Al-Mustaqbal University

College of Engineering & Technology

Electrical Engineering Fundamentals

Asst. Lec.   ALI IMAD

Lectur (2)

Ohm’s Law, Power and Energy

Ohm’s Law

Ohm‘s law states that the voltage V across a resistor is directly proportional to the current I flowing through the resistor.

$v \, \alpha \, I$ --(1)

$V = I \times R$ --(2)

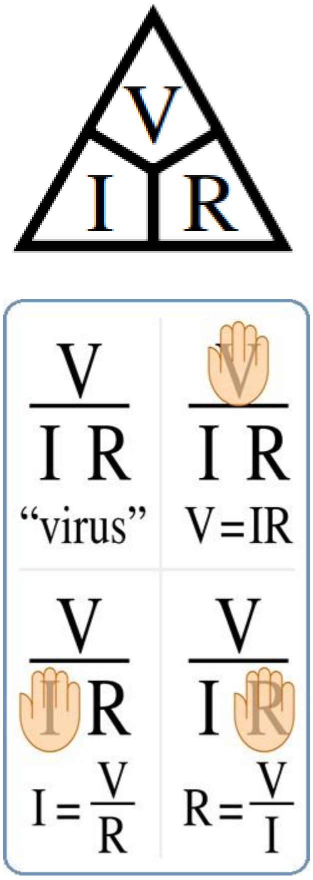


Fig.(1). Ohm's law triangle

Example 1: Find the resistance (R) for an electric conductor draws (4 A) at (240 V) ?

Solution:

From Ohm's law, ➔ $V = I.R$

$$R = \frac{V}{I} \quad ➔ \quad \mathrm{R} = \frac{240\ \mathrm{V}}{4\ A}$$

⇨ $R = 60\ \Omega$

Electric Power:

Electric Power defines the value of work done by an electric current per unit of time.

The unit of power is the WATT.

For a resistor in a DC-circuit

The power delivered to, or absorbed by, an electrical device or system can be found in terms of the current and voltage by first substituting equations

$$P = \frac{W}{t} = \frac{VQ}{t}$$

$$I = \frac{Q}{t}$$

$$P = V.I \text{ -- (3)}$$

The unit of power is the WATT.

By direct substitution of Ohm's law, the equation for power can be obtained in two other forms:

$$P = V.I$$

$$P = V \times (\frac{V}{R})$$

$$P = \frac{V^2}{R} \qquad \text{--------- (4)}$$

Or

$P = V \times I$

$P = (I \times R) \times I$

$P = I^2 \times R$--(5)

Energy:

The **energy** (W) lost or gained by any system is therefore determined by

$$\boxed{W = Pt} \qquad \text{-------(6)}$$

Watt . Second ➔ W.S Or Joules

$$\boxed{\text{Energy (Wh)} = \text{power (W)} \times \text{time (h)}} \qquad \text{-------(7)}$$

Or

$$\boxed{\text{Energy (kWh)} = \frac{\text{power (W)} \times \text{time (h)}}{1000}} \qquad \text{-------(8)}$$

Example 2: For the circuit shown in figure (2), calculate the current (I), voltage (V), conductance (G) and power (P)?

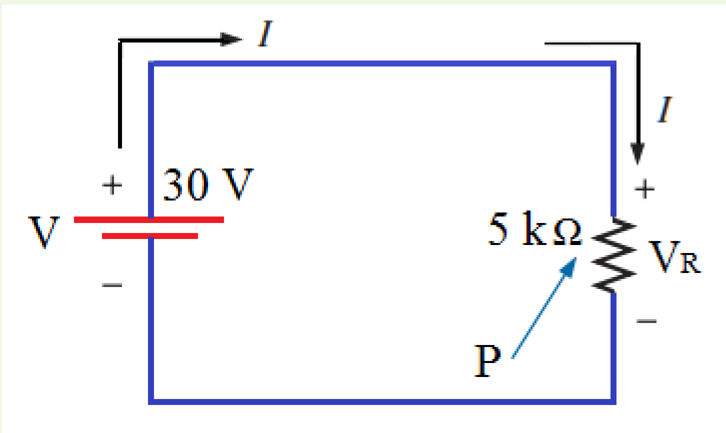


Fig. (2)

Solution:
From Ohm's law,

The current $\rightarrow$ $I = \frac{V}{R} = \frac{30}{5\times10^3} = 6 \times 10^{-3} = 6mA$

The conductance $\rightarrow$ $G = \frac{1}{R} = \frac{1}{5\times10^3} = 0.2 \times 10^{-3} = 0.2\ mS$

The Power (P) $\rightarrow$ $P = I \times V_R = 30(6 \times 10^{-3}) = 180\ mW$

Or

$\rightarrow$ $P = I^2 \times R = (6 \times 10^{-3})^2 \times (5 \times 10^3) = 180\ mW$

Or

$\rightarrow$ $P = \frac{V^2}{R} = V^2 \times G = (30)^2 \times (0.2 \times 10^{-3}) = 180\ mW$

Example2: How much energy (in kWh) is required to light a (60 W) bulb continuously for 1 year (365 days)?

