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Methods of X- Ray Interaction with Matter

Lecture 4

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1-Introduction

This chapter deals with the physics of events that occur when photons and electrons interact with matter. these are the radiations that are important for diagnostic radiology, and only those interactions that result in their attenuation, absorption and scattering are dealt with. other interactions, such as those with nuclei, are not considered here because they only occur for radiation that is higher in energy than that used for diagnostic radiology. X rays of energy of a few tens of kiloelectronvolts or so have a wavelength of a few nanometres. since this is also in the general range of atomic dimensions, one would expect interactions to take place between electromagnetic radiation and atoms — and this is indeed the case. electron dimensions (the 'classical radius of the electron' is 2.8 pm) correspond to the higher end of the diagnostic. X ray energy range and one would expect this to be the general region where interactions take place between electromagnetic radiation and the electrons that are constituents of atoms. this is also the case. The energy range used for diagnostic radiology is generally on the boundary between classical and quantum physics and, following the 'complementarity principle', the numerical details of the interactions will be treated by classical reasoning where appropriate and by quantum mechanical considerations where this gives superior results. The behaviour of photons and electrons as they traverse matter is very different. Photons in general have zero, one or a few interactions and are exponentially attenuated. Direct computation of the combined effects of several interactions is difficult, and Monte carlo techniques are usually used to study photon transport through bulk media. Photon interactions are expressed in terms of cross-sections for individual interactions and attenuation coefficients for passage through bulk media. electrons experience large numbers of interactions and in general gradually lose energy until they are stopped. this is expressed in terms of electron range and material stopping powers.

2. **Interactions of photons with matter**

The interactions of radiations such as photons and electrons are stochastic and obey the laws of chance. for photon radiation, the concept of cross-section, with its relation to probability, follows directly. this can be explained rather simply by considering a single photon to be incident on a slab of material of area A that contains one target of cross-sectional area σ. the probability of the photon interacting with the target will be the ratio of the two areas: σ/A. Next, let us say that there are Φ photons and that they are randomly directed at area A, and further, that area A contains n targets, each with area σ. it is easy to see that the expected number of interactions ΔΦ between photons and targets will be:

$$\Delta \Phi = \Phi\, n\, (\sigma/A) \quad \text{(2-1)}$$

another way of stating this is that the probability of a projectile making a hit is $n(\sigma/A)$, which is just the fraction of the area that is blocked off by the targets. now suppose that we change the geometrical description a little and we let the targets be atoms. their cross-section would be an atomic cross-section. This would not be an actual area of the atom but would be an effective area — effective for an interaction between the photon and the atom that is being considered. cross-sections are frequently represented by the symbol σ and conventionally expressed in a unit of area called the barn (1 barn = 10–28 m^2. this unit is commonly used in nuclear and radiation physics. It is not SI but is, somewhat reluctantly, accepted by that body).. There are four fundamental X ray interactions that we need to consider; each can be associated with a specific cross-section. it is useful to use different symbols to represent them: t is used to signify the cross-section for a photon to interact with an atom by the photoelectric effect, σ_{coh} is used to represent the cross-section for interaction by coherent scattering, σ_{incoh} for incoherent scattering and κ for pair and triplet production. the first three of these interactions are important in the diagnostic energy range up to 150 keV, whereas pair and triplet production are only important at much higher energies and are only treated here for completeness.

2.1 Photoelectric effect

In the photoelectric effect, the incident photon interacts with an atom, which is left in an excited state. the excess energy is released by the ejection of one of the electrons bound to the nucleus. this electron, called a photoelectron, leaves the atom with kinetic energy:

$T = h\nu - E_s$ ………………………………………..(2-2)

where

E_s is the binding energy of the electron shell from which the electron came;

h is Planck's constant;

and ν is the photon frequency.

