Al-Mustaqbal University
Department of Biomedical Engineering
Third Stage / 1st Course
"Transport Phenomena for BME"
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Lecture 3
"Viscosity"

Viscosity is a measure of a fluid's resistance to flow. It describes the internal friction of a moving fluid. A fluid with large viscosity resists motion because its molecular makeup gives it a lot of internal friction. A fluid with low viscosity flows easily because its molecular makeup results in very little friction when it is in motion. Gases also have viscosity, although it is a little harder to notice it in ordinary circumstances.

To understand viscosity, let fluid between two parallel infinites in width and length plates. See Fig. 1.1. The bottom plate A is fixed and the upper plate B is moveable. The vertical distance between the two plates is represented by h. A constant force F is applied to the moveable plate B causing it to move along at a constant velocity uB with respect to the fixed plate. This behavior is consistent with the definition of a fluid: a material that deforms continuously under the application of a shearing stress, regardless of how small the stress is.

After some infinitesimal time dt, a line of fluid that was vertical at time t= 0 will move to a new position, as shown by the dashed line in Fig. 1.1. The tan of angle between the line of fluid at t = 0 and t = t + dt is defined as the shearing strain du/dy.

The fluid that touches plate A has zero velocity u=0. The fluid that touches plate B moves with the same velocity as that of plate B, uB. That is, the molecules of the fluid adhere to the plate and do not slide along its surface. This is known as the no-slip condition. The no-slip condition is important in fluid mechanics. All fluids, including both gasses and liquids, satisfy this condition.

To understand viscosity, let us begin by imagining a hypothetical fluid between two parallel plates which are infinite in width and length. See Fig. 1.4.

The bottom plate A is a fixed plate. The upper plate B is a moveable plate, suspended on the fluid, above plate A, between the two plates. The vertical distance between the two plates is represented by h. A constant force F is applied to the moveable plate B causing it to move along at a constant velocity V_B with respect to the fixed plate.

If we replace the fluid between the two plates with a solid, the behavior of the plates would be different. The applied force F would create a displacement d, a shear stress τ in the material, and a shear strain γ. After a small, finite displacement, motion of the upper plate would cease.

If we then replace the solid between the two plates with a fluid, and reapply the force F, the upper plate will move continuously, with a velocity of V_B. This behavior is consistent with the definition of a fluid: a material that deforms continuously under the application of a shearing stress, regardless of how small the stress is.

After some infinitesimal time dt, a line of fluid that was vertical at time $t = 0$ will move to a new position, as shown by the dashed line in Fig. 1.4. The angle between the line of fluid at $t = 0$ and $t = t + dt$ is defined as the shearing strain. Shearing strain is represented by the Greek character γ (gamma).

The first derivative of the shearing strain with respect to time is known as the rate of shearing strain $d\gamma/dt$. For small displacements, $\tan(d\gamma)$ is approximately equal to $d\gamma$. The tangent of the angle of shearing strain can also be represented as follows:

$$\tan(d\gamma) = \frac{\text{opposite}}{\text{adjacent}} = \frac{V_B dt}{h}$$

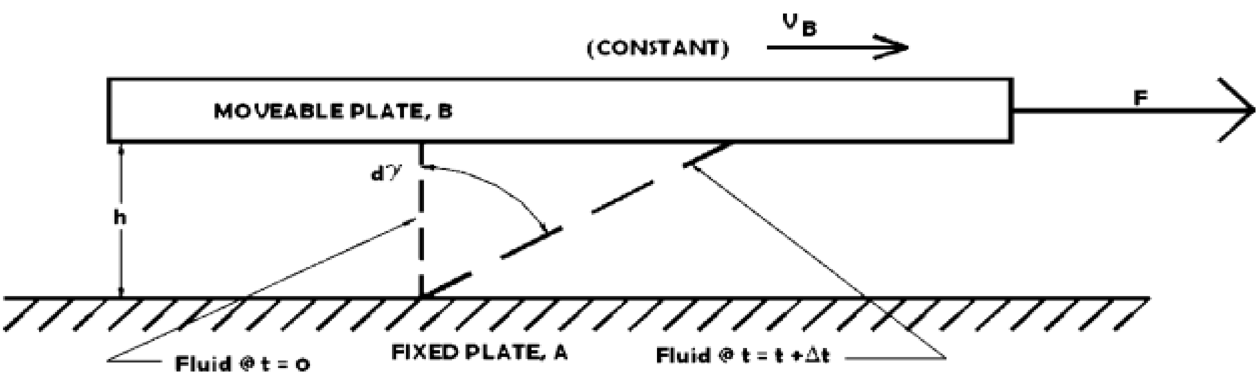

Figure 1.4 Moveable plate suspended over a layer of fluid.