Solution:

$$\text{Energy (Wh)} = \frac{pwoer(w)\times time\ (t)}{1000} \quad \text{---------} \ (*)$$

$$W = \frac{(60W)\ (\frac{24h}{day}) \times (365\ day)}{1000}$$

$W = 525.60$ kWh

Nodes, Branches, and Loops

- **A branch** represents a single element such as a voltage source or a resistor.
- **A node** is the point of connection between two or more branches.
- **A loop** is any closed path in a circuit.

The figure (2) show a network with (b) branches,(n) nodes, and (l) independent loops which will achieve the fundamental theorem of network techniques:

$$b = l + n - 1$$

Example 2:

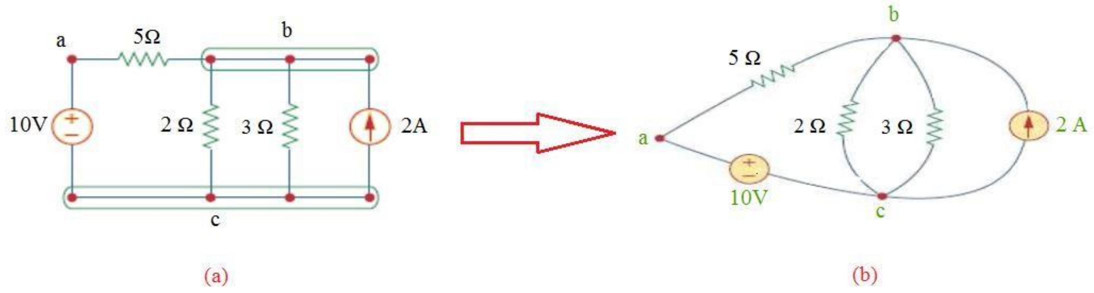


Figure (3),(a): Nodes, branches and loops Figure (3),(b): The three nodes circuit of Figure (3)(a) is redrawn.

Example 2: Determine the number of branches and nodes in the circuit shown below?

Solution:

Since there are 5 elements ➔ 10V, 2A, 5Ω, 2Ω, 3Ω,

- ⇨ Number of branches= 5
  Or
- ⇨ There are three nods :
  A, b, and c
- ⇨ The number of loops= 3

Notes:

⇨ There are more than 3 (dependent) loops in this example, we had only calculated the INDEPENDENT loop which are only (3).

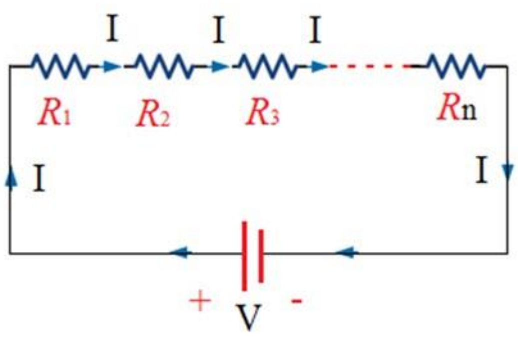
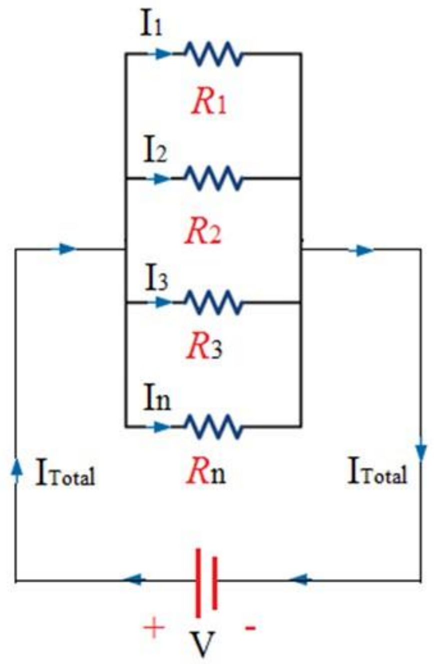
⇨ In general; Any circuit with b branches, n nodes and l independent loops, the following fundamental theorem of network topology:

$$b = l + n - 1$$

Series and Parallel Resistances

Two or more elements are in series if they exclusively share a single node and consequently carry the same current.

Two or more elements are in parallel if they are connected to the same two nodes and consequently have the same voltage across them.

	Series Circuit	Parralel Circuit
	 Resistors connected in series	 Resistors connected in parralel
1	The same current flows through each resistance. $I = I_1 = I_2 = \ldots = I_n$	The same voltage exists across all the resistances in parallel. $V = V_1 = V_2 = \ldots = V_n$
2	The voltage across each resistance is different.	The current through each resistance is different.
3	The sum of the voltages across all the resistances is the supply voltage. $V = V_1 + V_2 + \ldots + V_n$	The sum of the currents through all the resistances is the supply current. $I = I_1 + I_2 + \ldots + I_n$
4	The equivalent resistance is, $R_{eq} = R_1 + R_2 + \ldots + R_n$	The equivalent resistance is, $\frac{1}{R_{eq}} = \frac{1}{R1} + \frac{1}{R2} + \cdots + \frac{1}{Rn}$
5	The equivalent resistance is the largest than each of the resistances in series. $R_{eq} > R_1, R_{eq} > R_2 \ldots, R_{eq} > R_n$	The equivalent resistance is the smaller than the smallest of all the resistances in parallel.