The energy transferred to the recoiling atom is very small and can be neglected. the photoelectric effect can only take place if the photon energy, hν, exceeds the binding energy of the electron in that shell. the most probable electron shell to lose an electron is the one that satisfies this constraint and also has the highest binding energy. although this seems like a rather simple process, calculation of the probability of the interaction is very complicated and quantum mechanics is required. this is because it involves the wave function of the whole atomic electron cloud and these functions are available only for relatively simple atoms. in the diagnostic energy range up to 150 keV, the photoelectric effect cross-section per atom, τ, is given approximately by:

$$\tau(h\nu, Z) = k\frac{Z^n}{(h\nu)^m}$$ ……………………………..(2-3)

where

k is a constant;

Z is the atomic number;

n is an exponent in the range 3.6–5.3, being largest for low atomic numbers;

and m is an exponent in the range 2.5–3.5, again being largest for low atomic numbers.

$$\tau \sim \frac{Z^4}{(h\nu)^3} \qquad \text{……………………………………(2-4)}$$

This expression indicates a very strong dependence on atomic number as well as a strong inverse dependence on photon energy. figure 2.1 shows atomic cross-section data for the photoelectric process for photons irradiating tungsten, molybdenum and copper. they are plotted against photon energy on a log–log scale and cover the energy range from 1 keV to 300 keV. the sharp discontinuities correspond to the positions of the absorption edges for the different materials, which increase in energy with increasing atomic number and shell binding energy. for example, for tungsten, the discontinuity seen at 69.5 keV represents the effect of the k shell. at an energy just less than this, the cross-section is 6.4×10^2 barn/atom, while just above this energy, the coefficient is 3.3×10^3 barn/atom. this represents a sudden increase in cross-section of about a factor of five when the photon energy increases above that of the k shell (k edge). thus, the major contribution to the cross-section above the k edge comes from interactions with the two k shell electrons. The discontinuities in the tungsten cross-section at energies just greater than 10 keV represent the effect of the l shell, which is more complicated because it comprises three subshells. the effect of the M shell shows up at about 2.5 keV with an even more complex structure. for copper and molybdenum, the k absorption edges are at 8.98 and 20.00 keV, respectively

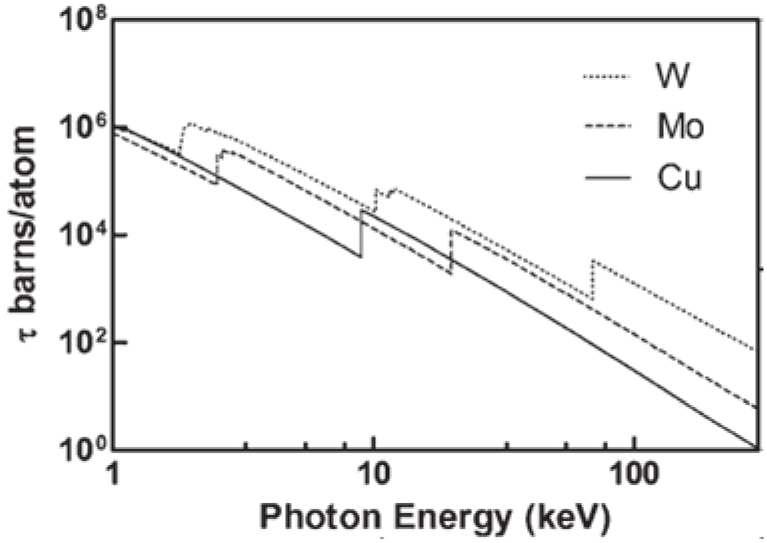

Figure (2.1): Atomic photoelectric cross-sections for copper (Cu), molybdenum (Mo) and tungsten (W).

The incident photon disappears in the photoelectric interaction. after the interaction, a vacancy is left in the atomic shell structure and this is filled by an electron from a higher shell, with the resulting energy difference being carried off either by a characteristic X ray (also known as a fluorescent X ray) or by another electron from a higher shell, known as an auger electron. after the initial vacancy is filled, the new vacancy or vacancies will themselves be filled and this process will continue with a cascade of events that may finally leave the atom in a highly ionized state.

