# 1 Equivalent Circuit of a Single-Phase Transformer

The analysis of a transformer can be carried out by using an equivalent circuit which can be derived by considering the following: -

- The primary and secondary windings have finite resistances considered as lumped parameters.
- The leakage fluxes are modelled as leakage reactance in the equivalent circuit.
- The core-loss component of current is modelled using a shunt resistance.
- The magnetization of the core is modelled using a magnetizing reactance as a shunt branch.

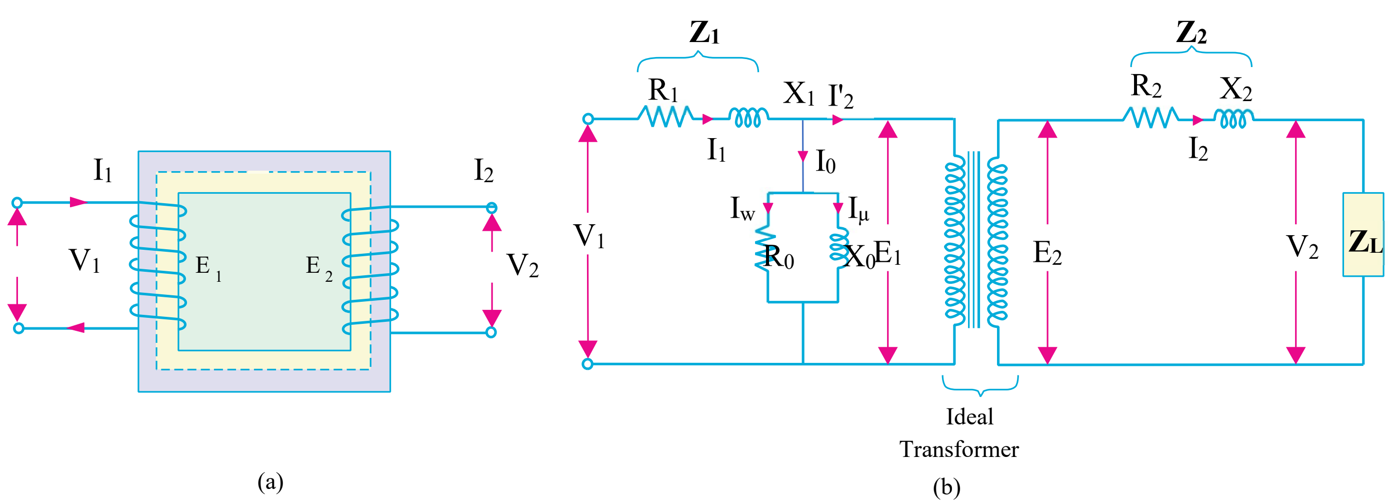


Fig.1 Diagram of equivalent circuit for the transformer with load

The transformer shown diagrammatically in Fig.1 (*a*) can be resolved into an equivalent circuit in which the resistance and leakage reactance of the transformer are imagined to be external to the winding whose only function then is to transform the voltage.

Generally, there are two modes of operations in the transformers which are:

1- No-load operation.
2- On load operation.

The voltages, currents and elements of the circuit can be defined as in Table 1.

| SYMBOL | DEFINITION |
|---|---|
| **$R_1$, $R_2$** | Primary and secondary resistances |
| **$X_1$, $X_2$** | primary and secondary leakage reactance |
| **$R_0$** | Exciting resistance |
| **$X_0$** | Exciting reactance |
| **$I_1$** | Primary side current |
| **$I_0$** | No-load/Excitation current component of primary current |
| **$I_w$ (or $I_{c1}$)** | Core-loss component of no-load current |
| **$I_\mu$ (or $I_{m1}$)** | Magnetizing component of no-load current |
| **$I'_2$** | Load component of primary current |
| **$I_2$** | Secondary side current (load current) |
| **$V_1$** | Applied primary side voltage |
| **$V_2$** | Secondary side terminal voltage |
| **$E_1$** | Primary side induced emf |
| **$E_2$** | Secondary side induced emf |

Table 1 Quantities of equivalent circuit.