Example 3: For the figure (4), find the equivalent resistance between the two points A & B?

Solution:

The resistances 3 Ω, 4 Ω, and 4 Ω are in parallel (as voltage across them same but current divides),

∴ Equivalent resistance is, $\frac{1}{R} = \frac{1}{3} + \frac{1}{4} + \frac{1}{4} = \frac{10}{12}$

$$\frac{1}{R} = \frac{10}{12} = 1.2\ \Omega$$

The resistances of 5Ω and 6 Ω are in series (as carry the same current).

∴ Equivalent resistance is 5+6 = 11 Ω

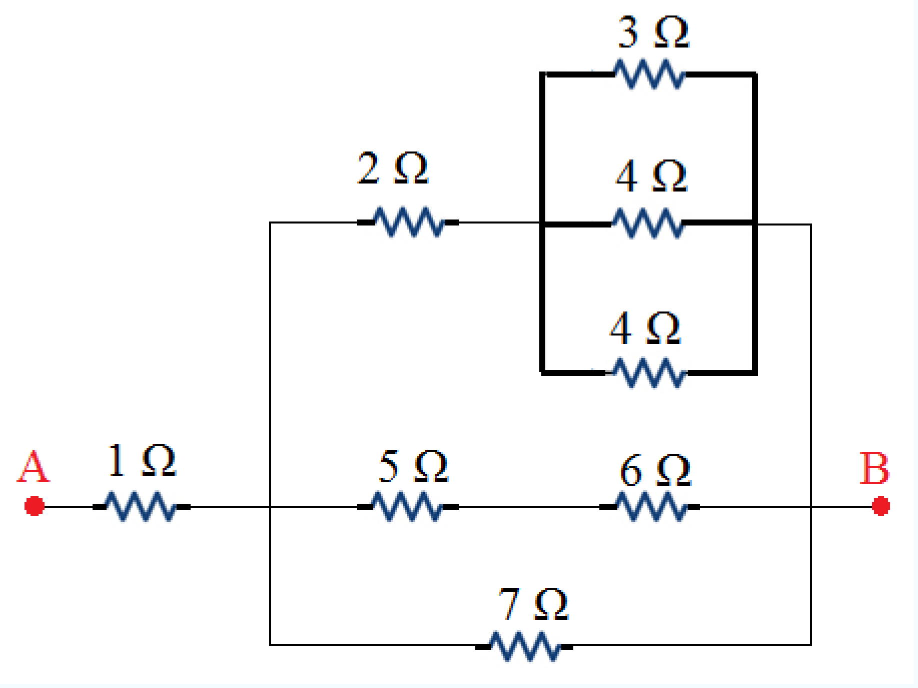


Fig.(4)

The circuit become as shown in figure (5):

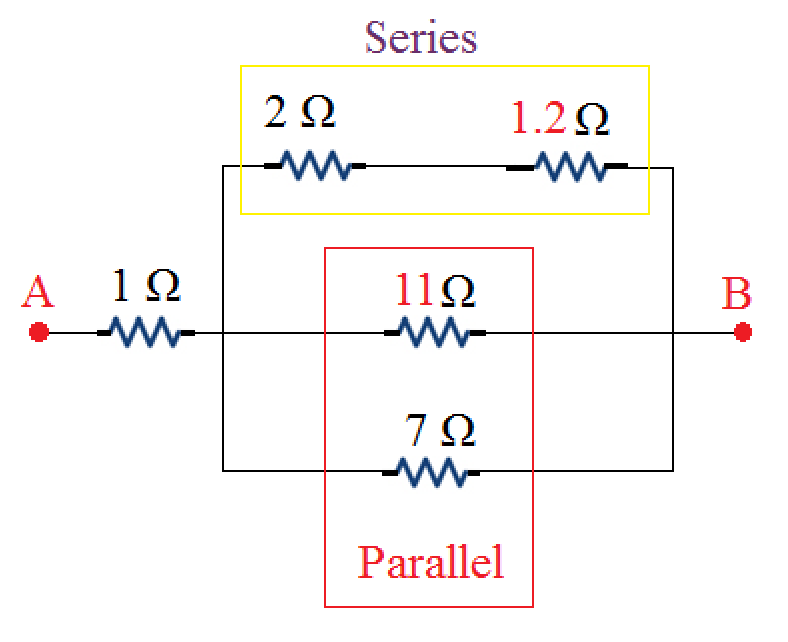


Fig. (5)

The resistors (2 Ω) and (1.2 Ω) are in series therefore, equivalent resistance is :

⇨ 2+1.2=3.2 Ω

While 11 Ω and 7 Ω are in parallel :

