Chapter Two: Resultant of Force Systems

Resultant: simplest force system which have same external effect of the original system.

2.1 Resultant of Coplanar Concurrent Force System

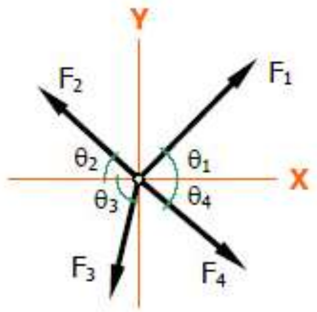
In x-y plane, the resultant of coplanar concurrent force system where the lines of action of all forces pass through a common point can be found by the following formulas:

$$R_x = \sum F_x \quad \rightarrow^{+}$$

$$R_x = F_{1x} - F_{2x} - F_{3x} + F_{4x}$$

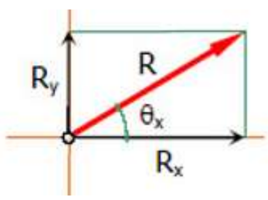
$$R_y = \sum F_y \quad \uparrow^{+}$$

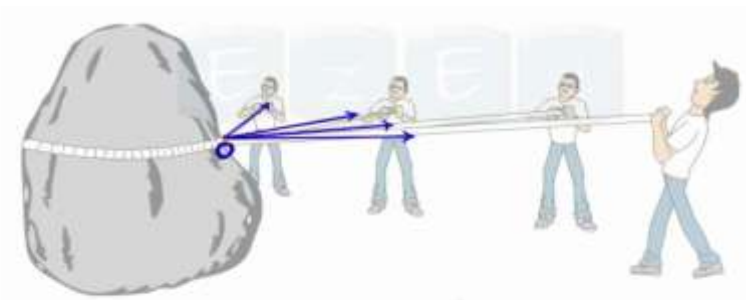
$$R_y = F_{1y} + F_{2y} - F_{3y} - F_{4y}$$



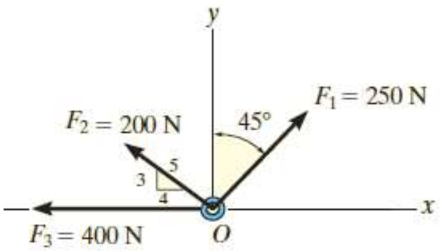
$$R = \sqrt{{R_x}^2 + {R_y}^2}$$

$$\theta_x = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

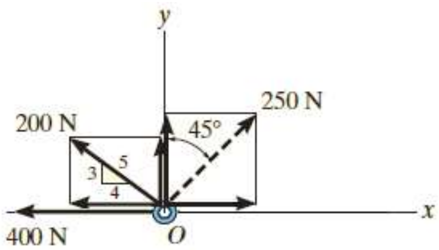




Example No. 1: Determine the magnitude and direction of the resultant forces system shown in Figure.



Solution:



$$F_{1x} = 250 \times \sin 45 = 176.8\ N \quad \rightarrow$$

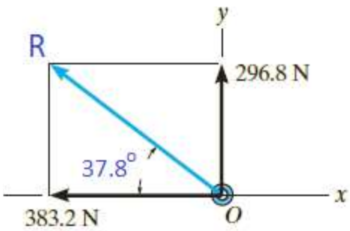
$$F_{1y} = 250 \times \cos 45 = 176.8\ N \quad \uparrow$$

$$F_{2x} = 200 \times \frac{4}{5} = 160\ N \quad \leftarrow$$

$$F_{2y} = 200 \times \frac{3}{5} = 120\ N \quad \uparrow$$

$$F_{3x} = 400\ N \quad \leftarrow$$

$$F_{3y} = 0$$



$$\rightarrow^{+}\ R_x = \sum F_x = 176.8 - 160 - 400$$

$$R_x = -383.2\ N = 383.2\ N \leftarrow$$

$$\uparrow^{+}\ R_y = \sum F_y = 176.8 + 120 + 0 = 296.8\ N \quad \uparrow$$

$$R = \sqrt{{R_x}^2 + {R_y}^2} = \sqrt{(383.2\ )^2 + (296.8)^2} = 484.7\ N$$

$$\theta_x = \tan^{-1}\left(\frac{R_y}{R_x}\right) = \tan^{-1}\left(\frac{296.8}{383.2}\right) = 37.8^{\circ}$$

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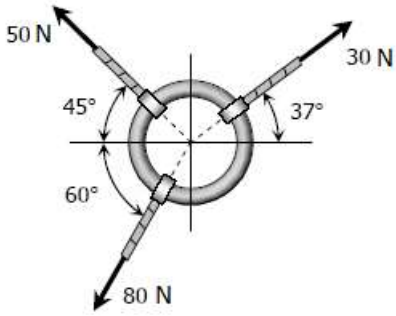
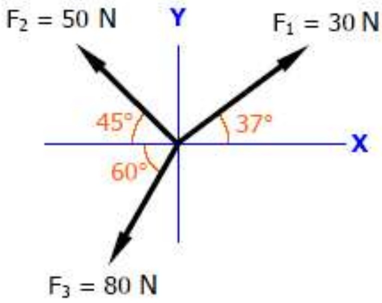
Example No. 2: Find the resultant force on the ring due to the three applied forces.



