Al-Mustaqbal University
Department of Medical Instrumentation Techniques Engineering
Class: Third
Subject: Medical Communication Systems
Lecturer: Dr. Bayan Mahdi Sabbar & M.Sc. Huda Wasfi Hassoon
Lecture: 1

Lecture 1

General Review in Electrostatic & Gauss's Law

1.1 Coulomb's Law:

Records from at least 600 B.C. show evidence of the knowledge of static electricity. The Greeks coined the term "electricity," derived from their word for amber. They spent many leisure hours rubbing a small piece of amber on their sleeves and observing how it would attract pieces of fluff and other small objects. However, their main interest lay in philosophy and logic, not in experimental science, and it took many centuries before the attracting effect was considered anything other than magic or a "life force."

Dr. Gilbert, physician to Queen Elizabeth I, was the first to conduct true experimental work with this effect. In 1600, he stated that glass, sulfur, amber, and other materials could attract straws and chaff, as well as metals, wood, leaves, stones, earth, even water and oil.

Shortly thereafter, Colonel Charles Coulomb, a precise and orderly-minded officer in the French Army Engineers, performed an elaborate series of experiments using a delicate torsion balance he invented. He determined quantitatively the force exerted between two objects, each having a static charge of electricity. His published result is now known to many high school students and bears a great similarity to Newton's gravitational law, discovered about a hundred years earlier.

Email: huda.wasfi@uomus.edu.iq

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Coulomb's law: Coulomb stated that the force between two very small objects, separated in a vacuum or free space by a distance that is large compared to their size, is directly proportional to the charge on each object and inversely proportional to the square of the distance between them.

$$F = K\frac{Q_1 Q_2}{R_{12}^2}\overrightarrow{a_{12}}$$

Where:

Q_1 and Q_2 are the positive or negative quantities of charge.

K: is constant ($8.99 \times 10^9\ Nm^2/C^2$)

$K = \frac{1}{4\pi\epsilon_0}$ $(\epsilon_0 = 8.85 \times 10^{-12})$

F: Force (N)

$\overrightarrow{a_{12}}: unit\ vector\ in\ the\ direction\ of\ R_{12}$

$$\overrightarrow{a_{12}} = \frac{R_{12}}{|R_{12}|} = \frac{r_2 - r_1}{|R_{12}|}$$

Coulomb’s Law is fundamental because it:

1. **Quantified Electrical Forces:** It allowed scientists to calculate the exact force between charged objects, moving beyond qualitative observations.
2. **Paralleled Gravitational Law:** It showed a similarity to Newton’s Law of Gravitation, suggesting a universal principle of inverse-square laws in nature.
3. **Foundation for Electromagnetism:** It laid the groundwork for later developments in electromagnetism, including Maxwell’s equations, which describe how electric and magnetic fields interact.

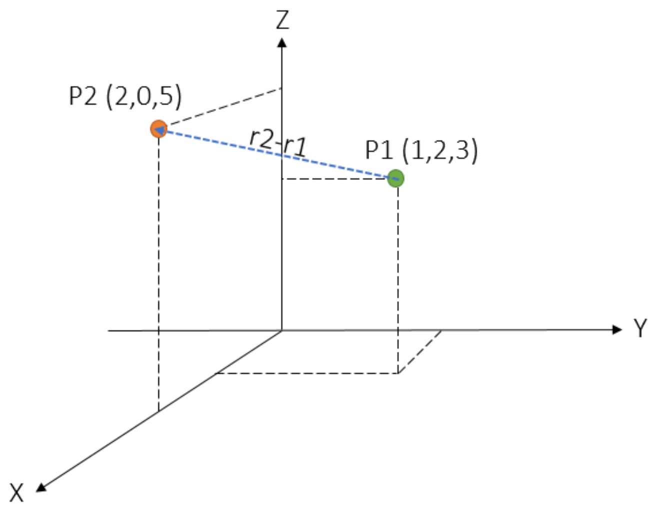
Overall, Coulomb's work was crucial in transforming electricity from a mysterious phenomenon into a well-understood scientific field.