## 1.1 Transformer at No-load

A transformer is said to be on no-load when its secondary winding is kept open and no-load is connected across it. As such, no current flows through the secondary i.e., $I_2 = 0$. Hence, the secondary winding is not causing any effect on the magnetic flux set-up in the core or on the current drawn by the primary. But the losses cannot be ignored. At no-load, a transformer draws a small current $I_0$ (usually 2 to 10% of the rated value). This current has to supply the iron losses (hysteresis and eddy current losses) in the core and a very small amount of copper loss in the primary (the primary copper losses are so small as compared to core losses that they are generally neglected moreover secondary copper losses are zero as $I_2$ is zero).

Therefore, current $I_0$ lags behind the voltage vector $V_1$ by an angle $I_0$ (called hysteresis angle of advance) which is less than 90, as shown in Fig. 2. The angle of lag depends upon the losses in the transformer.

The basic equations in this mode can viewed as follows: -

| | |
|---|---|
| **Working component,** | $I_w = I_0 \cos\phi_0$ |
| **Magnetizing component,** | $I_\mu = I_0 \sin\phi_0$ |
| **No-load current,** | $I_0 = \sqrt{I_\mu{}^2 + I_w{}^2}$ <br> $I_0 = I_w + jI_\mu$ |
| **Primary p.f. at no-load,** | $\cos\phi_0 = \dfrac{I_w}{I_0}$ |
| **No-load power input,** | $P_0 = V_1 I_0 \cos\phi_0$ |
| **Exciting resistance,** | $R_0 = \dfrac{V_1}{I_w}$ |
| **Exciting reactance** | $X_0 = \dfrac{V_1}{I_\mu}$ |

Table 2. No-load parameters in transformer.

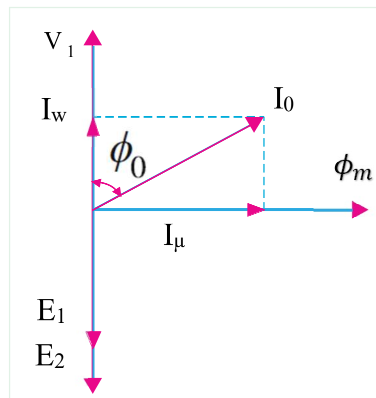

Fig.2 No-load current

**Examples 1:** A 230/110 V single-phase transformer has a core loss of 100 W. If the input under no-load condition is 400 VA, find core loss current, magnetizing current and no-load power factor angle.

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**Solution**

No- load current $V_1 I_0 = 400$ VA, $I_0 = \frac{400}{230} = \mathbf{1.739\ A}$

Core loss current $I_w = \frac{P_i}{V_1} = \frac{100}{230} = \mathbf{0.4348\ A}$

Magnetizing current $I_\mu = \sqrt{{I_0}^2 - {I_w}^2} = \sqrt{(1.739)^2 - (0.4348)^2} = \mathbf{1.684\ A}$

No-load power factor, $\cos\phi_0 = \frac{I_w}{I_0} = \frac{0.4348}{1.739} = \mathbf{0.25\ lag}$

No-load power factor angle $\phi_0 = \cos^{-1} 0.25 = \mathbf{75.52^o}$

**Example 2:** At open circuit, transformer of 10 kVA, 500/250 V, 50 Hz draws a power of 167 watt at 0.745 A, 500 V. Determine the magnetizing current, Ro current, no-load power factor, hysteresis angle of advance, equivalent resistance and reactance of exciting circuit referred to primary side.