2.2 Thomson scattering

J.J. thomson gave the first treatment of the scattering of photons by electrons in the very early years of the 20th century. it was an early attempt to investigate the way the waves described by Maxwell’s equations would be expected to interact with the newly discovered electron. his derivation is of historical interest, in that it is based on classical physics and results in a description of photon scattering that is only meaningful at the low energy limit of this interaction.

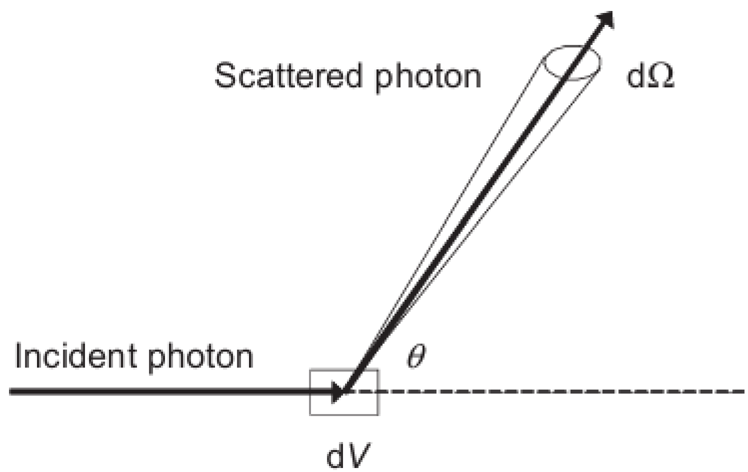

Figure (2.2): Scattering angles and solid angle. A photon incident on a small volume element dV is scattered though angle θ into the solid angle element dΩ

We give thomson's results here as a first step towards the treatment of the coherent and incoherent cross-sections for scattering from atoms. first we need to introduce the concept of the differential cross-section. Whereas the total cross-section is related to the probability that the photon will interact, the differential cross-section, $d\sigma/d\Omega$, is related to the probability that the photon will interact and be scattered into solid angle $d\Omega$ (Fig. 2.2). This probability is proportional to:

$$\frac{d\sigma}{d\Omega}d\Omega \qquad \text{……………………………………… (2-5)}$$

and the total cross-section is obtained by integrating over all solid angles:

$$\sigma = \int \frac{d\sigma}{d\Omega}d\Omega \qquad \text{…………………………………………… (2-6)}$$

In diagnostic radiology, the shape of the differential cross-section has an important influence on the amount of scattered radiation recorded by the image receptor.

For scattering of a photon by a single free electron, thomson showed that

the differential cross-section, at scattering angle q, is given by the rather simple

expression:

$$\frac{d\sigma_{Th}}{d\Omega} = \frac{r_0^2}{2}\left(1 + \cos^2\theta\right) \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2\text{-}7)$$

in this expression, r_0 is the 'classical radius of the electron' and is given by:

$$r_0 = \frac{k e^2}{m_0 c^2} = 2.81794 \times 10^{-15}\,\mathrm{m} \qquad \ldots\ldots\ldots\ldots\ldots\ldots.(2\text{-}8)$$

where

k is a proportionality constant from coulomb's law;

e is the charge on the electron;

m_0 is the rest mass of the electron;

and c is the speed of light.

It can be seen that eq. (2.7) predicts that the same amount of energy will be scattered forward as backward, and also that the energy scattered at right angles will be half this amount. except at low energies, this result does not agree with observation or with the predictions made by quantum mechanics.

equation (2.7) describes the probability of scattering radiation through a unit solid angle centred on scattering angle θ. an alternative way of expressing

The differential scattering cross-section involves describing the probability of scattering into a solid angle described by an annular ring of angular width dθ centred on angle q. for unpolarized incident photons, there is no dependence of the scattering probability on the azimuthal scattering angle, and we can then use the relationship between the two solid angles θ:

$$d\Omega = 2\pi \sin\theta \; d\theta \qquad \text{………………………(2-9)}$$

the total cross-section for thomson scattering is thus obtained by using Eq. (2.9) to convert dΩ to dθ and by integrating the differential scattering cross-section (eq. (2.7)) over all scattering angles from 0 to π. the integration is very simple and it gives:

$$\sigma_{\mathrm{Th}} = \frac{r_0^2}{2} \int_0^{\pi} 2\pi \left(1 + \cos^2\theta\right) \sin\theta d\theta = \frac{8\pi r_0^2}{3} = 66.52 \times 10^{-30}\ \mathrm{m}^2 \qquad \text{…………….. (2-10)}$$

which is constant, predicting that the classical scattering probability is independent of electromagnetic radiation energy. this, of course, is not correct, but it will be seen in section 2.2.4 that the integrand in eq. (2.10) is the first term of the much more accurate result obtained by using quantum mechanics (for the compton effect). in other words, the result obtained by classical physics is the result given by quantum mechanics when the photon energy approaches zero.

2.3 Coherent (Rayleigh) scattering

in deriving the expression for thomson scattering, it was assumed that the electron was free, alone and at rest. in reality, the photon is scattered collectively by the atomic electrons, which are not free, and their proximity to one another is not very different from the wavelength of the radiation. in coherent scattering, essentially no energy is lost by the photon as it transfers momentum to the atom (strictly speaking, the condition of no change in photon energy applies to the inertial frame in which the total momentum of atom plus photon is zero) and is scattered through angle θ. the scattering by the different electrons is in phase and the resultant angular distribution is determined by an interference pattern that is characteristic of the atom. the differential cross-section is then given by:

$$\frac{d\sigma_{coh}}{d\Omega} = \frac{d\sigma_{Th}}{d\Omega} F^2(x, Z) \qquad \text{……………………..(2-11)}$$

where

$d\sigma_{Th}/d\Omega$ is the thomson differential scattering coefficient from eq. (2.7) and the quantity F is known as the coherent form factor. it may be calculated using quantum mechanical models and is a function of the atomic number of the atom, Z, and the parameter *x*, which is given by:

$$x = \frac{\sin(\theta/2)}{\lambda} \qquad \text{……………………………….(2-12)}$$

where λ is the wavelength of the incident photon. the parameter x is proportional to the transfer of momentum between the initial and scattered photon directions. for scattering in the forward direction, all the atomic electrons act together, and F is equal to the atomic number and the differential cross-section depends upon Z^2. as the scattering angle increases, *F* decreases because it becomes increasingly difficult for all the electrons to scatter in phase without any energy transfer. however, for a given value of the scattering angle, the normalized coherent form factor, F/Z, increases with increasing atomic number. figure 2.3 shows the normalized form factor for three different elements

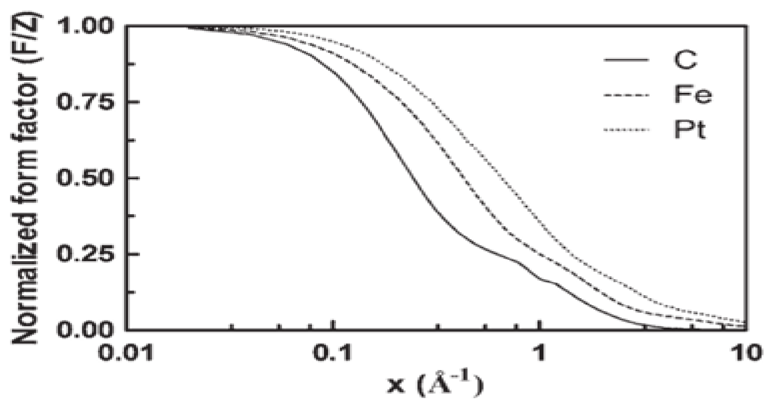

Figure (2.3): Variation of the normalized form factor F/Z for coherent scattering with the

momentum transfer parameter x. Values are shown for carbon (C), iron (Fe) and platinum (Pt)