Example: Consider a charge of $3 \times 10^{-4} C$ at point P1(1,2,3) and a charge of $-10^{-4} C$ at point P2(2,0,5), Find F.

Solution:

$$R_{12} = r_2 - r_1 = (2-1)\overrightarrow{a_x} + (0-2)\overrightarrow{a_y} + (5-3)\overrightarrow{a_z} = \overrightarrow{a_x} - 2\overrightarrow{a_y} + 2\overrightarrow{a_z}$$

$$|R_{12}| = \sqrt{1 + (-2)^2 + 2^2} = 3$$



$$\overrightarrow{a_{12}} = \frac{R_{12}}{|R_{12}|} = \frac{\overrightarrow{a_x} - 2\overrightarrow{a_y} + 2\overrightarrow{a_z}}{3}$$

$$F = K\frac{Q_1 Q_2}{R_{12}^2}\overrightarrow{a_{12}} = 8.99 \times 10^9 \times \frac{(3\times10^{-4})(-10^{-4})}{(3)^2} \times \frac{\overrightarrow{a_x} - 2\overrightarrow{a_y} + 2\overrightarrow{a_z}}{3}$$

$$= (-10\overrightarrow{a_x} + 20\overrightarrow{a_y} - 20\overrightarrow{a_z})N$$

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1.2 Electrical Field Intensity

➢ Presence of a fixed charge and its effect on another charge (Q_1 and Q_t):

If we consider one charge fixed at a specific position and move a second charge slowly around it, we observe that there is a continuous force acting on the second charge as it moves. This force indicates the presence of a force field in the region around the charge

We refer to the second charge Q_t as a "test charge" because it is used to test the force exerted by Q_1.

The force acting on the test charge Q_t can be determined using **Coulomb's Law**, which states that the force between two electric charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them.

The mathematical expression for this force is:

$$F_t = K\frac{Q_1 Q_t}{R_{1t}^2}\overrightarrow{a_{1t}}$$

where:

- F_t is the force acting on charge Q_t.
- K is Coulomb's constant.
- Q_1 and Q_t are the magnitudes of the charges.
- R_{1t} is the distance between the fixed charge Q_1 and the test charge Q_t .
- $\overrightarrow{a_{1t}}$ is the unit vector indicating the direction of the force (from Q_1 to Q_t).

To simplify things, we calculate the force per unit charge, i.e., the force acting on the test charge per coulomb of charge. This leads us to the concept of electric field

Email: huda.wasfi@uomus.edu.iq

intensity, which gives an idea of how strong the electric field is at a certain point. The mathematical expression for electric field intensity is:

$$\frac{F_t}{Q_t} = K\frac{Q_1}{R_{1t}^2}\overrightarrow{a_{1t}}$$

This expression represents the electric field intensity, denoted by $\vec{E}$.

$$\vec{E} = \frac{F_t}{Q_t} = K\frac{Q_1}{R_{1t}^2}\overrightarrow{a_{1t}}$$

$\vec{E}$ is the vector representing the electric field intensity produced by charge Q_1.

**It only depends on the charge Q_1 and the distance between Q_1 and the test charge Q_t.

➢ General Case for a Single Charge:

In the case of a single point charge Q_1 located at a specific point, the electric field intensity at any point a distance R from the charge is given by:

$$\vec{E} = K\frac{Q_1}{R^2}\overrightarrow{a_r}$$

$\overrightarrow{a_r}$ is the unit vector pointing from the charge Q_1 to the point where we want to calculate the field.

➢ Electric Field Intensity Due to Two Charges:

In the case of two point charges Q_1 and Q_2 located at different positions r_1 and r_2, the electric field intensity at a given point is the sum of the forces exerted by each charge.

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The mathematical expression is a sum:

$$\vec{E} = K\frac{Q_1}{|\vec{r} - r_1|^2}\overrightarrow{a_1} + K\frac{Q_2}{|\vec{r} - r_2|^2}\overrightarrow{a_2}$$

$\vec{E}$ is the total electric field intensity produced by the two charges Q_1 and Q_2 at the point at certain distances from both charges.