**Solution**

$V_1 = 500\,V$ , $I_0 = 0.745\,A$, $P_0 = 167\,W$

$$I_w = \frac{P_0}{V_1} = \frac{167}{500} = \mathbf{0.334\ A}$$

$$I_\mu = \sqrt{{I_0}^2 - {I_w}^2} = \sqrt{(0.745)^2 - (0.334)^2} = \mathbf{0.666\ A}$$

$$\cos\phi_0 = \frac{I_w}{I_0} = \frac{0.334}{0.745} = \mathbf{0.448\ lag}$$

$$\phi_0 = \cos^{-1} 0.448 = \mathbf{63.36^\circ\ lag}$$

$$R_0 = \frac{V_1}{I_w} = \frac{500}{0.334} = \mathbf{1497\ \Omega}$$

$$X_0 = \frac{V_1}{I_\mu} = \frac{500}{0.666} = \mathbf{750\ \Omega}$$

## 1.2 Transformer on load

When the secondary is loaded, the secondary current $I_2$ is set up. The magnitude and phase of $I_2$ with respect to $V_2$ is determined by the characteristics of the load. Current $I_2$ is in phase with $V_2$ if load is non-inductive, it lags if load is inductive and it leads if load is capacitive. A set of equation can be drawn using basic theory of electric circuit analysis.

Applying KVL in the primary side of equivalent circuit: -

$$V_1 = E_1 + I_1(R_1 + jX_1)$$

Applying KVL in the secondary side of equivalent circuit: -

$$E_2 = V_2 + I_2(R_2 + jX_2)$$

Since $\frac{E_1}{E_2} = \frac{I_2}{I_2'} = a = \frac{1}{k}$ using $E_1 = aE_2$ we get:

$$V_1 = aE_2 + I_1(R_1 + jX_1)$$

Then we can rewrite the equation of $V_1$ by substituting the equation of $E_2$.

$$V_1 = a[V_2 + I_2(R_2 + jX_2)] + I_1(R_1 + jX_1)$$

Since $I_2 = aI_2' \Longrightarrow V_1 = [aV_2 + I_2' a^2(R_2 + jX_2)] + I_1(R_1 + jX_1)$

Let $R_2' = a^2 R_2$ and $X_2' = a^2 X_2$

Where $R_2'$ is called secondary resistance referred to the primary side and $X_2'$ is called secondary leakage reactance referred to the primary side.

Another modification to $V_1$ applying the same rules above then:

$$V_1 = V_2' + I_2'(R_2' + jX_2') + I_1(R_1 + jX_1)$$

Where

$V_2' = aV_2$ is called secondary voltage referred to the primary side, $I_2' = \frac{I_2}{a}$ is called secondary current referred to the primary side.

In Table 3 shown the quantities of the secondary side with their referred quantities.

| **Referred to the primary** | **Referred to the secondary** |
|---|---|
| $E_1 = E_2' = \frac{E_2}{k}$ | $E_1' = E_2 = kE_1$ |
| $V_2' = \frac{V_2}{k}$ | $V_1' = kV_1$ |
| $I_2' = kI_2$ | $I_1' = \frac{I_1}{k}$ |
| $X_2' = \frac{X_2}{k^2}$ | $X_1' = k^2X_1$ |
| $R_2' = \frac{R_2}{k^2}$ | $R_1' = k^2R_1$ |

Table 3. Referred quantities.

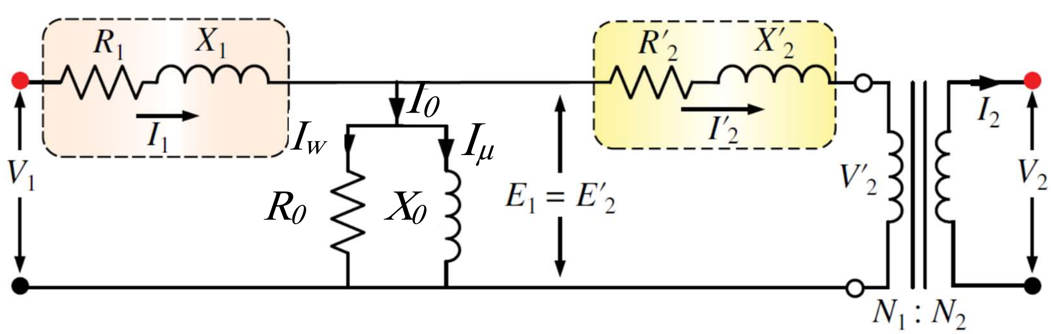


Fig 3. Modified equivalent circuit of a single-phase transformer.