Therefore, the rate of shearing strain $d\gamma/dt$ can be written as

$$d\gamma/dt = V_B/h$$

The rate of shearing strain is also denoted by $\dot{\gamma}$, and has the units of 1/s.

The fluid that touches plate A has zero velocity. The fluid that touches plate B moves with the same velocity as that of plate B, V_B. That is, the molecules of the fluid adhere to the plate and do not slide along its surface. This is known as the no-slip condition. The no-slip condition is important in fluid mechanics. All fluids, including both gasses and liquids, satisfy this condition.

Let the distance from the fixed plate to some arbitrary point above the plate be y. The velocity V of the fluid between the plates is a function of the distance above the fixed plate A. To emphasize this we write

$$V = V(y)$$

The velocity of the fluid at any point between the plates varies linearly between $V = 0$ and $V = V_B$. See Fig. 1.5.

Let us define the velocity gradient as the change in fluid velocity with respect to y.

$$\text{Velocity gradient} \equiv dV/dy$$

The velocity profile is a graphical representation of the velocity gradient. See Fig. 1.5. For a linearly varying velocity profile like that shown in Fig. 1.5, the velocity gradient can also be written as

$$\text{Velocity gradient} = V_B/h$$

1.2.2 Shear stress and viscosity

In cardiovascular fluid mechanics, shear stress is a particularly important concept. Blood is a living fluid, and if the forces applied to the fluid

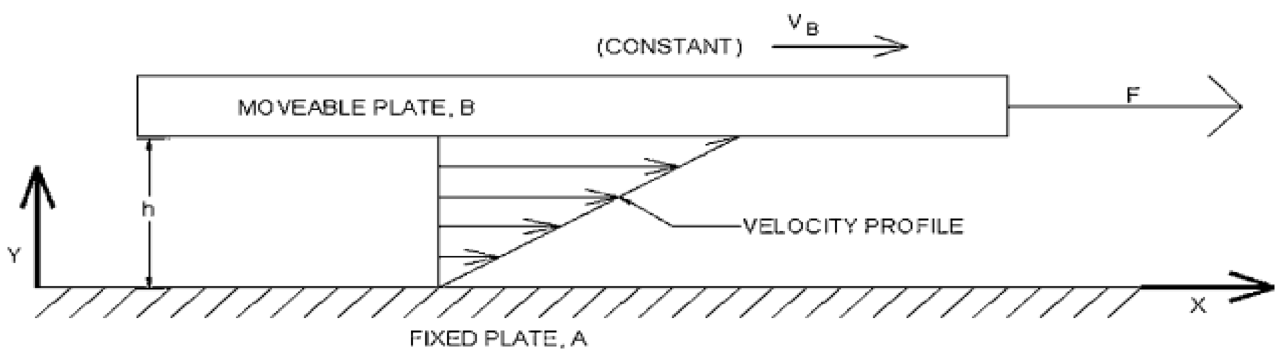

Figure 1.5 Velocity profile in a fluid between two parallel plates.

are sufficient, the resulting shearing stress can cause red blood cells to be destroyed. On the other hand, studies indicate a role for shear stress in modulating atherosclerotic plaques. The relationship between shear stress and arterial disease has been studied much, but is not yet very well understood.

Figure 1.6 represents the shear stress on an element of the fluid at some arbitrary point between the plates in Figs. 1.4 and 1.5. The shear stress on the top of the element results in a force that pulls the element "downstream." The shear stress at the bottom of the element resists that movement.

Since the fluid element shown will be moving at a constant velocity, and will not be rotating, the shear stress on the element τ' must be the same as the shear stress τ. Therefore,

$$d\tau/dy = 0 \text{ and } \tau_A = \tau_B = \tau_{\text{wall}}$$

Physically, the shearing stress at the wall may also be represented by

$$\tau_A = \tau_B = \text{force/plate area}$$

The shear stress on a fluid is related to the rate of shearing strain. If a very large force is applied to the moving plate B, a relatively higher velocity, a higher rate of shearing strain, and a higher stress will result. In fact, the relationship between shearing stress and rate of shearing strain is determined by the fluid property known as viscosity.