➢ Electric Field Intensity for More than Two Charges:

If there are more than two charges, the electric field intensity at a point is the sum of the fields produced by each charge individually. The expression becomes:

$$\vec{E} = K\frac{Q_1}{|\vec{r} - r_1|^2}\overrightarrow{a_1} + K\frac{Q_2}{|\vec{r} - r_2|^2}\overrightarrow{a_2} + \cdots \ldots\ldots + K\frac{Q_n}{|\vec{r} - r_n|^2}\overrightarrow{a_n}$$

Q_n is the point charge, r_n is the position of the charge, and $\overrightarrow{a_n}$ is the unit vector pointing from charge Q_n to the point where we want to calculate the field.

Example: Determine the electric field intensity produce by a point charge at P (1,1,1) caused by four identical 3nC point charges located at P1(1,1,0), P2(-1,1,0), P3(-1, -1,0), and P4(1, -1,0)

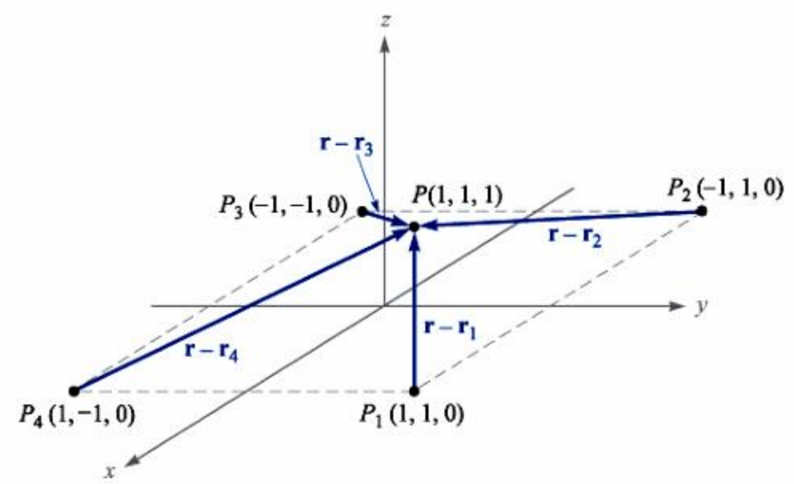
Solution:

$\vec{r} - r_1 = \overrightarrow{a_z}$

$\vec{r} - r_2 = 2\overrightarrow{a_x} + \overrightarrow{a_z}$

$\vec{r} - r_3 = 2\overrightarrow{a_x} + 2\overrightarrow{a_y} + \overrightarrow{a_z}$

$\vec{r} - r_4 = 2\overrightarrow{a_y} + \overrightarrow{a_z}$



$$\vec{E} = KQ_1\left[\frac{1}{|\vec{r}-r_1|^2}\overrightarrow{a_1} + \frac{1}{|\vec{r}-r_2|^2}\overrightarrow{a_2} + \frac{1}{|\vec{r}-r_3|^2}\overrightarrow{a_3} + \frac{1}{|\vec{r}-r_4|^2}\overrightarrow{a_4}\right]$$

$$\vec{E} = 27\left[\frac{1}{(\sqrt{1})^2}\left(\frac{a_z}{\sqrt{1}}\right) + \frac{1}{(\sqrt{5})^2}\left(\frac{2\overrightarrow{a_x}+\overrightarrow{a_z}}{\sqrt{5}}\right) + \frac{1}{(\sqrt{9})^2}\left(\frac{2\overrightarrow{a_x}+2\overrightarrow{a_y}+\overrightarrow{a_z}}{\sqrt{9}}\right) + \frac{1}{(\sqrt{5})^2}\left(\frac{2\overrightarrow{a_y}+\overrightarrow{a_z}}{\sqrt{5}}\right)\right]$$

$$\vec{E} = 6.82\overrightarrow{a_x} + 6.82\overrightarrow{a_y} + 32.8\overrightarrow{a_z}$$

1.3 Electric Flux Density

Around 1837, Michael Faraday became interested in the behavior of static electric fields and the influence of insulating materials (dielectrics). After a decade of experiments on electromotive force, he designed an experiment using two concentric metallic spheres. The key steps were:

1. The inner sphere was given a known positive charge.
2. The outer sphere, made of two hemispheres, was clamped around the inner sphere with about 2 cm of dielectric between them.