⇨ $\frac{1}{R} = \frac{1}{11} + \frac{1}{7} = \frac{18}{77}$

⇨ $R = \frac{77}{18} = 4.277\Omega$

Replacing the circuit to the figure (6) below:

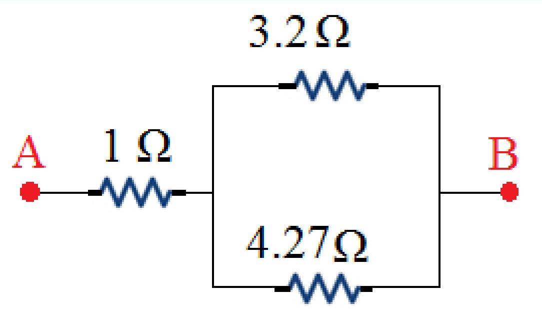


Fig.(6)

3.2Ω and 4.27Ω are in parallel:

$$R = \frac{3.2 * 4.27}{3.2 + 4.27} = 1.833\Omega$$

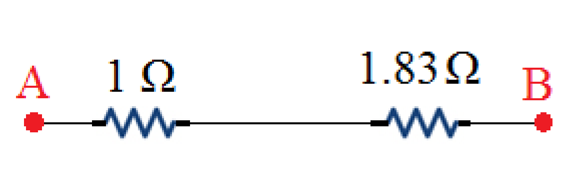


Fig.(7)

From Fig.(7):

$\therefore R = 1 + 1.83 = 2.83\Omega$

Example 4: Find the equivalent resistance between the two points A & B in the Figure (8)?

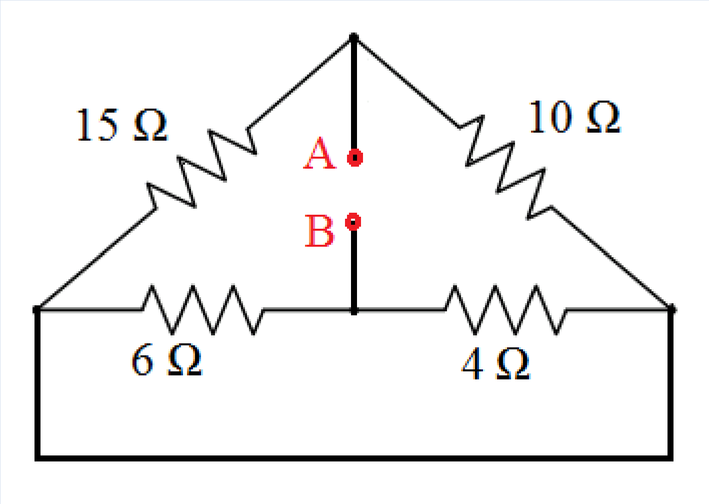
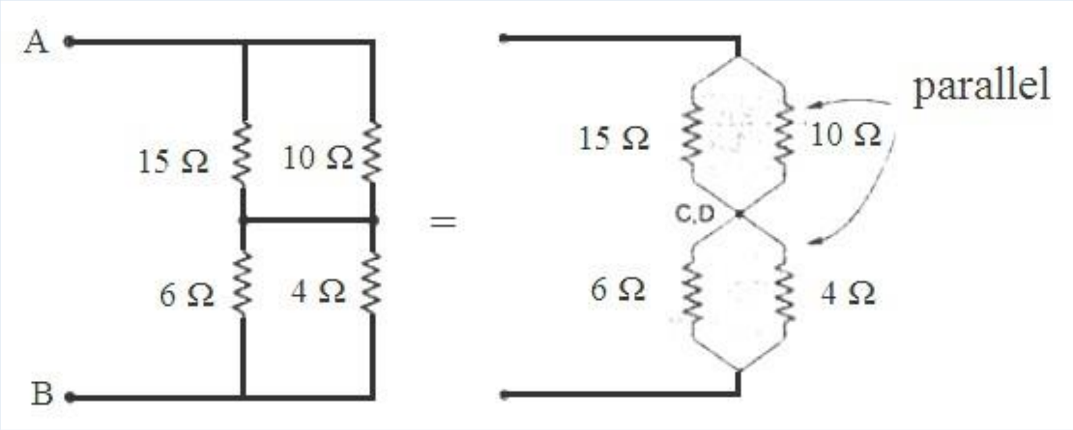
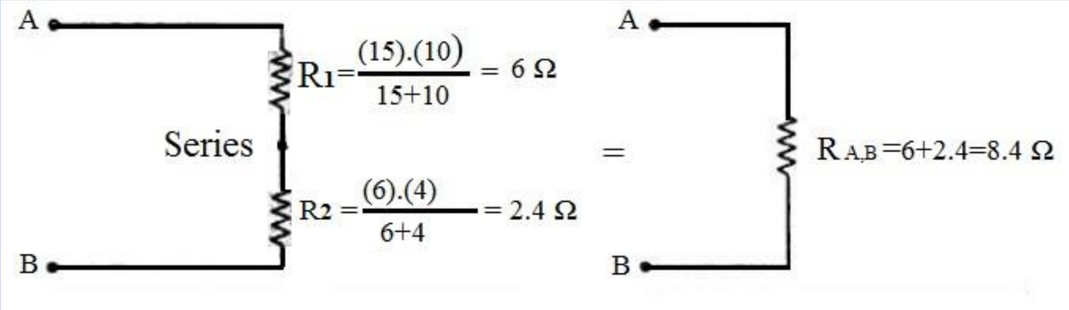


Fig.(8)

Solution:





$R_{AB} = 8.4\Omega$

POWER DISTRIBUTION IN A SERIES CIRCUIT

the power applied by the dc supply must equal that dissipated by the resistive elements.as shown in figure (2).