Solution:

$$\xrightarrow{+} R_x = \sum F_x$$

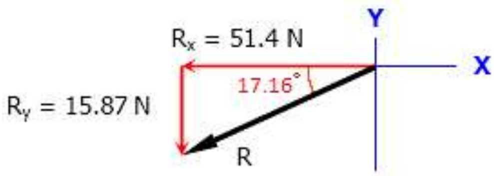


$$R_x = 30\cos 37 - 50\cos 45 - 80\cos 60$$

$$R_x = -51.40\ N = 51.40\ N \leftarrow$$

$$\uparrow^{+} R_y = \sum F_y$$

$$R_y = 30\sin 37 + 50\sin 45 - 80\sin 60$$



$$R_y = -15.87\ N = 15.87\ N \ \downarrow$$

$$R = \sqrt{{R_x}^2 + {R_y}^2} = \sqrt{(51.40\,)^2 + (15.87)^2}$$

$$R = 53.79\ N$$

$$\theta_x = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

$$\theta_x = \tan^{-1}\left(\frac{15.87}{51.40}\right) = 17.16^{\circ}$$

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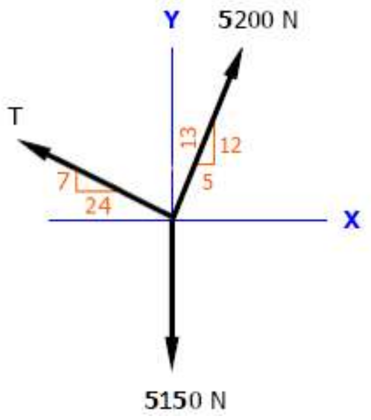
Example No. 3: The resultant of the three forces is horizontal. Determine the magnitude of the resultant.



Solution:

Since the resultant is horizontal, therefore:

$R_y = 0, \quad R = R_x$

$c = \sqrt{24^2 + 7^2} = 25$

$\uparrow^+ \; R_y = \sum F_y$

$$0 = T \times \frac{7}{25} + 5200 \times \frac{12}{13} - 5150$$

$\therefore T = 1250 \; N$

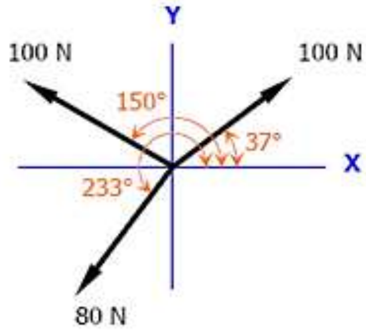
$\rightarrow^+ \; R_x = R = \sum F_x$

$$R = -T \times \frac{24}{25} + 5200 \times \frac{5}{13}$$

$R = 800 \; N \rightarrow$

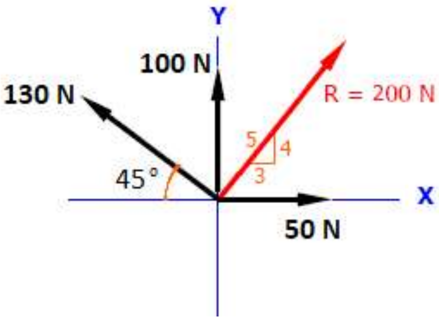
Problems:

1. Determine the magnitude and direction of the resultant forces system shown in Figure.



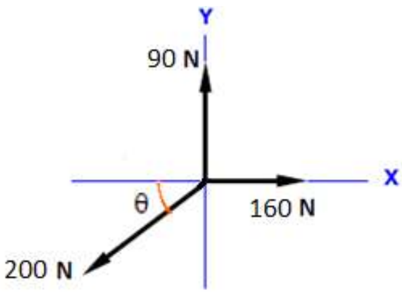
Answer: $R = 71.8\ N,\qquad \theta_x = 40.15^\circ$

2. If the resultant of fourth forces is 200 N as shown in figure. Find the unknown for the force.



Answer: $F_4 = 165\ N,\qquad \theta_x = 11.153^\circ$

3. The resultant of the three forces as shown in figure is vertical. determine the angle θ, and magnitude of the resultant.



Answer: $R = 30\ N \downarrow,\qquad \theta = 36.87^\circ$

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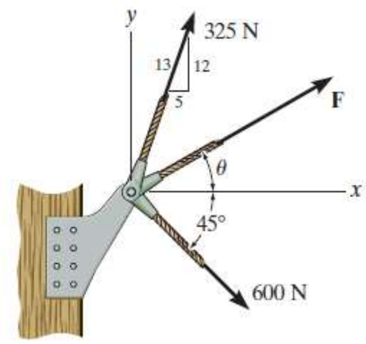
4. If the resultant force acting on the bracket is to be 750 N directed along the positive $x-\ axis$, determine the magnitude of **F** and its direction θ.



Answer: $F = 236.1\ N, \qquad \theta = 31.76^{\circ}$

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