Ministry of Higher Education and Scientific Research
Al-Mustaqbal University
Medical Instrumentation Techniques Engineering Department

Subject: Digital Signal Processing

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Lecture2: Classification of Signals

Class: 3rd

Content of Lecture

- **Periodic and Aperiodic Signals**
- **Energy and Power Signals**
- **Deterministic and Random Signals**
- **Symmetric Sequence**
  - ➢ **Even and Odd Signals**
  - ➢ **Conjugate Symmetric and Conjugate Antisymmetric Signals**
  - ➢ **Decomposing a Signal**

1.1. Periodic and Aperiodic Signals

A periodic signal is a signal that repeats itself after a certain time, for some positive real integer N. Mathematically, a signal $x(n)$is periodic if there exists a period N such that:

$$x(n)=x(n+N)$$

Where

- x(n) is the signal at discrete time n.
- N is the **fundamental period** of the signal.

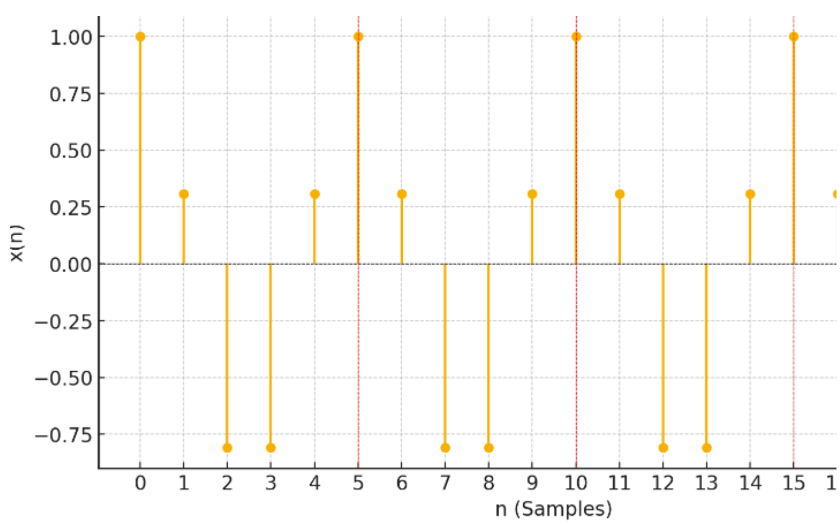

Fig.1 periodic signal

A non-periodic (or aperiodic) signal is a signal that does not repeat over time as shown in figure2.

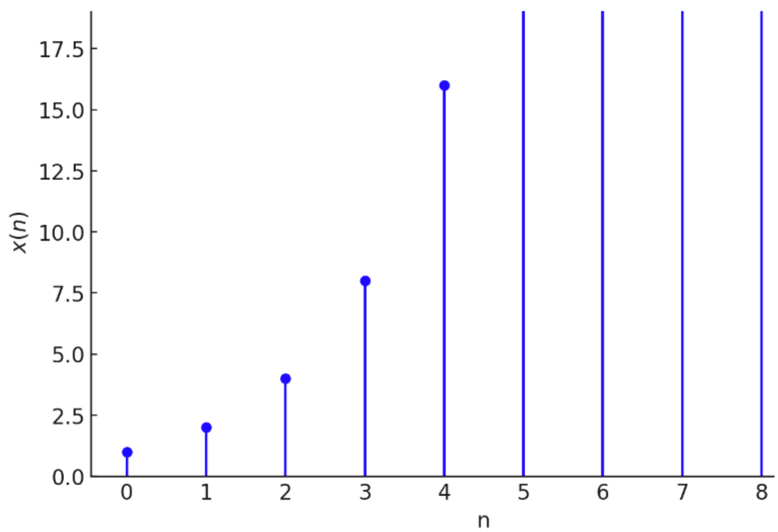

Fig.2 Aperiodic Signal

If $x1(n)$ is a sequence that is periodic with a period $N1$, and $x2(n)$ is another sequence that is periodic with a period $N2$.