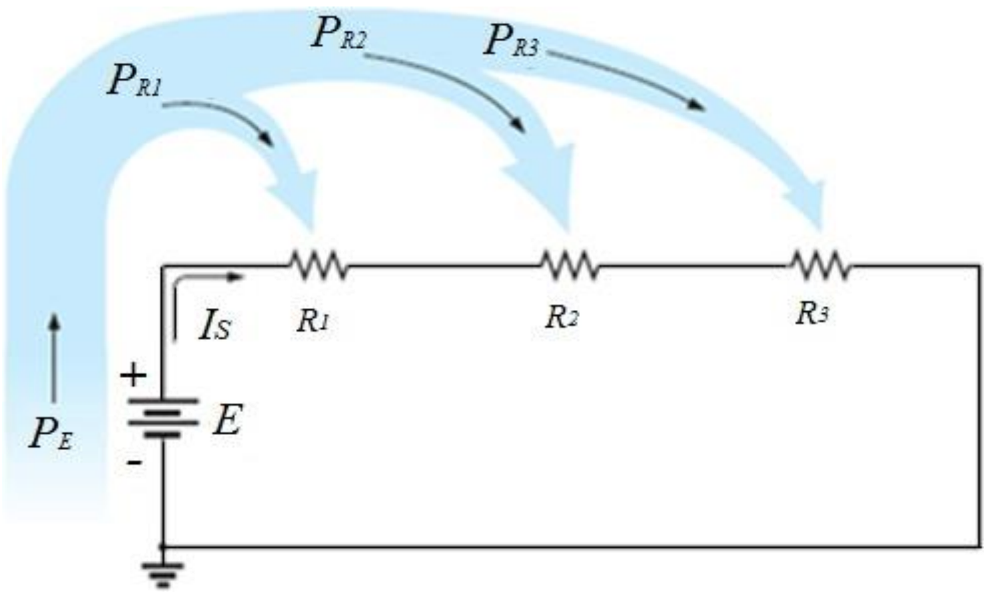


Fig.(9)

In equation form:

$$P_E = P_{R_1} + P_{R_2} + P_{R_3} \text{......} (9)$$

The power deliverd by the supply can be determine using:

$P_E = EI_S$ -- (10)

EXAMPLE 7 For the series circuit in Fig. 22 (all standard values):

a. Determine the total resistance R_T.
b. Calculate the current I_s.
c. Determine the voltage across each resistor.
d. Find the power supplied by the battery.
e. Determine the power dissipated by each resistor.
f. Comment on whether the total power supplied equals the total power dissipated.

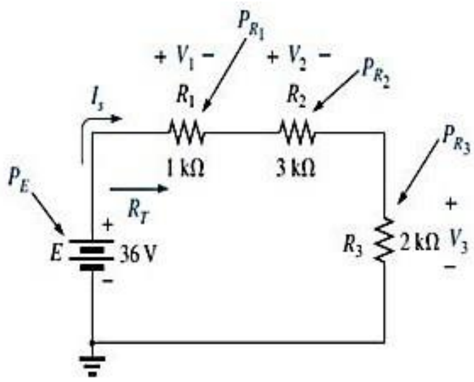


FIG. 22
Series circuit to be investigated in Example 7.

Solutions:

a. $R_T = R_1 + R_2 + R_3$
$= 1\ \text{k}\Omega + 3\ \text{k}\Omega + 2\ \text{k}\Omega$
$R_T = 6\ \text{k}\Omega$

b. $I_s = \dfrac{E}{R_T} = \dfrac{36\ \text{V}}{6\ \text{k}\Omega} = 6\ \text{mA}$

c. $V_1 = I_1R_1 = I_sR_1 = (6\ \text{mA})(1\ \text{k}\Omega) = 6\ \text{V}$
$V_2 = I_2R_2 = I_sR_2 = (6\ \text{mA})(3\ \text{k}\Omega) = 18\ \text{V}$
$V_3 = I_3R_3 = I_sR_3 = (6\ \text{mA})(2\ \text{k}\Omega) = 12\ \text{V}$

d. $P_E = EI_s = (36\ \text{V})(6\ \text{mA}) = 216\ \text{mW}$

e. $P_1 = V_1I_1 = (6\ \text{V})(6\ \text{mA}) = 36\ \text{mW}$
$P_2 = I_2^2R_2 = (6\ \text{mA})^2(3\ \text{k}\Omega) = 108\ \text{mW}$
$P_3 = \dfrac{V_3^2}{R_3} = \dfrac{(12\ \text{V})^2}{2\ \text{k}\Omega} = 72\ \text{mW}$

f. $P_E = P_{R_1} + P_{R_2} + P_{R_3}$
$216\ \text{mW} = 36\ \text{mW} + 108\ \text{mW} + 72\ \text{mW} = 216\ \text{mW}$ (checks)

Short and Open Circuits

The **short circuit** or **open circuit** in the network plays an important role to simplifying the network.

