Chapter Two

Pressure and Its Measurement

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2.1/ Fluid pressure at a point:

Consider a small area dA in large mass of fluid. If the fluid is static, then the force exerted by fluid on the area dA will always be perpendicular to the surface dA. Let dF is the force acting on the area dA in the normal direction.

Then the ratio of $\frac{dF}{dA}$ is known as the pressure (P). Hence mathematically the pressure at a point in a fluid at rest (static) is:

$$P = \frac{dF}{dA}$$

If the force (F) is uniformly distributed over the area (A), the pressure at any point is given by:

$$P = \frac{F}{A} \qquad (2.1)$$

The unit of pressure are (1) kgf / cm^2 (in MKS) (meter – kilogram – second)

(2) Newton / m^2 (N/ m^2) (in SI unit). N / m^2 is known as Pascal (1 bar = 100 kpa = 10^5 Pascal)

2.2 / Pascal Law:

It states that the pressure or intensity of pressure at a point in a static fluid is equal in all directions. This is proved as:

The fluid element is of very small directions, i.e., (dx, dy and ds).

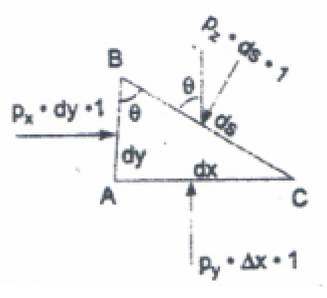

Fig. (2.1) Forces on a fluid element.

Consider an arbitrary fluid element of wedge shape in a fluid mass at rest, as shown in Fig. (2.1). Let the width of the element perpendicular to the plane of

paper is unity and P_x , P_y and P_z are the pressure acting on the face AB , AC and BC respectively . Let angle ABC is ϴ . Then the forces acting on the element are:

1. **Pressure force normal to the surfaces.**
2. **Weight of the element in the vertical direction.**

Force on the face AB = $P_x \times$ area of face AB

$$= P_x \times dy \times 1$$

Force on the face AC = $P_y \times dx \times 1$

Force on the face BC = $P_z \times ds \times 1$

Weight of element = mass of element × g

$$= (\text{volume} \times \rho) \times g = \left(\frac{AB \times AC}{2} \times 1\right) \times \rho \times g$$

$$\sum F_x = 0$$

$$P_x \times dy \times 1 - P_z (ds \times 1) \sin(90 - \Theta) = 0$$

$$P_x \times dy - P_z \times ds \times \cos\Theta = 0$$

But, from Fig. (2.1), $ds \cos\Theta = AB = dy$

$$P_x \times dy - P_z \times dy = 0$$

$$P_x = P_z$$

Similarly, $\sum F_y = 0$

$$P_y \times dx \times 1 - P_z \times ds \times 1 \times \cos(90 - \Theta) - \frac{dx \times dy}{2} \times 1 \times \rho \times g = 0$$

$$P_y \times dx - P_s\, ds \sin\Theta - \frac{dx\, dy}{2} \times \rho \times g = 0$$

But, $ds \sin\Theta = dx$, and the element has very small, therefore the weight is negligible (third term), therefore,

$$P_y = P_s$$

Therefore, $Px = P_y = P_z$ (2.2)

This equation shows that the pressure at ant point in x , y and z direction is equal .

2.3 / Pressure variation in a fluid at rest (fluid static) :

The pressure at any point in a fluid at rest is obtained by the hydrostatic law which states that the rate of increase of pressure in a vertically downward direction must be equal to the specific weight (weight density) of the fluid at the point. This is proved as:

Consider a small fluid element as shown in Fig. (2.2).

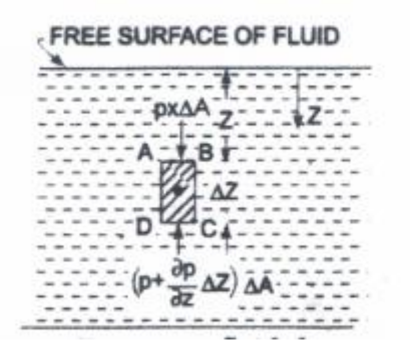

Fig. (2.2) Forces on a fluid element

Let, ∆ A - cross – section area of element.

∆ Z - Height of fluid element.

P - pressure on face AB.

Z – distance of fluid element from free surface.