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3. The outer sphere was discharged by grounding it.
4. The outer sphere was separated, and the induced charge on each hemisphere was measured.

Faraday discovered that the total charge on the outer sphere equaled the charge on the inner sphere, regardless of the dielectric. He concluded that "electric displacement" occurred, which we now call electric flux. His experiments showed that a larger charge on the inner sphere induced a proportionally larger negative charge on the outer sphere.

If the electric flux is denoted by (psi) and the total charge on the inner sphere by Q, then for Faraday's experiment

$$\psi = Q$$

and the electric flux is measured in coulombs.

We can obtain more quantitative information by considering an inner sphere of radius a and an outer sphere of radius b, with charges of $+Q$ and $-Q$, respectively (figure(1-1)). The paths of electric flux ψ extending from the inner sphere to the outer sphere are indicated by the symmetrically distributed streamlines drawn radially from one sphere to the other. At the surface of the inner sphere, ψ coulombs of electric flux are produced by the charge $Q = \psi$ coulombs distributed uniformly over a surface having an area of $4\pi a^2\ m^2$. The density of the flux at this surface is $\psi/4\pi a^2$ or $Q/4\pi a^2\ C/m^2$, and this is an important new quantity.

Electric flux density, measured in coulombs per square meter (sometimes described as "lines per square meter," for each line is due to one coulomb), is given the letter D, which was originally chosen because of the alternate names of displacement flux density or displacement density. Electric flux density is more descriptive, however, and we shall use the term consistently.

The electric flux density D is a vector field and is a member of the "flux density" class of vector fields, as opposed to the "force fields" class, which includes the electric field intensity E. The direction of D at a point is the direction of the flux lines at that point, and the magnitude is given by the number of flux lines crossing a surface normal to the lines divided by the surface area.

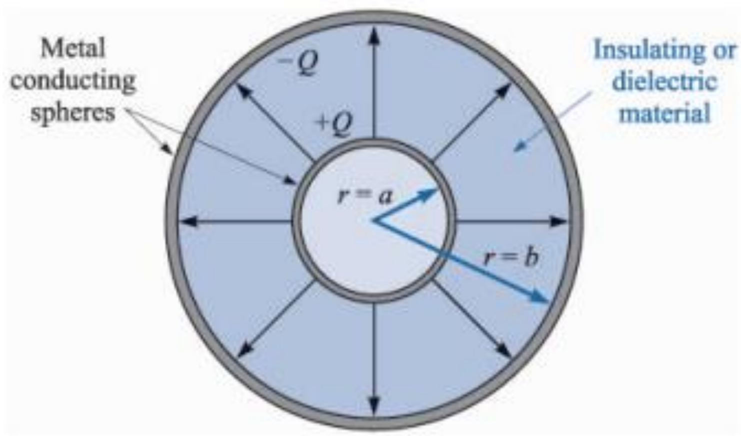

Figure (1-1) The electric flux in the region between a pair of charged concentric spheres. The direction and magnitude of Dare not functions of the dielectric between the spheres.

Referring again to Fig. (1-1), The **electric flux density** (denoted as D) is a vector field that represents the amount of electric flux passing through a given area in space. In this scenario, the flux density is **radial** because we are dealing with spherical symmetry (the system is centered around a sphere).

For two concentric spheres:

- The inner sphere has radius a and a charge Q_1.
- The outer sphere has radius b, and there's dielectric material in between.

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The electric flux density at a radial distance r from the center, where $a \leq r \leq b$, is given by the following formulas:

- **At the surface of the inner sphere (radius a):**

$$D|_{r=a} = \frac{Q_1}{4\pi a^2}\overrightarrow{a_r}$$

This means the flux density at radius a depends on the charge Q_1 and the surface area of the sphere $4\pi a^2$. The direction is radial (denoted by $\overrightarrow{a_r}$).