The sum $x(n) = x1(n) + x2(n)$ or the product $x(n) = x1(n)x2(n)$ will always be periodic and the **fundamental period** is:

$$N = \frac{N1N2}{\gcd(N1, N2)}$$

Where gcd ($N1$, $N2$) means *the* ***greatest common divisor*** of $N1$ and, $N2$

Ex: The signals x(n)=a^n u(n)
and x2=cos(n^2)
are not periodic, whereas the signal

$$x3 = e^{jn\pi/8}$$
$$x3 = \cos(n\pi/8) + j\sin(n\pi/8)$$
$$x3 = \cos(n\pi/8 + 2\pi)$$
$$N = 16$$

Ex: Determine whether or not the signals below are periodic and, for each signal that is periodic, determine the fundamental period.

a- $x(n) = \cos(n\pi/17)\, e^{jn\pi/16}$

b- $x(n) = \mathrm{Re}[e^{jn\pi/12}] + \mathrm{Im}[e^{jn\pi/18}]$

a- $x(n)=\cos(n\pi/17)\cos(n\pi/16)+j\sin(n\pi/16)\cos(n\pi/17)$

$N1=34 \quad N2=32$

$N=\frac{32\ 34}{2}=544$

The system is periodic in N=544

b- Here we have the sum of two periodic signals,

$x(n)=\cos(n\pi/12)+\sin(n\pi/18)$

$N1=24, N2=36$

$N=\frac{24\ 36}{12}=72$

The system is periodic in N=72

H.W

1- $x(n)=\sin(\pi+0.2n)$

2- $x(n)=\cos(n\pi 0.125)$

1.2. Energy and Power Signals

In signal processing and electrical engineering, signals are classified into energy signals and power signals based on their energy and average power over time.

1. Energy Signal

An energy signal is a signal that has a finite amount of energy but zero average power. The energy of a signal is a measure of how much total "work" or "strength" is contained in the signal over time. Mathematically, the energy E of a signal x(t) is defined as:

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$E < \infty \text{ and } P=0$$

- If $0 < E < \infty$, the signal is classified as an energy signal.
- $E < \infty$ and $P=0$, The average power P of an energy signal is **zero**.

Example of Energy Signals:

Pulse signals (such as a square pulse) or any signal that exists for a short time and then disappears (e.g., a Gaussian pulse).

2. Power Signals

A power signal is a signal that has infinite energy but finite average power over time. Power signals are continuous signals that do not have a limited duration but exist indefinitely, so their energy becomes infinite over time.

Mathematically, the average power P of a signal x(t) is defined as:

$$P = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |x(t)|^2 dt$$

- If $0 < P < \infty$, the signal is classified as a power signal.
- $E=\infty$ and $0 < P < \infty$

Example of Power Signals:

Periodic signals such as **sine waves** and **cosine waves**, or any continuous signals that repeat indefinitely.

1.3. Deterministic and Random Signals

Deterministic signals can be completely represented by mathematical equations at any time, for example (triangular wave, square pulse etc.) as shown in figure3.

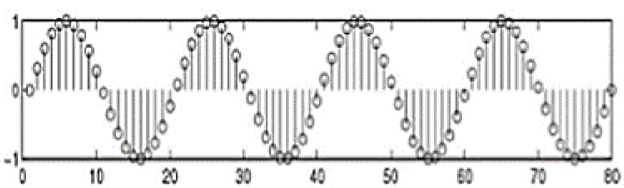

Fig.3. Deterministic Signal

Signals cannot be represented by mathematical equation is called random signals, for example (noise generated in electronic components, transmission channels, etc) as shown in figure4.

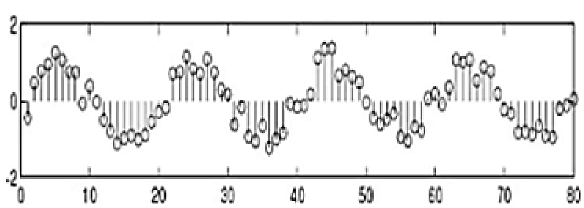

Fig.4. Random Signal

1.4. Symmetric Sequence

Any signal x(n) can be decomposed into its even part xe(n) and odd part xo(n). This decomposition is important in signal processing as it allows us to analyze the symmetry properties of signals.

1. Even and Odd Signals:

- **Even signal**: A signal x(n) is called **even** if it satisfies:

$$x(n)=x(-n)$$

This means the signal is symmetric with respect to the y-axis, meaning it looks the same when reflected around n=0

Example:

Consider the signal:

$$x(n)=\{3,2,1,0,1,2,3\}$$

For n=−3,−2,−1,0,1,2,3 the values are:

- x(−3)= x(3)=3
- x(−2)=x(2)=2
- x(−1)=x(1)=1
- x(0)=0

Since x(n)=x(−n) all n, this is an even signal.