The forces acting on the fluid element are:

1. **Pressure force on AB = p × ∆ A (acting perpendicular to face AB in the downward direction).**
2. **Pressure force on CD = $(p + \frac{\partial p}{\partial z} \Delta Z) \times \Delta A$ (acting perpendicular to face CD vertically upward direction) .**
3. **Weight of fluid element = γ × volume = ρ g (∆A × ∆Z).**
4. **Pressure forces on surface BC and AD are equal and opposite. For equilibrium of fluid element, we have**

$$p\Delta A - (p + \frac{\partial p}{\partial Z} \Delta Z) \Delta A + \rho g (\Delta A \times \Delta Z) = 0$$

$$p\Delta A - p\Delta A - \frac{\partial p}{\partial z} \Delta Z \Delta A + \rho g \times \Delta A \times \Delta Z = 0$$

$$- \frac{\partial p}{\partial z} \Delta Z \Delta A + \rho g \times \Delta A \Delta Z = 0$$

$$\frac{\partial p}{\partial z}\ \Delta Z\ \Delta A = \rho\ g \times \Delta A\ \Delta Z$$

$$\frac{\partial P}{\partial Z} = \gamma$$

$$\frac{dP}{dZ} = \gamma \quad , \quad d\,p = \gamma\ dz \quad , \quad \int dp = \gamma \int dz$$

$$P = \gamma\ Z \qquad (2.3)$$

Equation (2.3) states that the rate of increase of pressure in vertical direction is equal to weight density (γ) of the fluid at that point. This is Hydrostatic Law. (Z is called pressure head).

2.4 / Absolute, Gauge, Atmospheric, And Vacuum Pressures

The pressure on the fluid is measured in two difference systems. In one system, it is measured above the absolute zero or complete vacuum and it is called the absolute pressure and in other system, pressure is measured above the atmospheric pressure and is called gauge pressure.

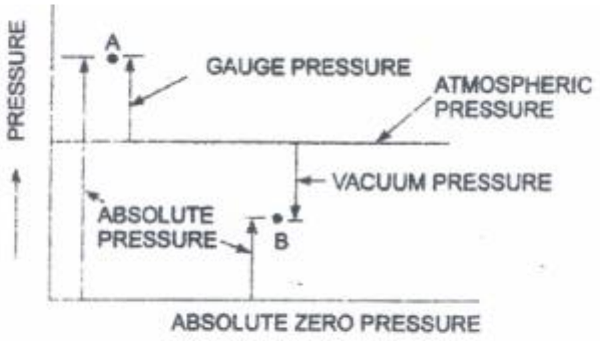

Fig. (2.3) Relationship between pressure.

The relationship between the absolute pressure, gauge pressure and vacuum pressure are shown in Fig. (2.3).

Mathematically:

$$P_{abs.} = P_{atm.} \pm P_{gauge} \qquad (2.4)$$

$$P_{A(abs)} = P_{atm.} + P_{gauge}$$

$$P_{B(abs)} = P_{atm.} - P_{gauge(vacuum)}$$

The values of atmospheric pressure at sea level at 15°c:

$P_{atm.} = 101.3\ KN/m^2$ **(kpa)**, $P_{atm.} = 10^5\ N/m^2$ **(Pascal)**

$P_{atm.}$ = 76 cm Hg. $P_{atm.}$ = 10 m (water), $P_{atm.}$ = 14.7 psi

$P_{atm.}$ = 14.7 psi. $P_{atm.}$ = 1bar.

2.5/ Measurement of pressure:

The pressure of a fluid is measured by the following devices:

1. **Manometers.**
2. **Mechanical Gauges.**

2.5.1/ Manometers:

Manometers are defined as the devices used for measuring the pressure at a point in a fluid by balancing the column of fluid by the same or another column of the fluid. They are classified as:

(1) simple manometers , (2) Differential manometers

2.5.2 / Simple Manometers:

A simple manometer consists of a glass tube having one of its ends connected to a point where pressure is to be measured and other end remains open to atmosphere. Common types of simple manometers are :

1. Piezometer. 2. U – tube Manometer. , 3. Single Column Manometer.

1. Piezometer:

It is simple form of manometer, used for measuring gauge pressures, as shown in Fig. (2.4)

$$P_A = \rho\, g\, h = \gamma\, h \quad N/m^2 \quad (Pascal) \qquad (2.5)$$

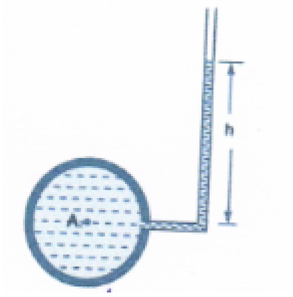

Fig. (2.4) Piezometer.

2. U– tube Manometer:

It consists of glass tube bent in U- shape, one end of which is connected to a point at which pressure is to be measured and other end remains open to

the atmosphere as shown in Fig. (2.5). In this manometer, we can measure positive pressure (gauge pressure) and negative pressure (vacuum).