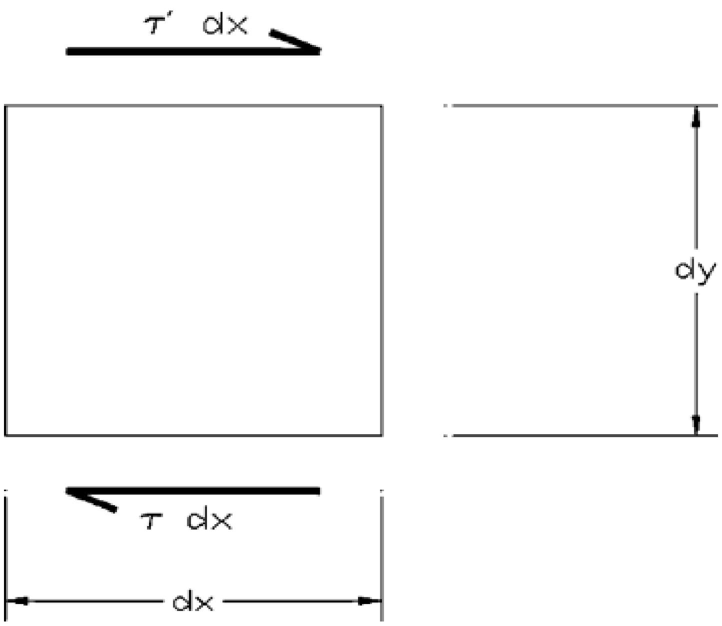

Figure 1.6 Shear stress on an element of the fluid.

1.2.3 Example problem: shear stress

Wall shear stress may be important in the development of various vascular disorders. For example, the shear stress of circulating blood on endothelial cells has been hypothesized to play a role in elevating vascular transport in ocular diseases such as diabetic retinopathy.

In this example problem, we are asked to estimate the wall shear stress in an arteriole in the retinal circulation. Gilmore et al. have published a related paper in the *American Journal of Physiology: Heart and Circulatory Physiology,* volume 288, in February 2005. In that article, the authors published the measured values of retinal arteriolar diameter and blood velocity in arterioles. For this problem, we will use their published values: 80 μm for a vessel diameter and 30 mm/s for mean retinal blood flow velocity. Later in Sec. 1.4.4, we will see that, for a parabolic flow profile, a good estimate of the shearing rate is

$$\dot{\gamma} = \frac{8V_{\mathrm{m}}}{D}$$

where V_{m} is the mean velocity across the vessel cross section and D is the vessel inside diameter.

We will also see in the next section that the shear stress is equal to the viscosity multiplied by the rate of shearing strain, that is,

$$\tau = \mu\dot{\gamma}$$

Therefore, to estimate the shear stress on the wall of a retinal arteriole, with the data from Gilmore's paper, we can calculate

$$\tau = \frac{\mu\, 8V_{\mathrm{m}}}{D} = \frac{0.0035\,\dfrac{\mathrm{Ns}}{\mathrm{m}^2}\,8(3)\,\dfrac{\mathrm{cm}}{\mathrm{s}}}{0.008\ \mathrm{cm}} = 10.5\,\frac{\mathrm{N}}{\mathrm{m}^2}$$

Although 10.5 Pa seems like a low shear stress when compared to the strength of aluminum or steel, it is a relatively high shear stress when compared to a similar estimate in the aorta, 0.5 Pa. See Table 1.1.

TABLE 1.1 Estimate of Wall Shear Stress in Various Vessels in the Human Circulatory System

Vessel	ID, cm	V_{m}, cm/s	Shear rate[3]	Shear stress,[4] N/m^2
Aorta	2.5	48	154	0.5
Large arteriole[5]	0.05	1.4	224	0.8
Arteriole (retinal microcirculation[6])	0.008	3	3000	10.5
Capillary	0.0008	0.7	7000	24.5

Note the increasing values for shear rate and shear stress as vessel inside diameter decreases.

