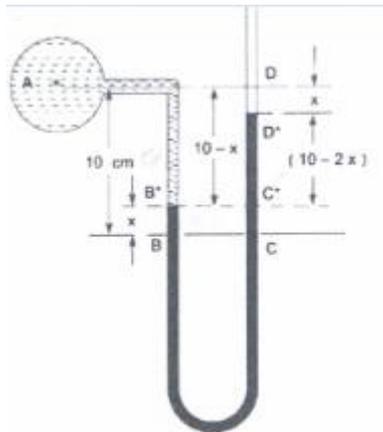
Case ( 1 ) : Pressures at B = Pressures at C

$$P_A + \rho_w g h_w = \rho_m g h_m$$

$$P_A + 1000 \times 9.81 \times 0.1 = 13.6 \times 1000 \times 9.81 \times 0.1$$

$$P_A = 12360.6 \text{ N/m}^2$$

Case ( 2 ) :



If the value of  $P_A$  becomes is  $9810 \text{ N/m}^2$ , i.e is less than the  $12360.6 \text{ N/m}^2$ , hence mercury in left limb will rise , this rise of mercury in the left limb is equal to the fall of mercury in the right limb, because the volume of mercury remains same .

Let  $X$  rise of mercury in left limb in cm , the points  $B$  ,  $C$  and  $D$  , show the initial conditions whereas points  $B^*$  ,  $C^*$  and  $D^*$  show the final conditions .

Pressure at  $B^*$  = Pressure at  $C^*$

$$P_A + \rho_w g h_w = \rho_m g h_m$$

$$9810 + 1000 \times 9.81 \times \frac{(10-x)}{100} = 13.6 \times 1000 \times 9.81 \times \frac{(10-2x)}{100}$$

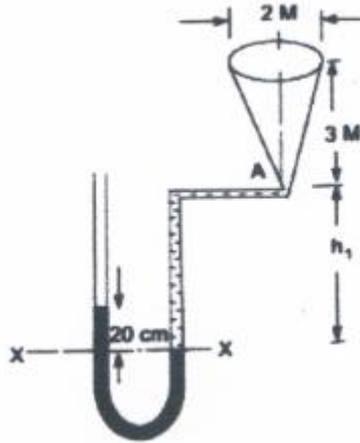
$$X = 1 \text{ cm}$$

$$\text{New difference of mercury} = 10 - 2x = 10 - 2 \times 1 = 8 \text{ cm} .$$

### Problem 2.9 /

A conical vessel having its outlet at A to which a U- tube manometer is connected. The reading of the manometer given , when the vessel is empty. Find the reading of the manometer when the vessel is completely filled with water.

Solution :



1. When vessel is empty :

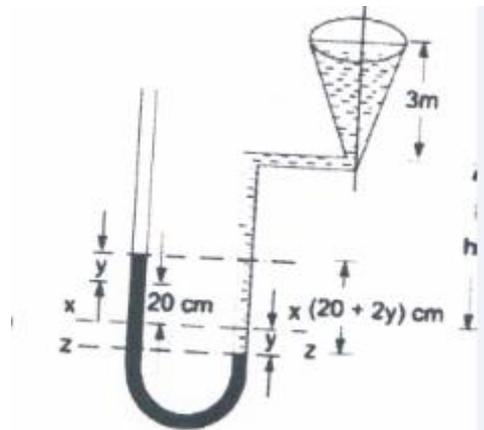
Equating the pressure above the datum X - X , we have

$$\rho_m g h_m = \rho_w g h_w$$

$$13.6 \times 1000 \times 9.81 \times 0.2 = 1000 \times 9.81 \times h_1$$

$$h_1 = 2.72 \text{ m of water}$$

2. When vessel is full of water :



At datum ( Z - Z ) :

Pressure in left limb = pressure in right limb

$$\rho_m g \left( 0.2 + \frac{2y}{100} \right) = \rho_w g \left( 3 + h_1 + \frac{y}{100} \right)$$

$$13.6 \times 1000 \times 9.81 \left( 0.2 + \frac{2y}{100} \right) = 1000 \times 9.81 \left( 3 + 2.72 + \frac{y}{100} \right)$$

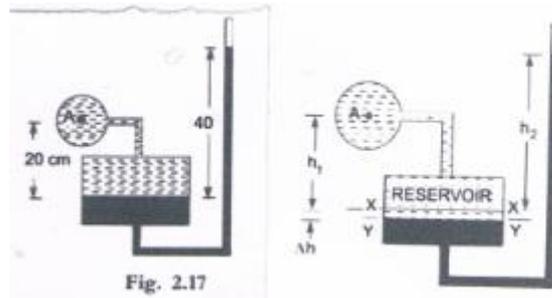
$$y = 11.45 \text{ cm}$$

The difference of mercury level in two limbs =  $20 + 2y$

$$= 20 + 2 \times 11.45 = 42.9 \text{ cm of Hg.}$$

**Problem 2.10 /**

A single column manometer is connected to a pipe containing a liquid of sp. gr. 0.9 . Find the pressure in the pipe , if the area of the reservoir is 100 times the area of the tube for the manometer reading . The sp. gr.of mercury is 13.6.