Let B is the point at which pressure is to be measured (p). The datum line is A – A.

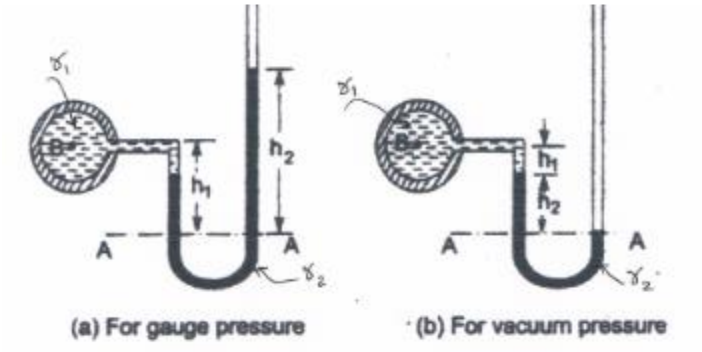

Fig. (2.5) U- tube Manometer.

If we want to measure the pressure (p) at point B .

There are two legs in the manometer, if there is equilibrium between two legs (right and left legs) over the datum (A – A), i. e the pressure at each leg over the datum are equal.

Mathematically,

(a) For gauge pressure :

Pressure at left leg = Pressure at right leg

$$P + \gamma_1 h_1 = \gamma_2 h_2$$

$$P = \gamma_2 h_2 - \gamma_1 h_1 \quad N/m^2 \qquad (2.6)$$

(b) For vacuum (negative) pressure:

Pressure at left leg = pressure at right leg

$$P + \gamma_1 h_1 + \gamma_2 h_2 = 0$$

$$P = -\gamma_1 h_1 - \gamma_2 h_2 \quad N/m^2 \qquad (2.7)$$

2.6/ Differential Manometers :

Differential manometer are the devices used for measuring the difference of pressures between between two points in a pipe or in two different pipes. A differential manometer consists of a U – tube, containing a heavy liquid (liquid manometer), frequently is mercury (Hg). Most commonly types of differential manometers are:

1. U-tube differential manometer., 2 – Inverted U-tube differential manometer. Fig. (2.6) shows the differential manometer of U-tube type.

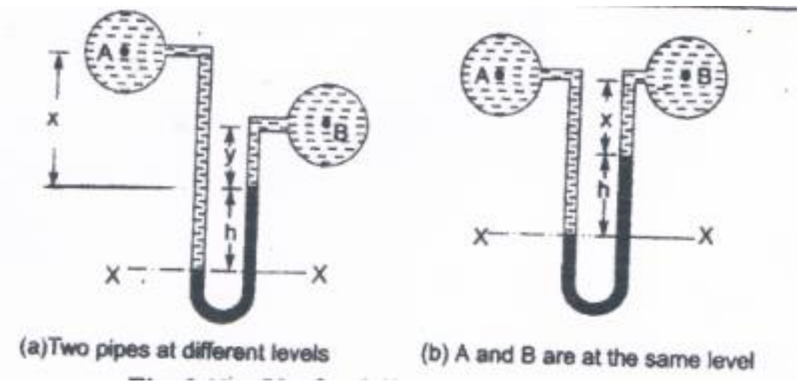

Fig. (2.6) U-tube differential manometer.

In Fig. (2.6) (a), let the two points A & B are at different level and also contains liquids at different specific gravity (S) (sp. gr.).

Level X - X, level of equilibrium, the pressures in the left leg equal the pressures in right leg:

$$p_A + \gamma_A (x + h) = p_B + \gamma_B y + \gamma_m h$$

$$p_A - p_B = \gamma_B y + \gamma_m h - \gamma_A (x + h) \qquad (2.8)$$

In Fig. (2.6) (b),

$$p_A + \gamma_A (x + h) = p_B + \gamma_B x + \gamma_m h$$

$$p_A - p_B = \gamma_B x + \gamma_m h - \gamma_A (x + h) \qquad (2.9)$$

2.Inverted U- tube differential manometer:

It consists of an inverted U-tube. The two ends of the tube are connected to the points whose difference of pressure is to be measured. It is used for measured difference of low pressure. Fig. (2.7) shows an inverted U- tube differential manometer connected to the two points A & B. Let the pressure at A is more than the pressure at B.