1.2.4 Viscosity

A common way to visualize material properties in fluids is by making a plot of shearing stress as a function of the rate of shearing strain. For the plot shown in Fig. 1.7, shearing stress is represented by the Greek character τ, and the rate of shearing strain is represented by $\dot{\gamma}$.

The material property that is represented by the slope of the stress–shearing rate curve is known as viscosity and is represented by the Greek letter μ (mu). Viscosity is also sometimes referred to by the name absolute viscosity or dynamic viscosity. For common fluids like oil, water, and air, viscosity does not vary with shearing rate. Fluids with constant viscosity are known as Newtonian fluids. For Newtonian fluids, shear stress and rate of shearing strain may be related by the following equation:

$$\tau = \mu\dot{\gamma}$$

where τ = shear stress
μ = viscosity
$\dot{\gamma}$ = the rate of shearing strain

For non-Newtonian fluids, τ and $\dot{\gamma}$ are not linearly related. For those fluids, viscosity can change as a function of the shear rate (rate of shearing strain). Blood is an important example of a non-Newtonian fluid. Later in this book, we will investigate the condition under which blood behaves as, and may be considered, a Newtonian fluid.

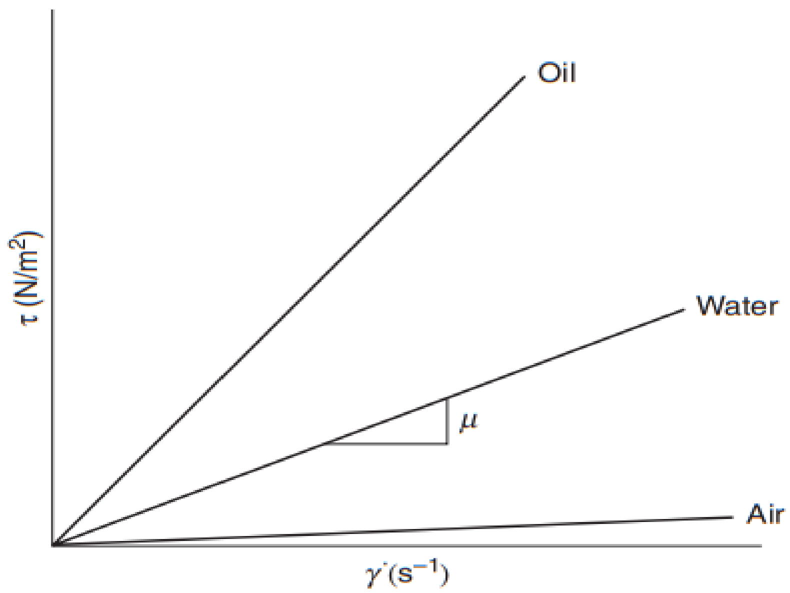

Figure 1.7 Stress versus rate of shearing strain for various fluids.

Shear stress and shear rate are not linearly related for non-Newtonian fluids. Therefore, the slope of the shear stress/shear rate curve is not constant. However, we can still talk about viscosity if we define the apparent viscosity as the instantaneous slope of the shear stress/shear rate curve. See Fig. 1.8.

Shear thinning fluids are non-Newtonian fluids whose apparent viscosity decreases as shear rate increases. Latex paint is a good example of a shear thinning fluid. It is a positive characteristic of the paint that the viscosity is low when one is painting, but that the viscosity becomes higher and the paint sticks to the surface better when no shearing force is present. At low shear rates, blood is also a shear thinning fluid. However, when the shear rate increases above 100 s^{-1}, blood behaves as a Newtonian fluid.

Shear thickening fluids are non-Newtonian fluids whose apparent viscosity increases when the shear rate increases. Quicksand is a good example of a shear thickening fluid. If one tries to move slowly in quicksand, then the viscosity is low and the movement is relatively easy. If one tries to move quickly, then the viscosity increases and the movement is difficult. A mixture of cornstarch and water also forms a shear thickening non-Newtonian fluid.

A Bingham plastic is neither a fluid nor a solid. A Bingham plastic can withstand a finite shear load and flow like a fluid when that shear stress is exceeded. Toothpaste and mayonnaise are examples of Bingham plastics. Blood is also a Bingham plastic and behaves as a solid at shear rates very close to zero. The yield stress for blood is very small, approximately in the range from 0.005 to 0.01 N/m^2.

