5 Elementary Theory of an Ideal Transformer

An ideal transformer is one which has no losses, i.e. its windings have no ohmic resistance and there is no magnetic leakage. In other words, an ideal transformer consists of two coils which are purely inductive and wound on a loss-free core.

It may, however, be noted that it is impossible to realize such a transformer in practice, yet for convenience, we will first analyze such a transformer and then an actual transformer. Consider an ideal transformer [Fig. 5(a)] in which the secondary is open and whose primary is connected to a sinusoidal alternating voltage V_1. Under this condition, the primary draws current from the source to build up a counter electromotive force equal and opposite to the applied voltage.

Since the primary coil is purely inductive and there is no output, the primary draws the magnetizing current Iμ only. The function of this current is merely to magnetize the core, it is small in magnitude and lags V1 by 90°. This alternating current Iμ produces an alternating flux f which is proportional to the current and hence is in phase with it. This changing flux is linked with both the windings. fore, it produces self-induced emf in the primary. This self-induced emf e_1 is, at any instant, equal to and in opposition to V1. It is also known as counter emf of the primary.

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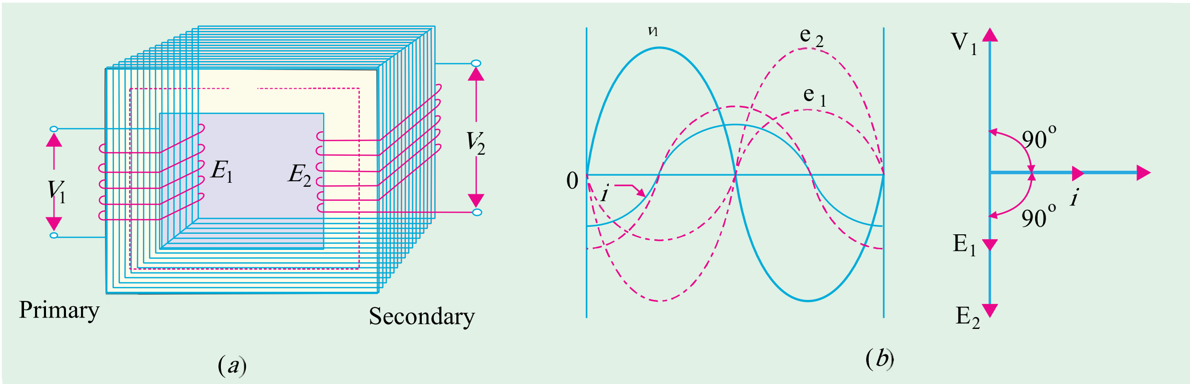

Fig 5. Single phase transformer.

Similarly in the secondary winding, an induced emf e_2 is produced which is known as mutually induced emf. The emf is in phase opposition with V1 and its magnitude is proportional to the rate of change of flux and the number of secondary turns. Figure 5(b) shows the vectorial representations of the above quantities.

6 E.M.F. Equation of a Transformer

Let N_1 = Number of turns in primary, N_2 = Number of turns in secondary

ϕ_m is Maximum flux in the core in Weber, B_m = Flux density in weber/ m^2 (Tesla)

A = Net cross-sectional area of core in m^2, f = Frequency of ac input in Hz

v_1 = Instantaneous value of applied voltage in primary winding in volts.

v_{1m} = Maximum value of applied voltage in volts.

The instantaneous value of electromotive force e_1 is:

$$e_1 = -N_1 \frac{d\phi}{dt}$$

The emf e_1 is equal and opposite to applied voltage V_1, i.e.

$$v_1 = N_1 \frac{d\phi}{dt}$$

If the applied voltage is sinusoidal, that is

$$v_1 = v_{1m} \sin 2\pi f t$$

Then

$$\phi = \phi_m \sin 2\pi f t$$

$$e_1 = -N_1 \phi_m 2\pi f \times \cos 2\pi f t$$

These equations are expressed as vectors as shown in Fig. 5(b), where V_1 and E_1 are the rms values of v_1 and e_1. To obtain the RMS value of counter emf e_1, divide its maximum value given above by $\sqrt{2}$.

$$E_{m1} = N_1 \phi_m \frac{2\pi f}{\sqrt{2}}$$

The cosine term has no significance except to derive the instantaneous values.

$$E_{m1} = 4.44 N_1 \phi_m f$$

$$E_{m1} = 4.44 N_1 B_m A f$$

Similarly, rms value of emf induced in secondary is,

$$E_{m2} = 4.44 N_2 B_m A f$$

7 Voltage Transformation Ratio (K)

The ratio K is defined as the division of the secondary emf over the emf of the primary side.

$$\frac{E_2}{E_1} = \frac{N_2}{N_1} = \mathrm{K}$$

This constant is known as voltage transformation ratio.

1- If $N_2 > N_1$, i.e., K > 1, then the transformation is called as step-up transformer.