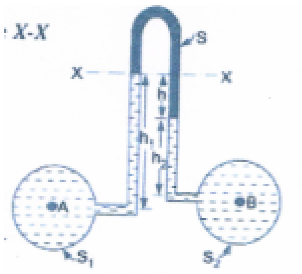

Fig. (2.7)

Taking X –X as datum line, then,

Pressure at the left leg below the X – X = $p_A - \gamma_1 h_1$

Pressure at the right leg below the X- X = $p_B - \gamma_2 h_2 - \gamma_m h$

Pressure at the left leg = pressure at the right leg

$$P_A - \gamma_1 h_1 = p_B - \gamma_2 h_2 - \gamma_m h$$

$$P_A - p_B = \gamma_1 h_1 - \gamma_2 h_2 - \gamma_m h \qquad (2.10)$$

1.7 / Inclined Single Column Manometer:

Fig. (2.8) shows the inclined single column manometer. This manometer is more sensitive. Due to inclination the distance moved by the heavy liquid in the right side will be more.

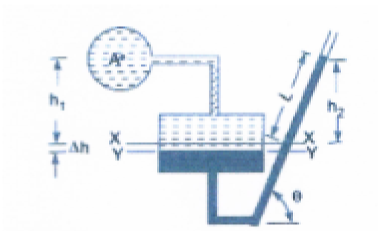

Fig. (2.8) Inclined manometer

Let L – length of heavy liquid moved in right side from X – X

Θ - inclination of right leg with horizontal.

h_2 - vertical rise of heavy liquid in right leg from X – X

(L sin Θ)

$$P_A = \gamma_2 h_2 - \gamma_1 h_1 \qquad (\text{ but } h_2 = L \sin\Theta)$$

$$P_A = \gamma_2 L \sin \Theta - \gamma_1 h_1 \qquad (2.11)$$

2.8/ Micromanometer:

It is used for determine small differences in pressure. With two gage liquids, immiscible in each other and in the fluid to be measured, a large gage difference R , as shown in Fig.(2.9) can be produced for a small pressure difference. The heaver gage liquid fills the lower U-tube up to O - O then the lighter gage liquid is added to both sides, filling the larger reservoir up to 1 – 1. The gas or liquid in the system fills the space above 1 – 1. When the pressure at C is slightly greater than at D, the menisci move as indicated in Fig. (2.9). The volume of liquid displaced in each reservoir equals the displacement in the U –tube, thus,

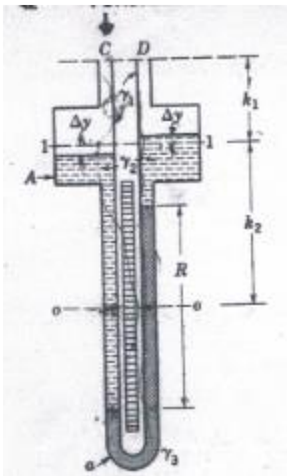

Fig. (2.9) Micromanometer

$$\Delta y . A = \frac{R}{2} . a \quad , \quad \Delta y = \frac{R\, a}{2A}$$

In which, A is area of reservoir, a is area of U – tube.

The manometer equation may be written starting at C,

$$P_c + (k_1 + \Delta y)\gamma_1 + (k_2 - \Delta y + \frac{R}{2}) \gamma_2 - R \gamma_3 - (k_2 - \frac{R}{2} + \Delta y) \gamma_2 -$$

$(k_1 - \Delta y)\,\gamma_1 = p_D$

In which γ_1, γ_2 γ_3 are the specific weights . Simplifying and substituting for Δy gives:

$$p_C - p_D = R\left[\gamma_3 - \gamma_2\left(1 - \frac{a}{A}\right) - \gamma_1\frac{a}{A}\right] \qquad (2.12)$$

The quantity in bracket is a constant for specified gage and fluids, hence, the pressure difference is directly proportional to R.

1.8 / Bourdon Gage (Mechanical):

The bourdon pressure gage as shown in Fig. (2.10) is typical of the devices used for measuring gage pressure.

The bourdon gage (shown schematically) in Fig. (2.11). In the gage, a bent tube (A) of elliptical cross section is held rigidly at (B) and its free end is connected to a pointer (C) by a link (D). When pressure is

admitted to the tube, its cross section tends to become circular, causing the tube to straighten and move the pointer to the right over the graduated scale.

Fig. (2.10) typical of Bourdon gage

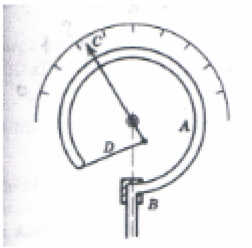

Fig. (2.11) Schematically shown of Bourdon gage

The pointer rests at zero on the scale, when the gauge is disconnected, in this condition the pressure inside and outside of the tube are the same.