AL- Mustaqpal University
Science College
Dep. Medical physics

Medical physics
Third Stage

Lec 4

Boltzmann distribution

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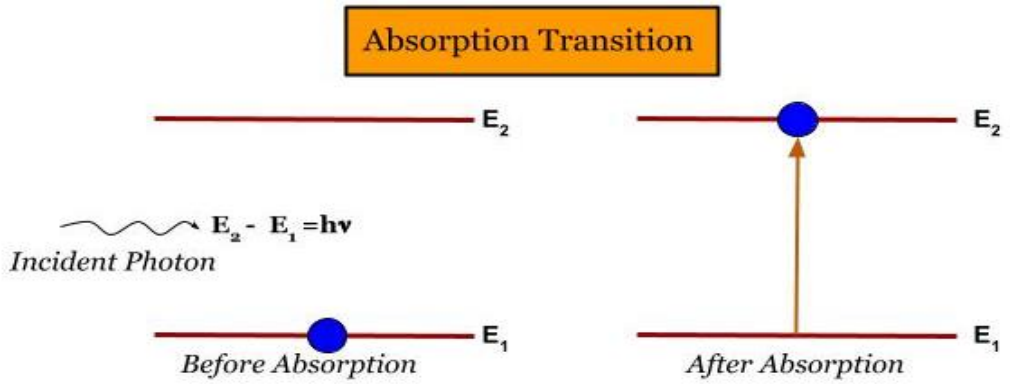
Boltzmann distribution

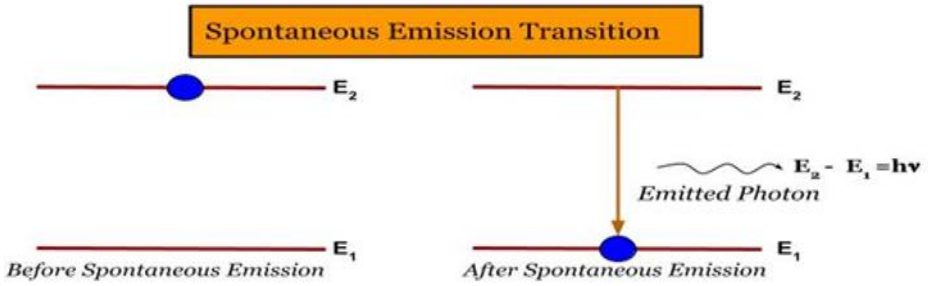
Induced absorption

It is the transition of an atom from a low energy level (E_1) to an excited energy level called (E_2) by absorbing a photon whose energy is equal to the energy difference between these two levels ($hf = E_2 - E_1$)



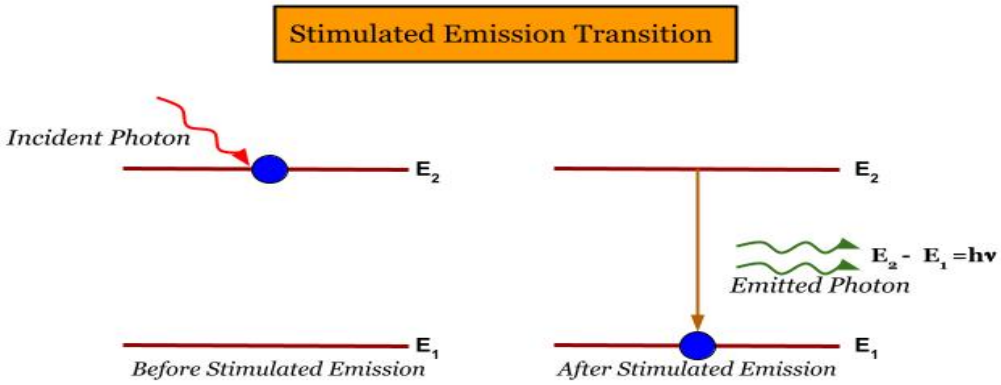
Spontaneous emission

When the atom becomes at the highest energy level (excited), it always tends to a stable state, so it automatically returns after a short period of time to the ground level, and this is accompanied by the emission of a photon... (This transition is called spontaneous emission, and its photons are different in terms of phase and direction).



Stimulated emission

When a photon impacts an excited atom while it is at the energy level (E_2), its energy is exactly equal to the energy difference between (E_1) and (E_2), it stimulates the unstable electron to descend to the level (E_1) and emit a photon similar to the stimulated photon with energy, frequency, phase, and direction (This means that we get two identical photons).



What is Boltzmann distribution ?

The Boltzmann distribution gives the probability that a system will be in a certain state as a function of that state's energy

If we have a system consisting of (molecules, atoms, or ions) in a state of thermal equilibrium, the number of atoms in the lower energy levels will be greater than the number of atoms in the higher (excited) levels, meaning that ($N_1 > N_2$).

But if the atoms gain a certain energy, they become agitated, and the number of atoms in the lower energy levels is less than the number of atoms in the higher (excited) levels, meaning that ($N_1 < N_2$)

What is meant by inverse distribution ?

It is when the number of atoms at high energy levels is greater than their number at low levels... This happens when the atomic system is not thermally balanced (which means that the inverted distribution contradicts the Boltzmann distribution).