Kinematic viscosity is another fluid property that has been used to characterize flow. It is the ratio of absolute viscosity to fluid density and

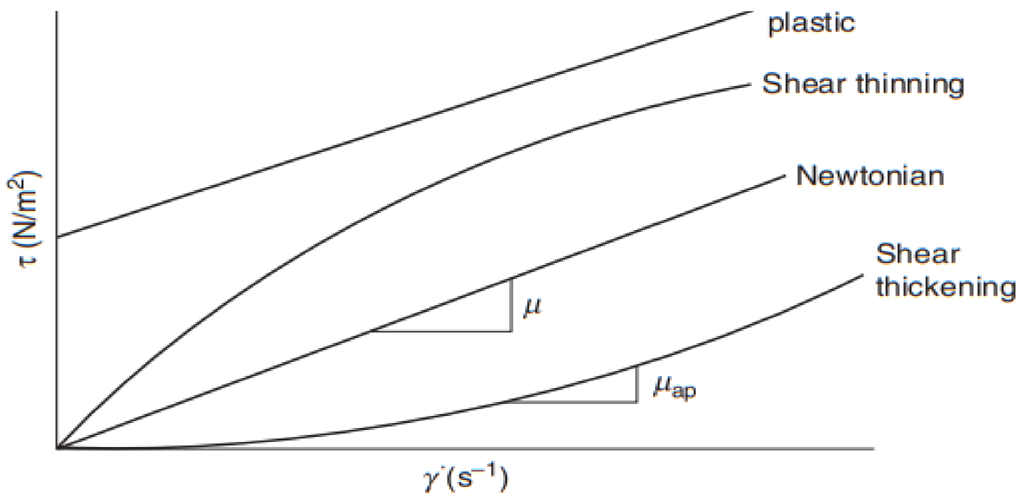

Figure 1.8 Shear stress versus rate of shearing strain for some non-Newtonian fluids.

1.3 Fundamental Method for Measuring Viscosity

A fundamental method for measuring viscosity involves a viscometer made from concentric cylinders. See Fig. 1.9. The fluid for which the viscosity is to be measured is placed between the two cylinders. The torque generated on the inner fixed cylinder by the outer rotating cylinder is determined by using a torque-measuring shaft. The force required to cause the cylinder to spin and the velocity at which it spins are also measured. Then the viscosity may be calculated in the following way:

The shear stress τ in the fluid is equal to the force F applied to the outer cylinder divided by the surface area A of the internal cylinder, that is,

$$\tau = \frac{F}{A}$$

The shear rate $\dot{\gamma}$ for the fluid in the gap, between the cylinders, may also be calculated from the velocity of the cylinder, V, and the gap width h as