- **Odd signal**: A signal x(n) is called **odd** if it satisfies:

$$x(n) = -x(-n)$$

An odd signal is antisymmetric, meaning it flips its sign when reflected around n=0

Example:

$x(n) = \{-3, -2, -1, 0, 1, 2, 3\}$

For $n = -3, -2, -1, 0, 1, 2, 3$ the values are:

- $x(-3) = -3$, and $x(3) = 3 \rightarrow x(3) = -x(-3)$
- $x(-2) = -2$, and $x(2) = 2 \rightarrow x(2) = -x(-2)$
- $x(-1) = -1$, and $x(1) = 1 \rightarrow x(1) = -x(-1)$
- $x(0) = 0$, which remains the same, as $x(0) = 0 = -x(0)$. Since this signal satisfies $x(n) = -x(-n)$ for all values of n, it is an odd signal.

2. Conjugate Symmetric and Conjugate Antisymmetric Signals:

- **Conjugate symmetric**: A signal is called conjugate symmetric if it satisfies:

x(n) = x*(-n)

where x*(n) represents the complex conjugate of x(n).

Example:

$$x(n) = \{1 + j, 2 - j, 3, 2 + j, 1 - j\}$$

For $n = -2, -1, 0, 1, 2$, we have the following signal values:

- $x(-2) = 1 - j$
- $x(-1) = 2 + j$
- $x(0) = 3$
- $x(1) = 2 - j$
- $x(2) = 1 + j$

The **complex conjugate** of the signal would be:

- $x^*(-2) = 1 + j$
- $x^*(-1) = 2 - j$
- $x^*(0) = 3$
- $x^*(1) = 2 + j$
- $x^*(2) = 1 - j$

We see that:

- $x(2) = x^*(-2) = 1 + j$
- $x(1) = x^*(-1) = 2 - j$
- $x(0) = x^*(0) = 3$ (no change since it's real)
- $x(-1) = x^*(1) = 2 + j$
- $x(-2) = x^*(2) = 1 - j$

Thus, this signal is **conjugate symmetric** because $x(n) = x^*(-n)$ for all n.

- **Conjugate antisymmetric**: A signal is conjugate antisymmetric if it satisfies:

x(n) = -x*(-n)

This involves a sign change and a conjugate reflection.

Example:

Consider the complex-valued signal:

$$x(n) = \{3j, 1-j, 0, -1+j, -3j\}$$

For $n = -2, -1, 0, 1, 2$, the signal values are:

- $x(-2) = -3j$
- $x(-1) = -1 + j$
- $x(0) = 0$
- $x(1) = 1 - j$
- $x(2) = 3j$

Complex Conjugate of $x(n)$:

The complex conjugate $x^*(n)$ flips the sign of the imaginary part:

- $x^*(-2) = 3j$
- $x^*(-1) = 1 - j$
- $x^*(0) = 0$ (since it's real, no change)
- $x^*(1) = -1 + j$
- $x^*(2) = -3j$

let's verify the condition $x(n)=-x^*(-n)$:

- $x(2)=3j$, and $-x^*(-2)=-3j$ ⟶ $x(2)=-x^*(-2)$
- $x(1)=1-j$, and $-x^*(-1)=-(1-j)=-1+j$ ⟶ $x(1)=-x*(-1)$
- $x(0)=0$, and $-x^*(0)=0$ ⟶ $x(0)=-x^*(0)$
- $x(-1)=-1+j$, and $-x^*(1) = -(1 - j) = -1 + j$ ⟶ $x(-1)=-x^*(1)$
- $x(-2)=-3j$, and $-x^*(2)=-(-3j)=3j$ ⟶ $x(-2)=-x^*(2)$

3. Decomposing a Signal:

Any signal x(n) can be decomposed into a **sum of its even part** xe(n) and **odd part** xo(n). The relationship is given by:

$$x(n)=xe(n)+xo(n)$$

Where:

- xe(n) is the even part of the signal.
- xo(n) is the odd part of the signal.

- **Even Part xe(n)**:

$$x_e(n) = \frac{1}{2}[x(n) + x(-n)]$$

- **Odd Part xo(n)**:

$$x_o(n) = \frac{1}{2}[x(n) - x(-n)]$$

- **The conjugate symmetric part of $x(n)$ is**

$$\text{xe}=\frac{1}{2}\,[\text{x}(\text{n}) + x*(-n)\,]$$

- **The conjugate antisymmetric part of $x(n)$is**

$$\text{xo}=\frac{1}{2}\,[\text{x}(\text{n}) - x*(-n)\,]$$