(i) Short Circuit: If any two points in a network are connected directly to each other with a conducting wire, then the two points are said to be short circuited. The resistance of such short circuit is zero.

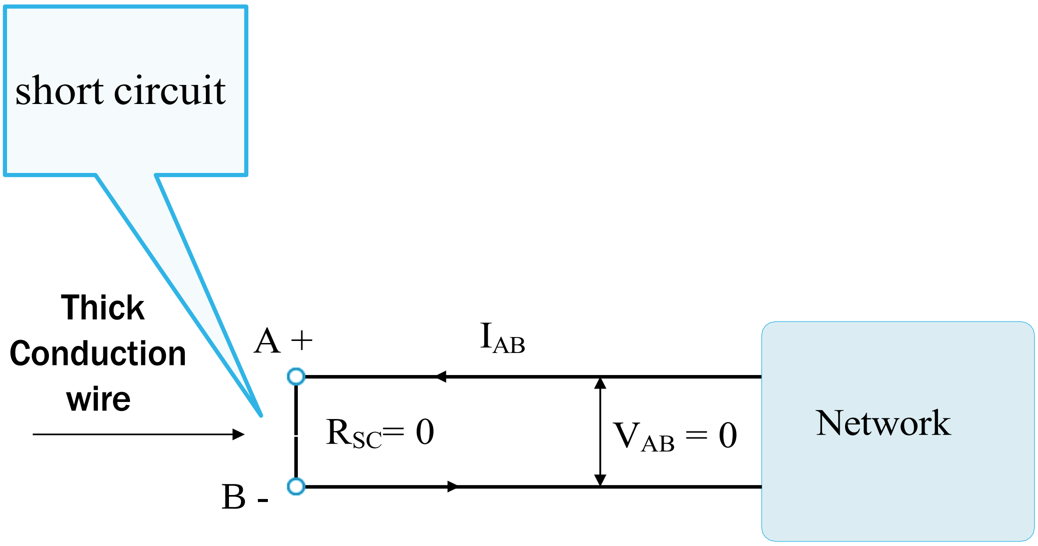


Fig.(10) Short Circuit.

$V_{AB} = R_{SC} \times I_{AB} = 0 \times I_{AB} = 0\ V$

(ii) Open Circuit: When there is no connection between the two points of a network, having some voltage across the two points then the two points are said to be open circuited.

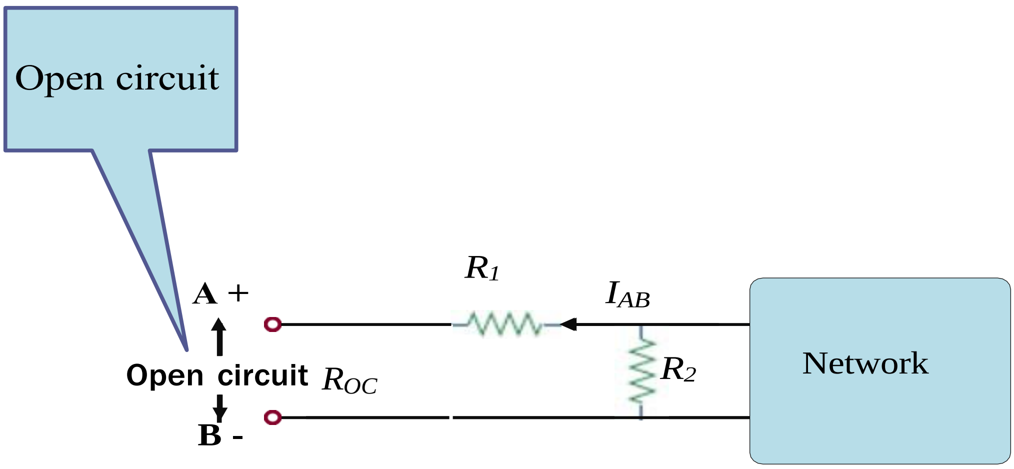


Fig.(11). Open circuit

$$I_{0C} = \frac{V_{AB}}{R_{0C}} = \frac{V_{AB}}{\infty} = 0A$$

Observe that this current through open circuit is always zero through there exist a voltage across open circuit terminals.

Redundant Branches

- If in a circuit there are branches of elements which do not carry any current then such branches are called redundant from circuit point of view.
- The redundant means plus and undesirable.
- The redundant branches can be removed and these branches do not affect the performance of the circuit.