2- If $N_1 > N_2$, i.e., K < 1, then the transformation is called as step-down transformer.

8 Electrical Power in a Transformer

Another one of the transformer basics parameters is its power rating. The power rating of a transformer is obtained by simply multiplying the current by the voltage to obtain a rating in **Volt-amperes**, (VA). Small single-phase transformers may be rated in volt-amperes only, but much larger power transformers are rated in units of **Kilo volt-amperes**, (kVA) where 1 kilo volt-ampere is equal to 1,000 volt-amperes, and units of **Mega volt-amperes**, (MVA) where 1 mega volt-ampere is equal to 1 million volt-amperes.

In an ideal transformer (ignoring any losses), the power available in the secondary winding will be the same as the power in the primary winding, they are constant wattage devices and do not change the power only the voltage to current ratio. Thus, in an ideal transformer the **Power Ratio** is equal to one (unity) as the voltage, V multiplied by the current, I will remain constant.

That is the electric power at one voltage/current level on the primary is "transformed" into electric power, at the same frequency, to the same voltage/current level on the secondary side. Although the transformer can step-up (or step-down) voltage, it cannot step-up power. Thus, when a transformer steps-up a voltage, it steps-down the current and vice-versa, so that the output power is always at the same value as the input power. Then we can say that primary power equals secondary power, ($P_1= P_2$).

9 Transformer Efficiency

The transformer's efficiency has a direct effect on its performance and aging. In general, the efficiency of a transformer is in the range of 95 – 99 %. The efficiency of large power transformers with very low losses can be as high as 99.7%. The output and input of a transformer are not measured under loaded conditions when the wattmeter readings inevitably suffer errors of 1 – 2%.

The copper losses depend on the currents through the transformer primary and secondary windings and the core losses depend on the transformer rated voltage. Therefore, transformer efficiency plays an important role in operating it under constant voltage and frequency conditions. The temperature rise of the transformer due to heat generated has an effect on the life of transformer oil properties and decides the reasonable type of cooling method. The rating of the equipment is limited by the temperature rise. The **transformer' efficiency** is simply shown as

$$\eta = \frac{Output\ Power}{Input\ Power} \times 100\%$$

For a practical transformer, the input power is given by,

$$\text{Input power} = \text{Output power} + \text{Losses}$$

Therefore, the transformer efficiency can also be calculated using the following expression:-

$$\eta = \frac{Output\ Power}{Output\ Power + Losses} \times 100\%$$

Examples 1: The maximum flux density in the core of a 250/3000-volts, 50-Hz single phase transformer is 1.2 Wb/m^2. If the e.m.f. per turn is 8 volts, determine

1- Primary and secondary turns

2- Area of the core.

Solution

1- $E_1 = N_1 \times \text{e.m.f/turn}$

N_1= 250/8 = 32; N_1 = 3000/8 = 375

2- We may use $E_2 = 4.44 N_2 B_m A f$

$3000 = 4.44 \times 50 \times 375 \times 1.2 \times A \Longrightarrow A = 0.03\, m^2$.

Example 2: A single-phase transformer has 400 primary and 1000 secondary turns. The net cross-sectional area of the core is 60 cm^2. If the primary winding be connected to a 50-Hz supply at 520 V, calculate:

1- Voltage induced in the secondary winding.

2- Peak value of flux density in the core.

Solution

1- $\mathrm{K} = \frac{N_2}{N_1} = \frac{1000}{400} = \mathbf{2.5}$

$$\frac{E_2}{E_1} = \mathrm{K} \Longrightarrow E_2 = \mathrm{K}E_1 = 2.5 \times 520 = \mathbf{1300\ V}$$

2- $E_1 = 4.44 N_1 B_m A f$

$$520 = 4.44 \times 50 \times 400 \times B_m \times (60 \times 10^{-4})$$

$$B_m = \mathbf{0.976\ Wb/m^2}$$

Example 3: The Secondary of a 500 kVA, 4400/500 V, 50 Hz, single-phase transformer has 500 turns. Determine

1- e.m.f Per Turn.

2- Primary turns.

3- Secondary full load current.

4- Maximum flux.

5- Gross cross-sectional area of the core for flux density of 1.2 tesla.

Solution

VA = 500 kVA , $E_1 = 4400\ V$, $E_2 = 500\ V$, $f = 50\ Hz$, $N_2 = 500$, $B_m = 1.2\ T$

1- $\frac{E_2}{N_2} = \frac{500}{500} = \mathbf{1\ V/turn}$

2- $\frac{E_1}{N_1} = 1 = \frac{4400}{N_1} \Longrightarrow N_1 = \mathbf{4400}$

3- Secondary full load current $I_2 = \frac{kVA}{V_2} = \frac{500 \times 1000}{500} = \mathbf{1000\ A}$

4- Maximum flux , $\phi_m = \frac{E_2}{4.44 \times N_2 \times f} = \frac{500}{4.44 \times 500 \times 50} = \mathbf{4.5\ mWb}$

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