## 1.3 Exact Equivalent Circuit

The transformer circuit can be moved to the right or left by referring all quantities to the primary or secondary side, respectively. This is almost invariably done. The equivalent circuit moved to primary is shown in Fig. 3.

If we shift all the impedances from one winding to the other, the transformer core is eliminated and we get an equivalent electrical circuit. Various voltages and currents can be readily obtained by solving this electrical circuit.

## 1.4 Approximate Equivalent Circuit

The equivalent circuit can be simplified by assuming small voltage drop across the primary impedance and $V_1 = E_1$. If the applied voltage and the induced emf are the same then the shunt branch can be moved across the source voltage and the approximate equivalent circuit is drawn as shown in Figure 4.

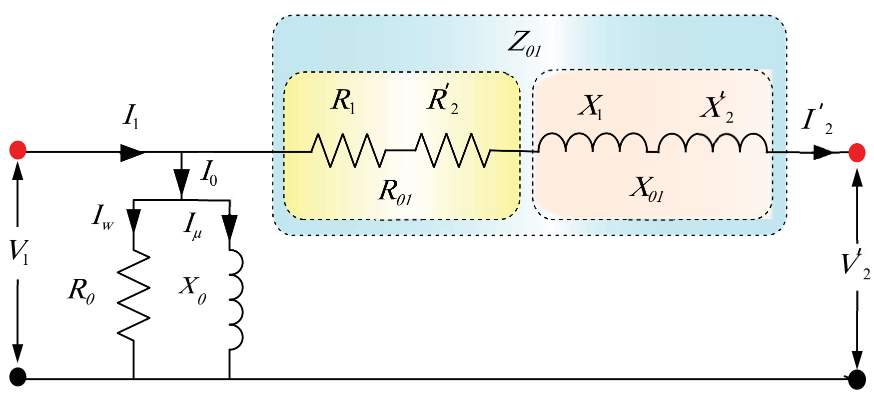


Fig 4. Approximate equivalent circuit

From the equivalent circuit the following relations are obtained

$$R_{01} = R_1 + R'_2 \qquad , X_{01} = X_1 + X'_2$$

$$\boldsymbol{Z_{01}} = R_{01} + jX_{01}$$

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**Example 3:** A 30 kVA, 2400/120 V, 50-Hz transformer has a high voltage winding resistance of 0.1 Ω and a leakage reactance of 0.22 Ω. The low voltage winding resistance is 0.035 Ω and the leakage reactance is 0.012 Ω. Find the equivalent winding resistance, reactance and impedance (only magnitude) referred to:

1- High Voltage Side.

2- Low-Voltage Side.

**Solution**

$k = \frac{120}{2400} = 1/20, R_1 = 0.1\ \Omega\ , X_1 = 0.22\ \Omega$

$R_2 = 0.035\ \Omega$ and $X_2 = 0.012\ \Omega$

1- For high voltage side

$$R_{01} = R_1 + R_2' = R_1 + \frac{R_2}{k^2} = 0.1 + \frac{0.035}{\left(\frac{1}{20}\right)^2} = \mathbf{14.1\ \Omega}$$

$$X_{01} = X_1 + X_2' = X_1 + \frac{X_2}{k^2} = 0.22 + \frac{0.012}{\left(\frac{1}{20}\right)^2} = \mathbf{5.02\ \Omega}$$

$$Z_{01} = \sqrt{R_{01}^2 + X_{01}^2} = \sqrt{14.1^2 + 5.02^2} = \mathbf{15\ \Omega}$$