$$\dot{\gamma} = \frac{V}{h}$$

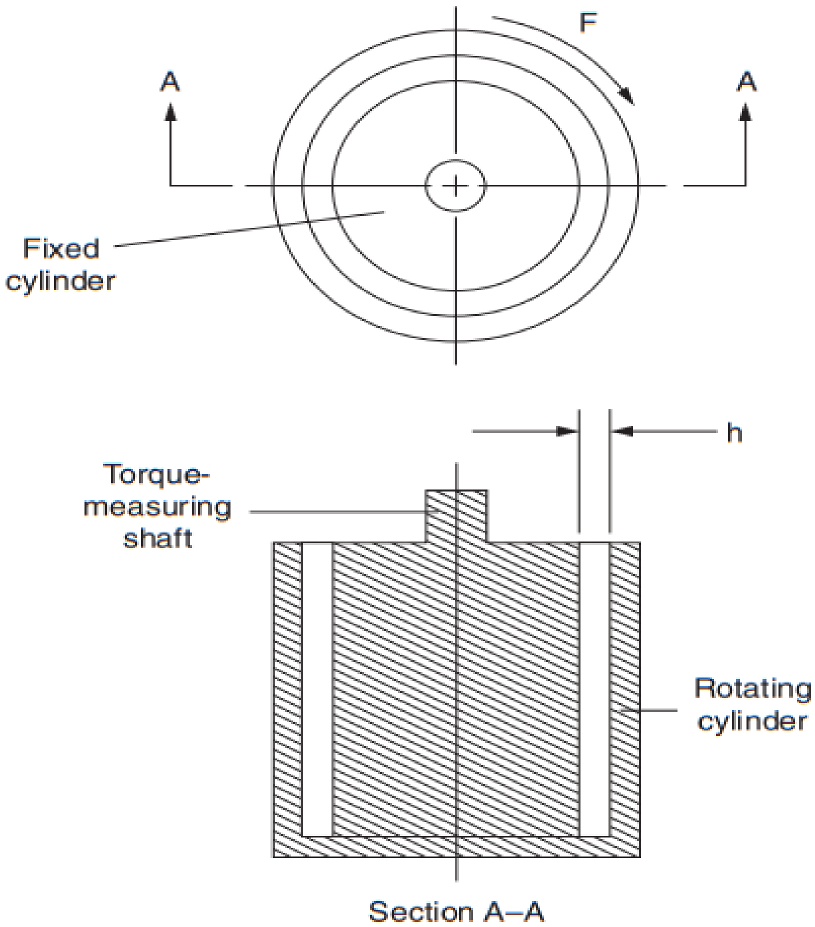

Figure 1.9*a* Cross section of a rotating cylinder viscometer.

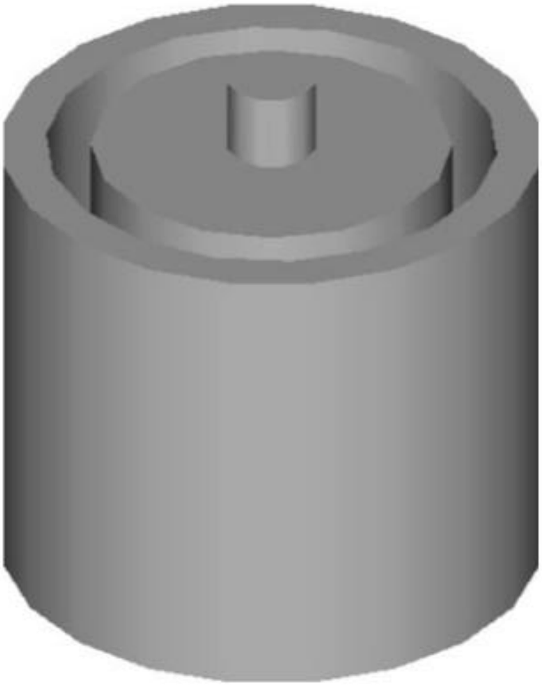

Figure 1.9*b* Rotating cylinder viscometer.

From the shear stress and the shear rate, the viscosity and/or the kinematic velocity may be obtained as

$$\mu = \frac{\tau}{\dot{\gamma}} \quad \text{and} \quad \nu = \frac{\mu}{\rho}$$

where μ = viscosity
ν = kinematic viscosity
ρ = density

A typical value for blood viscosity in humans is 0.0035 Ns/m^2, or 0.035 poise (P), or 3.5 cP. Note that 1 P = 1 dyne s/cm^2, or 0.1 Ns/m^2. Another useful pressure unit conversion is that 1 mmHg = 133.3 N/m^2.

Let T represent the measured torque in the viscometer shaft, and ω is its angular velocity in rad/s. Assume that D is the radius of the inner viscosimeter cylinder, and L is its length. The fluid velocity at the inner surface is

$$V = \omega \frac{D}{2}$$

It can be shown that

$$T = F \frac{D}{2}$$

leading to an equation which relates the torque, the angular velocity, and the geometric parameters of the device.

$$\mu = \frac{4T\,h}{\pi\,D^3\,L\,\omega}$$

1.3.1 Example problem: viscosity measurement

Whole blood (assume $\mu = 0.0035$ Ns/m^2) is placed in a concentric cylinder viscometer. The gap width is 1 mm and the inner cylinder radius is 30 mm. Estimate the wall shear stress in the fluid. Assume the angular velocity of the outer cylinder to be 60 rpm.