$$\frac{\text{Area of reservoir}}{\text{Area of right limb}} = \frac{A}{a} = 100$$

Consider the datum Y –Y , and equating the pressure of two sides :

$$P_A + \rho_1 g ( h_1 + \Delta h ) = \rho_2 g ( h_2 + \Delta h )$$

$$P_A = \rho_2 g ( h_2 + \Delta h ) - \rho_1 g ( h_1 + \Delta h )$$

$$P_A = \rho_2 g h_2 + \rho_2 g \Delta h - \rho_1 g h_1 - \rho_1 g \Delta h$$

$$P_A = \Delta h ( \rho_2 g - \rho_1 g ) + \rho_2 g h_2 - \rho_1 g h_1$$

But ,  $A \Delta h = a h_2$

$$\Delta h = \frac{a h_2}{A}$$

$$P_A = \frac{a h_2}{A} ( \rho_2 g - \rho_1 g ) + \rho_2 g h_2 - \rho_1 g h_1$$

$$P_A = \frac{0.4}{100} ( 13600 \times 9.81 - 900 \times 9.81 ) + 13600 \times 9.81 \times 0.4$$

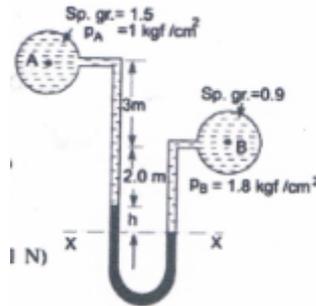
$$- 900 \times 9.81 \times 0.2$$

$$P_A = 52134 \text{ N / m}^2$$

**Problem 2.11 /**

A differential manometer is connected at the two points A and B of two pipes. The pipe A contains a liquid of sp. gr. 1.5 while pipe B contains a liquid of sp. gr. 0.9. The pressures at A and B are  $1 \text{ kgf/cm}^2$  and  $1.8 \text{ kgf/cm}^2$  respectively. Find the difference in mercury level in the differential manometer ( h ).

**Solution :**



$$P_A = 1 \text{ kgf/cm}^2 = 1 \times 10^4 \text{ kgf/m}^2 = 1 \times 10^4 \times 9.81 \text{ N/m}^2$$

( because  $1 \text{ kgf} = 9.81 \text{ N}$  )

$$P_B = 1.8 \times 10^4 \times 9.81 \text{ N/m}^2$$

Taking X - X as datum line

$$P_A + \rho_A g ( 3 + 2 ) + \rho_m g h = P_B + \rho_B g ( 2 + h )$$

$$1 \times 10^4 \times 9.81 + 1500 \times 9.81 \times 5 + 13600 \times 9.81 \times h = 1.8 \times 10^4 \times 9.81 + 900 \times 9.81 ( 2 + h )$$

$$h = 0.181 \text{ m} = 18.1 \text{ cm}$$

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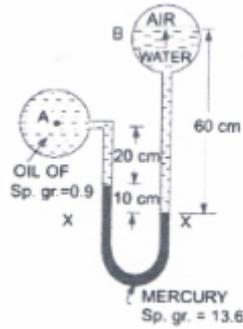
**Problem 2.12 /**

A differential manometer is connected at the two points A and B as shown in Fig. At B air pressure is  $9.81 \text{ N/cm}^2$  ( abs. ) . Find the absolute pressure at A .

**Solution :**

$$P_B = 9.81 \times 10^4 \text{ N/m}^2$$

Taking datum line at X - X



$$P_A + \rho_{oil} g \times 0.2 + \rho_m g \times 0.1 = P_B + \rho_w g \times 0.6$$

$$P_A + 900 \times 9.81 \times 0.2 + 13600 \times 9.81 \times 0.1 = 9.81 \times 10^4$$

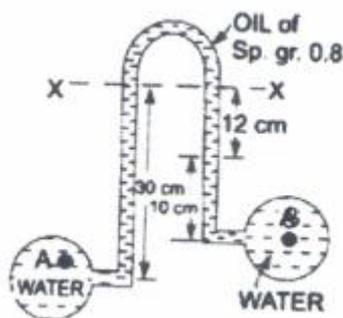
$$+1000 \times 9.81 \times 0.6$$

$$P_A = 88876.8 \text{ N / m}^2$$

### Problem 2.13 /

Water is flowing through two different pipes to which an inverted differential manometer having an oil of sp.gr. 0.8 is connected. The pressure head in the pipe A is 2 m of water. Find the pressure in the pipe B for the manometer reading as shown in Fig. ?