The two important situations of redundancy which may occur in practical circuits are,

Case 1 : Any branch across which there exists a short circuit, becomes redundant as it does not carry any current. As shown in Figure (12)

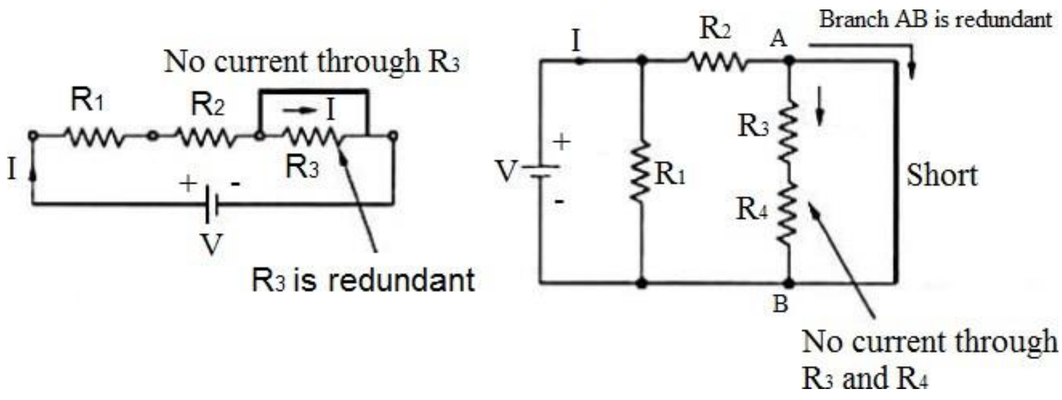
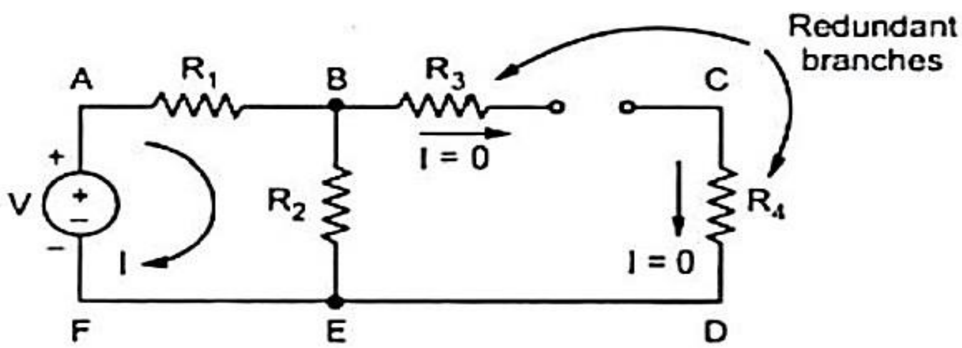


Fig.(12)

Situation 2 : If there is open circuit in a branch, it can not carry any current and becomes redundant as shown in figure (13).



Fig(13)

Voltage Division in Series Circuit Resistors

1- $R_T = R_1 + R_2$
2- $I_T = I_1 = I_2$
3- $V_T = V_1 + V_2$

V=I.R ➔ Ohms law

$V_T = I_T R_1 + I_T R_2 = I_T (R_1 + R_2)$

$$\therefore I_T = \frac{V_T}{R_1 + R_2}$$

$$V_1 = I_T R_1 = \left(\frac{V_T}{R_1 + R_2}\right) R_1$$

$$V_1 = \left(\frac{R_1}{R_1 + R_2}\right) V_T$$

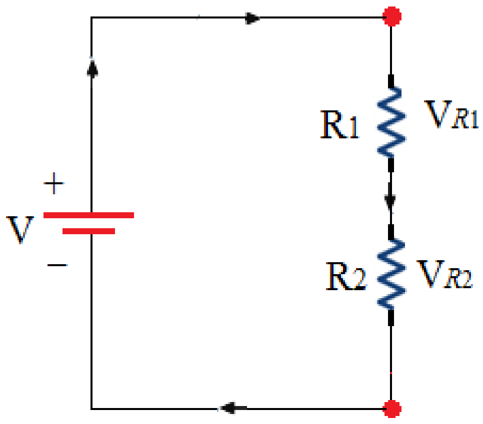
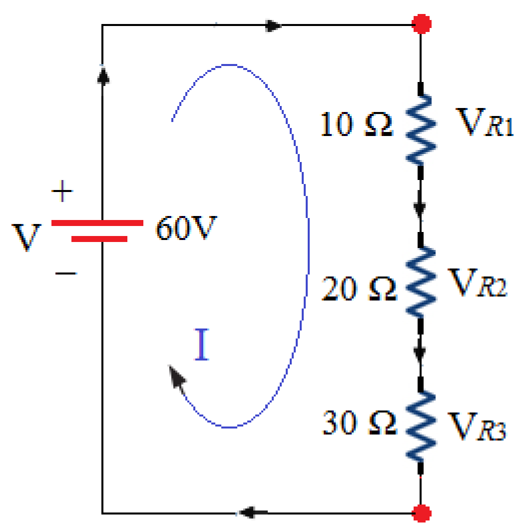


Fig.(15)