We can begin by calculating the shear rate based on the angular velocity of the cylinder, its radius, and the gap between the inner and outer cylinders. The shear rate is equal to the velocity of the outer cylinder multiplied by the gap between the cylinders (see Fig. 1.9*a*). That is,

$$\dot{\gamma} = \frac{V}{h}$$

The wall shear stress is equal to the viscosity multiplied by the shear rate. Thus,

$$\tau = \mu\dot{\gamma} = \frac{\mu(\mathrm{r}\omega)}{h} = 0.0035\,\frac{\mathrm{Ns}}{\mathrm{m}^2}\,\frac{\left(\frac{31}{1000}\right)\mathrm{m}\,(60)\left(\frac{\pi}{30}\right)\frac{\mathrm{rad}}{\mathrm{s}}}{(1/1000)\mathrm{m}} = 0.682\,\frac{\mathrm{N}}{\mathrm{m}^2}$$

Reynolds Number:

The Reynolds number is a dimensionless parameter named after Professor Reynolds. The number is defined as

$$\mathrm{Re} = \frac{\rho V D}{\mu}$$

where ρ = fluid density in kg/m^3
V = fluid velocity in m/s
D = pipe diameter in m
μ = fluid viscosity in Ns/m^2

Unless otherwise specified, this V will be considered to be the average velocity across the pipe cross section. Physically, the Reynolds number represents the ratio of inertial forces to viscous forces.

The Reynolds number helps us to predict the transition between laminar and turbulent flows. Laminar flow is highly organized flow along streamlines. As velocity increases, flow can become disorganized and chaotic with a random 3-D motion superimposed on the average flow velocity. This is known as turbulent flow. Laminar flow occurs in flow environments where $\mathrm{Re} < 2000$. Turbulent flow is present in circumstances under which $\mathrm{Re} > 4000$. The range of $2000 < \mathrm{Re} < 4000$ is known as the transition range.

The Reynolds number is also useful for predicting entrance length in pipe flow. I will denote the entrance length as X_E. The ratio of entrance length to pipe diameter for laminar pipe flow is given as

$$\frac{X_E}{D} \cong 0.06 \text{ Re}$$

Consider the following example: If Re = 300, then $X_E = 18\ D$, and an entrance length equal to 18 pipe diameters is required for fully developed flow. In the human cardiovascular system, it is not common to see fully developed flow in arteries. Typically, the vessels continually branch, with the distance between branches not often being greater than 18 pipe diameters.

Although most blood flow in humans is laminar, having a Re of 300 or less, it is possible for turbulence to occur at very high flow rates in the descending aorta, for example, in highly conditioned athletes. Turbulence is also common in pathological conditions such as heart murmurs and stenotic heart valves.

Stenotic comes from the Greek word "stenos," meaning narrow. Stenotic means narrowed, and a stenotic heart valve is one in which the narrowing of the valve is a result of the plaque formation on the valve.

1.4.2 Example problem: Reynolds number

Estimate the Reynolds number for blood flow in a retinal arteriole, using the published values from Gilmore et al. Assume that the blood density is 1060 kg/m^3. Is there any concern that blood flow in the human retina will become turbulent?

From Table 1.1, we see that the inside diameter of the arteriole is 0.008 cm, the mean velocity in the vessel is 3 cm/s, and the viscosity measured as 0.0035 Ns/m^2. The Reynolds number can be calculated as

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{1060 \dfrac{\text{kg}}{\text{m}^3} \dfrac{3}{100} \dfrac{\text{m}}{\text{s}} \dfrac{0.008}{100} \text{m}}{0.0035 \dfrac{\text{Ns}}{\text{m}^2}} = 0.73$$

For this flow condition, the Reynolds number is far, far less than 2000, and there is no danger of the flow becoming turbulent.

Newton´s law of viscosity

$$\tau = \mu \cdot du/dy$$

where, τ= shear stress
μ: viscosity
du/dy: rate of shear deformation

Kinematic viscosity

$$v = \mu/\rho$$

where, v: kinematic viscosity
μ: dynamic viscosity
ρ: density of fluid

Example: If the velocity distribution over a plate is given by **u=** $\mathbf{2y/3 - y^2}$ in which u is the velocity in meters per second at a distance y meter above the plate. Determine the shear stress at y=0 and y=0.15m. Take the dynamic viscosity of the fluid as 0.863 N.s/ m^2 .

Sol: u= $\frac{2y}{3} - y^2$

$\frac{du}{dy} = \frac{2}{3}$-2y , at y=0 ⟹ $\frac{du}{dy} = \frac{2}{3}$ =0.66 , τ =μ· $\frac{du}{dy}$ ⟹τ= 0.57 N/ m^2

or

at y=0.15 ⟹ $\frac{du}{dy} = \frac{2}{3}$ - 2*0.15= 0.36 , τ =μ· $\frac{du}{dy}$ ⟹ τ=0.31 N/ m^2