Solution :



$$P_A = \rho_w g h = 1000 \times 9.81 \times 2 = 19620 \text{ N / m}^2$$

Equating the pressures in two sides , below X - X ,

$$P_A - \rho_w g h_w = P_B - \rho_w g h_w - \rho_{oil} g h_{oil}$$

$$19620 - 1000 \times 9.81 \times 0.3 = P_B - 1000 \times 9.81 \times 0.1 - 800 \times 9.81 \times 0.12$$

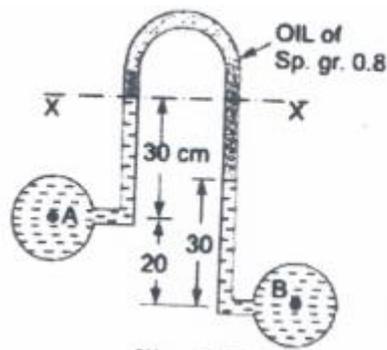
$$P_B = 18599.76 \text{ N / m}^2$$


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### Problem 2.14 /

An inverted differential manometer is connected to two pipes A and B which convey . The fluid in manometer is oil of sp.gr. 0.8. For the manometer reading , find the pressure difference between A and B .

Solution :



Equating the pressures in two sides , below X - X ,

$$P_A - \rho_w g h_w = P_B - \rho_w g h_w - \rho_{oil} g h_{oil}$$

$$P_A - 1000 \times 9.81 \times 0.3 = P_B - 1000 \times 9.81 \times 0.3 - 800 \times 9.81 \times 0.2$$

$$P_A - 2943 = P_B - 4512.6$$

$$P_B - P_A = 4512.6 - 2943 = 1569.6 \text{ N / m}^2$$


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### Problem 2.15 /

In the micromanometer, the pressure difference is wanted in Pascal , when air is in the system ,  $S_2 = 1$  ,  $S_2 = 1.1$  ,  $a / A = 0.01$  ,  $R = 5 \text{ mm}$  ,  $t = 20^\circ \text{c}$  , and the barometer reads 760 mm Hg .

Solution :

Equation of micromanometer is :

$$P_C - P_D = R \left[ \gamma_3 - \gamma_2 \left( 1 - \frac{a}{A} \right) - \gamma_1 \frac{a}{A} \right]$$

$$\rho_{air} = \frac{p}{RT} \quad , \quad \rho_{air} = \frac{13.6 \times 9810 \times 0.76}{287 (20 + 273)} = 1.205 \text{ kg / m}^3$$

$$\begin{aligned} \gamma_3 - \gamma_2 \left( 1 - \frac{a}{A} \right) &= 1.1 \times 9810 - 9810 ( 1 - 0.01 ) \\ &= 9810 ( 1.1 - 0.99 ) = 1078 \text{ N/m}^3 \end{aligned}$$

The term  $\gamma_1 \frac{a}{A}$  may be neglected , because it is small ,

$$P_C - P_D = 0.005 \times 1078 = 5.39 \text{ pa}$$


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### Problem 2.16/

A Bourdon gage register a vacuum of 310 mm of mercury when the atmospheric pressure is 100 kpa. Absolute. Calculate the corresponding absolute pressure ?

Solution :

$$P_{\text{abs.}} = P_{\text{atm.}} - P_{\text{gage (vacuum)}}$$

$$P_{\text{gage}} = \gamma_m h_m = S_m \gamma_w h_m = 13.6 \times 9810 \times 0.310 = 41.36 \text{ kpa}$$

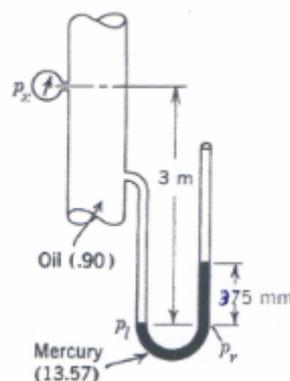
$$P_{\text{abs.}} = 100 - 41.36 = 58.64 \text{ kpa}$$


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### Problem 2.17 :

Vertical pipeline with attached bourdon gage and manometer as shown in Fig. The manometer is open to the atmospheric and there is no flow in the pipe. What will be the bourdon gage reading ( $P_x$ ) ?

Solution :



The pressures over the datum  $P_L - P_R$  in two sides are equal :

$$P_x + \gamma_{\text{oil}} \times h_{\text{oil}} = \gamma_m \times h_m$$

$$P_x + 0.9 \times 9810 \times 3 = 13.6 \times 9810 \times 0.375$$

$$P_x = 23544 \text{ Pa.} = 23.544 \text{ kpa}$$