Similarily

$$V_2 = I_T R_2 = \left(\frac{V_T}{R_1 + R_2}\right) R_2 = \left(\frac{R_2}{R_1 + R_2}\right) V_T$$

Example 4: For the circuit shown in Figure (16). Find the voltage across the three resistances



Solution:

$$I = \frac{V_T}{R_1+R_2+R_3} = \frac{60}{10+20+30} = 1A$$

$$V_{R1} = I.R_1 = (1A)(10\,\Omega) = 10V$$

$$V_{R2} = I.R_2 = (1A)(20\,\Omega) = 20V$$

$$V_{R3} = I.R_3 = (1A)(30\,\Omega) = 30V$$

V.D.R

$$V_1 = I_T R_1 = \left(\frac{V_T}{R_1+R_2+R_3}\right) R_1 = (1A)(10\Omega) = 10V$$

Current Division in Parallel Circuit Resistors

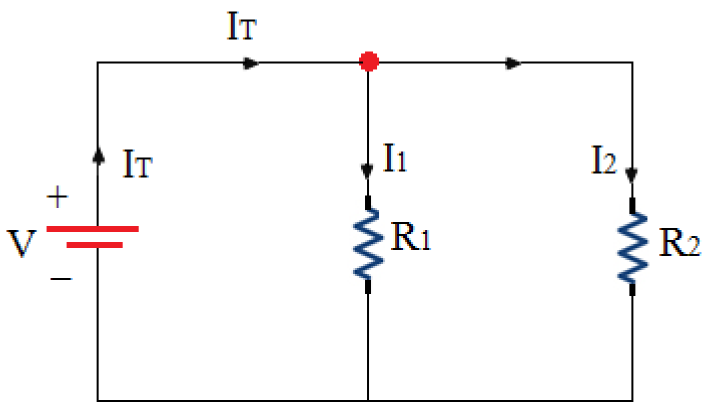


Fig.(17)

1. $V_T = V_1 = V_2$
2. $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

$$\boxed{R_{eq} = \frac{R_1 R_2}{R_1 + R_2}}$$

3. $I_T = I_1 + I_2$

$I = \frac{V}{R}$ (Ohms law)

$$\therefore I_T = \frac{V}{R_1} + \frac{V}{R_2} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right) V$$

$$I_T = \left(\frac{R_1 + R_2}{R_1 . R_2}\right) V$$

$$\therefore V = \left(\frac{R_1 . R_2}{R_1 + R_2}\right) I_T$$

$$I_1 = \frac{V}{R_1} = \left[\left(\frac{R_1 . R_2}{R_1 + R_2}\right) I_T\right] \frac{1}{R_1} = \left(\frac{R_2}{R_1 + R_2}\right) I_T \qquad \text{--------- (11)}$$

$$I_2 = \frac{V}{R_2} = \left[\left(\frac{R_1 . R_2}{R_1 + R_2}\right) I_T\right] \frac{1}{R_2} = \left(\frac{R_1}{R_1 + R_2}\right) I_T \qquad \text{- ---------(12)}$$

Example 5: Find the I_T, I_1 & I_2. If $R_1 = 10\ \Omega$, $R_2 = 20\Omega$ & $V = 50$ V

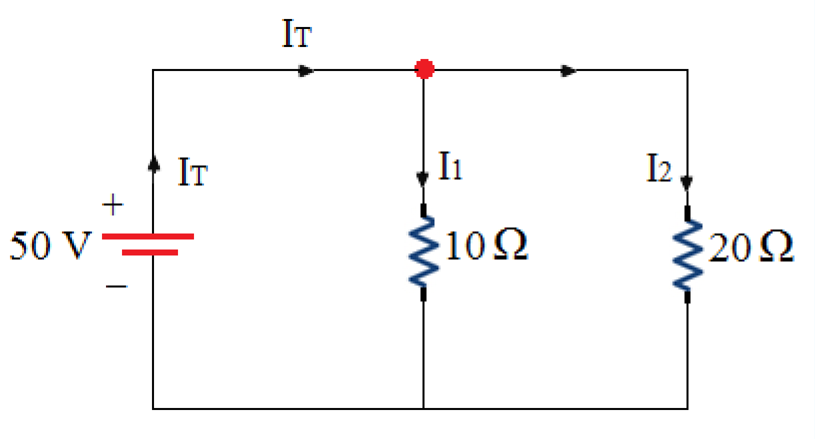


Fig.(18)

Solution:

The equivalent resistance for the circuit:

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2} = \frac{(10).(20)}{10 + 20} = 6.67\Omega$$

$$I_T = \left(\frac{V}{R_{eq}}\right) = \frac{50}{6.67} = 7.5A$$

$$I_1 = \left(\frac{R_2}{R_1 + R_2}\right) I_T = 7.5 \left(\frac{20}{10 + 20}\right) = 5A$$

$$I_2 = \left(\frac{R_1}{R_1 + R_2}\right) I_T = 7.5 \left(\frac{10}{10 + 20}\right) = 2.5A$$

$I_T = I_1 + I_2 = 5 + 2.5 = 7.5$ A