- **At the surface of the outer sphere (radius b):**

$$D|_{r=b} = \frac{Q_1}{4\pi b^2}\overrightarrow{a_r}$$

The flux density at radius b follows the same principle.

- **At any distance r between the inner and outer spheres, where $a \leq r \leq b$:**

$$D = \frac{Q_1}{4\pi r^2}\overrightarrow{a_r}$$

The flux density decreases with the square of the distance r, as it spreads out over the surface area of an imaginary sphere with radius r.

If you make the inner sphere smaller and smaller while keeping its charge Q_1constant, it eventually becomes a **point charge**. In this limit, the electric flux density at a distance r from the point charge is still given by the same equation:

$$D = \frac{Q_1}{4\pi r^2}\overrightarrow{a_r}$$

This is because the flux lines radiate symmetrically outward from the point charge, passing through a spherical surface of area $4\pi r^2$.

Email: huda.wasfi@uomus.edu.iq

The **electric field intensity** $\vec{E}$ represents the force experienced by a unit positive charge at a point in space. For a point charge in free space, the electric field intensity at a distance r from the charge is given by:

$$\vec{E} = \frac{Q_1}{4\pi\varepsilon_0 r^2}\overrightarrow{a_r}$$

Here, ε_0 is the **permittivity of free space**. The electric field also points radially outward from the point charge.

In free space, there is a simple relationship between the electric flux density D and the electric field intensity E. They are related by the permittivity of free space:

$$D = \varepsilon_0 E \qquad \text{(Free Space Only)}$$

This relationship holds only in free space (a vacuum or air, where the permittivity ε_0).

1.4 Gauss's Law

The results of Faraday's experiments with concentric spheres can be summarized by an experimental law stating that the electric flux passing through any imaginary spherical surface between the two conducting spheres equals the charge enclosed within that surface. This charge is distributed on the surface of the inner sphere or might be concentrated as a point charge at the center. Since one coulomb of electric flux is produced by one coulomb of charge, the shape of the inner conductor (whether a cube or a brass key) doesn't affect the total induced charge on the outer sphere, which remains the same. The flux distribution might change, but

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the induced charge will always be equal in magnitude and opposite in sign to the charge on the inner conductor.

These generalizations of Faraday's experiment lead to the following statement, which is known as Gauss's law:

The electric flux passing through any closed surface is equal to the total charge enclosed by that surface.

We then have the mathematical formulation of Gauss's law,

$$\psi = \oint D_s . dS = Charge\ enclosed = Q$$

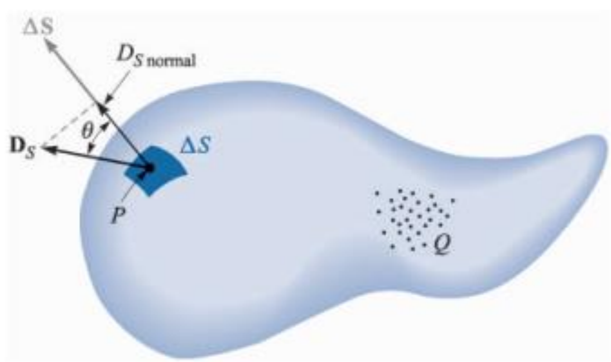
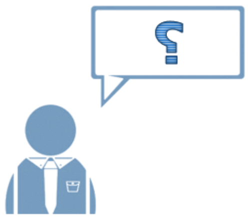


Figure (1-2): The electric flux density D_s at P due to charge Q. The total flux passing through ΔS is $D_s.\Delta S$.



Homework 1

1. A 2mC positive charge is located in vacuum at P1(3, -2, -4) and a 5 µC negative charge is at P2(1, -4, 2). Find the vector force on the negative charge.
2. Calculate $\vec{E}$ at point M(3,-4,2) in free space caused by :-

   a- A charge Q1=2µC at P1(0,0,0)

   b- A charge Q2=3µC at P2(-1,2,3)

   c- A charge Q1=2µC at P1(0,0,0) and a charge Q2=3µC at P2(-1,2,3)