2- For low voltage side

$$R_{02} = R_2 + R_1' = R_2 + k^2 R_1 = 0.035 + \left(\frac{1}{20}\right)^2 \times 0.1 = \mathbf{0.03525\ \Omega}$$

$$X_{02} = X_2 + X_1' = X_2 + k^2 X_1 = 0.012 + \left(\frac{1}{20}\right)^2 \times 0.22 = \mathbf{0.0125\ \Omega}$$

$$Z_{02} = \sqrt{R_{02}^2 + X_{02}^2} = \sqrt{0.0325^2 + 0.01255^2} = \mathbf{0.0374\ \Omega}$$

**Example 4:** The parameters of a 2300/230 V, 50-Hz transformer are given below:

$$R_1 = 0.286\,\Omega, \quad R_2' = 0.319\,\Omega\,, \quad R_0 = 250\,\Omega$$

$$X_1 = 0.73\,\Omega\,, X_2' = 0.73\,\Omega\,, \quad X_0 = 1250\,\Omega$$

The secondary load impedance $Z_L = 0.387 + j\,0.29$ Ω. Using the exact equivalent circuit Calculate the following.

1- Primary power factor.
2- Power input.
3- Power output.
4- Cu loss.
5- Efficiency.

**Solution**

$k = 230/2300 = 0.1$ , $\boldsymbol{Z_L} = 0.387 + j\,0.29\,\Omega$

$\boldsymbol{Z'_L} = \boldsymbol{Z_L}/k^2$ =100 (0.387 + j 0.29) = 38.7 + j 29 = **48.4 ∠36.8° Ω**

$\boldsymbol{Z'_2} + \boldsymbol{Z'_L}$ =(38.7 + 0.319) + j(29 + 0.73) = 39.02 + j29.73 = **49∠ 37.3° Ω**

$\boldsymbol{1}/\boldsymbol{Z_m} = 1/R_0 + 1/jX_0 \Longrightarrow \boldsymbol{Z_m}$ =240 + j48 = **245 ∠11.3° Ω**

$\boldsymbol{Z_m}//(\boldsymbol{Z'_2} + \boldsymbol{Z'_L})$ = 245∠11.3° // 49 ∠ 37.3° = **41.4∠33° Ω**

$\boldsymbol{Z_{t1}} = \boldsymbol{Z_1} + (\boldsymbol{Z_m}//(\boldsymbol{Z'_2} + \boldsymbol{Z'_L}))$=(0.286+j 0.73)+ 41.4 ∠33° = **42∠33.7° Ω**

$\boldsymbol{I_1} = \boldsymbol{V_1}/\boldsymbol{Z_{t1}}$=2300∠0 / 42∠33.7° = **54.8∠−33.7° A**

$\boldsymbol{Z_m} + \boldsymbol{Z'_2} + \boldsymbol{Z'_L}$=245 ∠11.3° + 49 ∠ 37.3° = **290∠15.6° Ω**

$\boldsymbol{I'_2} = \boldsymbol{I_1} \times \left(\boldsymbol{Z_m}/\boldsymbol{Z'_2} + \boldsymbol{Z'_L} + \boldsymbol{Z_m}\right)$ = 54.8∠−33.7 × 0.845∠−4.3 = **46.2 ∠ − 38° A**

$\boldsymbol{I_0} = \boldsymbol{I_1} \times \left(\boldsymbol{Z'_2} + \boldsymbol{Z'_L}/\boldsymbol{Z'_2} + \boldsymbol{Z'_L} + \boldsymbol{Z_m}\right)$ = 54.8∠−33.7 × $\frac{49\angle 37.3}{290\angle 15.6}$ = **9.26∠−12° A**

1- Primary power factor = cos(33.7) = **0.832 lag**

2- Power input = $V_1 I_1 \cos\phi_1$ = 2300 × 54.8 ×0.832 = **105 kW**

3- Power output = $I_2'^2 R_L'$ = $46.2^2 \times 38.7$ = **82.7 kW**

4- Primary Cu loss = $54.8^2 \times 0.286$ = **860 W**

Secondary Cu loss = $46.2^2 \times 0.319$ = **680 W**

Core Cu loss = $9.26^2 \times 240$ = **20.6 kW**

5- Efficiency = $\frac{82.7}{105} \times 100$ = **78.8 %**

**Example 5:** The equivalent circuit parameters of a single-phase 240/2400, 50 Hz, transformer are $R_0$ =600 Ω, $X_0$ =300 Ω, $R_{01}$ =0.25 Ω, $X_{01}$ =0.75 Ω. The transformer is supplying a load of 400+ *j* 200 Ω. Keeping the primary voltage of 240V, calculate the