The **scientist Boltzmann** established a law regarding this :

$$\frac{N_2}{N_1} = exp\left(\frac{-(E_2 - E_1)}{KT}\right)$$

N_1 is the number of atoms in the lower energy state, N_2 is the number of atoms in the excited state. The process of making $N_2 > N_1$ is called population inversion. **K** is Boltzmann's constant equal to ($1.38064852 \times 10^{-23}$ m^2 kg s^{-2} K^{-1})

T absolute temperature and is measured in Kelvin

The distribution of the atoms between the levels is given by Boltzmann's Law

This equation means that at thermal equilibrium, N1 > N2.

To achieve laser operation, one must upset the thermal equilibrium N2 > N1 in some way so as to produce the nonequilibrium situation of a population inversion.

With a number of energy levels the amount of energy is (E1 < E2), the energy population ($E_1 < E_2$)

In the Boltzmann Distribution, The Boltzmann equation determines the relation between the population number of a specific energy level and the temperature

Conclusions :

1. The relation between two population numbers (N_2 / N_1) does not depend on the values of the energy levels E1 and E2, but only on the difference between them:

   (E_2 - E_1)

2. For a certain energy difference, the higher the temperature, the bigger the relative population.

3. If N_2 is greater than N_1, the inverse distribution will be achieved and laser action will occur , If N_2 is smaller than N_1, the inverse distribution will not be achieved and the laser action will not occur.

Example 1

mathematically that the inverse distribution is not achieved when the thermal energy (**KT**) is equal to the energy of the incident photon?

$$\frac{N_2}{N_1} = exp\left(\frac{-(E_2 - E_1)}{KT}\right)$$

$$But \quad E_2 - E_1 = hf \qquad And \qquad KT = hf$$

$$\therefore \frac{N_2}{N_1} = exp - \left(\frac{hf}{hf}\right)$$

$$\therefore \frac{N_2}{N_1} = exp(-1)$$

$$\therefore N_2 = 0.37\, N_1$$

That is, $N_1 > N_2$, and therefore the inverse distribution is not achieved.

Example 2

Calculate the number of atoms in the highest energy level in thermal equbrium if the number of atoms in the ground energy level is 500 atoms ?

$$\frac{N_2}{N_1} = exp\left(\frac{-(E_2 - E_1)}{KT}\right)$$

In thermal equbrium $KT = (E_2 - E_1)$

$$\therefore \frac{N_2}{N_1} = exp - \left(\frac{(E_2 - E_1)}{(E_2 - E_1)}\right)$$

$$\therefore \frac{N_2}{N_1} = exp - 1$$

$$\therefore \frac{N_2}{N_1} = 0.37$$

$$\therefore \frac{N_2}{500} = 0.37$$

$$\therefore N_2 = 185 \quad Atoms$$

So the inverted distribution does not occur because the number of atoms in the upper level is less than in the ground level.

Discussion

Question 1

What is the transition of an atom from a low energy level to an excited energy level called?

A) Spontaneous emission

B) Stimulated emission

C) Induced absorption

D) Boltzmann distribution

E) None of the above

Question 2

What happens when an excited atom returns to its ground state?

A) It absorbs a photon

B) It emits a photon

C) It remains in the excited state

D) It stays at the same energy level

E) None of the above

Question 3

What is the Boltzmann distribution?

A) The probability of an atom being in a certain energy state

B) The energy difference between two energy levels

C) The number of atoms in a specific energy state

D) The temperature of a system

E) None of the above

Question 4

What is the condition for population inversion?

A) N1 > N2

B) N2 > N1

C) N1 = N2

D) N1 > N1×N2

E) None of the above

Question 5

What is the purpose of upsetting the thermal equilibrium in a laser operation?

A) To achieve population inversion

B) To maintain thermal equilibrium

C) To increase the energy difference between levels

D) To decrease the temperature of the system

E) None of the above

Question 6

What is the relation between the population numbers N2 and N1 in the Boltzmann distribution?

A) N2 depends on the values of E1 and E2

B) N2 depends only on the difference between E1 and E2

C) N2 is always greater than N1

D) N2 is always less than N1

E) None of the above

Question 7

What happens when the thermal energy (KT) is equal to the energy of the incident photon?

A) Inverse distribution is achieved

B) Inverse distribution is not achieved

C) The system is in thermal equilibrium

D) The system is not in thermal equilibrium

E) None of the above

Question 8

What is stimulated emission?

A) When an atom absorbs a photon

B) When an atom emits a photon

C) When a photon impacts an excited atom

D) When an atom remains in its ground state

E) None of the above

Question 9

What is the significance of the Boltzmann constant (K)?

A) It is a measure of the energy difference between levels

B) It is a measure of the temperature of the system

C) It is a constant used in the Boltzmann equation

D) It is a measure of the number of atoms in a specific energy state

E) None of the above

Question 10

What is the conclusion regarding the relation between N2 and N1?

A) N2 is always greater than N1

B) N2 is always less than N1

C) N2 depends on the temperature of the system

D) N1 is always greater than N2

E) None of the above