1- The secondary terminal voltage
2- Current in the primary winding
3- Power factor of the primary side
4- Power output
5- Power Input

**Solution**

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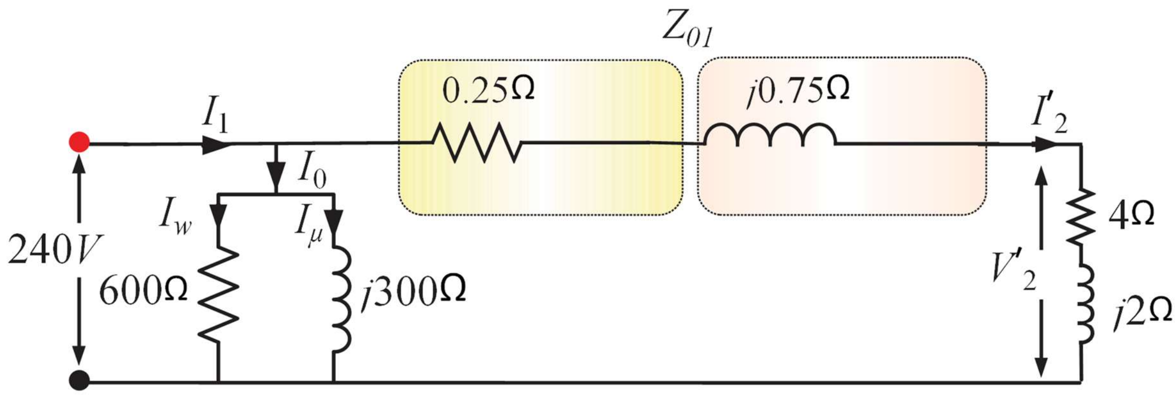


Since the equivalent circuit is referred to the low voltage (primary side), the load

impedance is also transformed to the low voltage side.

$k$ = 2400/240 = 10

$\boldsymbol{Z'_L} = {}^{\boldsymbol{Z_L}}/_{k^2}$ =(400 + j200) (0.1)$^2$ = **4 + j2 Ω**

$\boldsymbol{Z'_L} + \boldsymbol{Z_{01}}$ =0.25 + j0.75 + 4 + j2 = 4.25 + j2.75 = **5.062∠32.9 Ω**

$$\boldsymbol{I'_2} = {}^{\boldsymbol{V_1}}/_{(\boldsymbol{Z'_L} + \boldsymbol{Z_{01}})} = \frac{240\angle 0}{5.062\angle 32.9} = 47.412\angle -32.9 = \mathbf{39.8 - j25.753\ A}$$

1- Secondary terminal voltage (without phase)

$V'_2 = I'_2 Z_L$ =5.062 × $\sqrt{4^2 + 2^2}$ = **212.03V**

2- Primary current:

The core loss component of current $I_w = \frac{V_1}{R_0} = \frac{240}{600}$ = **0.4 A**

The magnetizing component of current $I_\mu = \frac{V_1}{X_0} = \frac{240}{300}$ = **0.8 A**

The no-load current $I_0 = I_w + jI_\mu$ = **0.4 − j0.8 A**

The primary current

$I_1 = I_0 + I_2' = 39.8 - j25.753 + 0.4 - j0.8 = 40.2 - j26.553 =$ **48.178∠−33.44 A**

3- Power factor of the primary current pf = cos(33.44) = **0.834 lagging**

4- Power output = $I_2'^2 R_L' = 47.412^2 \times 4 =$ **8.99 kW**

5- Power Input = $V_1 I_1 \cos\phi_1 = 240 \times 48.178 \times \cos(33.44) =$ **9.65 kW**

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