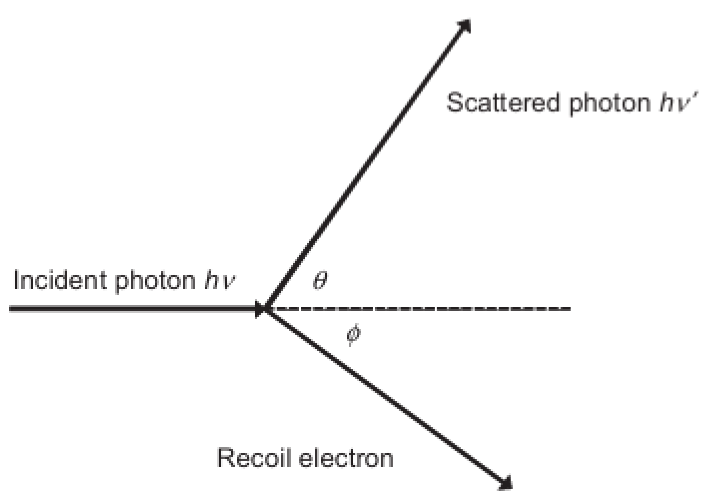

Figure (2.4): Geometry for Compton scattering.

2.4. Compton scattering by free electrons

compton scattering, like thomson scattering, is the interaction between electromagnetic radiation and a free electron, but in this case there is an energy transfer to the electron. We consider this case before treating incoherent scattering by an atom. the energy range is such that relativity and quantum mechanics must be used to derive expressions for the cross-section. both the photon and the electron must be considered as particles. the geometrical arrangement is shown in fig. 2.4, where the photon is coming in from the left with energy hν and momentum *hν/c*. it is a billiard-ball-like collision with the electron and is scattered through angle θ, with energy *hν′* and momentum *hν′/c*. the electron recoils at angle ϕ with kinetic energy T_e and momentum p_e.

using conservation of energy and momentum, we can derive several useful relations, such as the ratio of the scattered photon energy, *hν′,* to the incident photon energy, *hν:*

$$\frac{h\nu'}{h\nu}=\frac{1}{1+\alpha(1-\cos\theta)}$$ ……………………….. (2-13)

where α is the dimensionless *ratio* $h\nu/m_0 c^2$. the relationship between the scattered photon angle and the scattered electron angle is:

$$\cot\phi=(1+\alpha)\tan\left(\frac{\theta}{2}\right)$$ ……………………………….(2-14)

and the scattered electron has kinetic energy given by:

$$T_e = h\nu - h\nu' = \frac{\alpha(1-\cos\theta)h\nu}{1+\alpha(1-\cos\theta)}$$ …………………………(2-15)

these are the compton relations. they describe the kinematics of the interaction but say nothing about the probability of interaction, or the cross-section. in the diagnostic energy range, the parameter α is small and, as a consequence, the energy transfer to the recoil electron is also small, being zero in the forward direction and taking its largest value when the photon is backscattered. this is demonstrated in fig. 2.5, which shows the relationship between the incident and scattered photon energies. for 20 keV, 50 keV and 100 keV incident photons, the maximum energy transfers to the recoil electron are 1.5 keV, 8.2 keV and 28.1 keV, respectively. the cross-section for the scattering of a photon, with energy hn through a given angle *θ*, was first derived in 1928 by klein and nishina using the Dirac theory of the electron. klein and nishina obtained the following expression for the differential cross-section for scattering of photons by a single free electron:

$$\frac{d\sigma_{KN}}{d\Omega} = \frac{r_0^2}{2}\left(1+\cos^2\theta\right) f_{KN}$$ ……………………………(2-16)

Where

$$f_{KN} = \left\{\frac{1}{1+\alpha(1-\cos\theta)}\right\}^2 \left\{1 + \frac{\alpha^2(1-\cos\theta)^2}{[1+\alpha(1-\cos\theta)][1+\cos^2\theta]}\right\}$$ …………..(2-17)

this cross-section reduces to the thomson cross-section when α → 0 (that is, *hv′/hv* →1).

figure 2.6 shows the differential scattering cross-section plotted as a function of the photon scattering angle plotted in two ways. the lower curve is a graph of the differential coefficient per steradian and the upper curve is a graph of the differential coefficient per unit scattering angle. the differential scattering cross-section $d\sigma/d\theta$ is zero in the forward direction because sin θ is zero (see eq. (2.9))

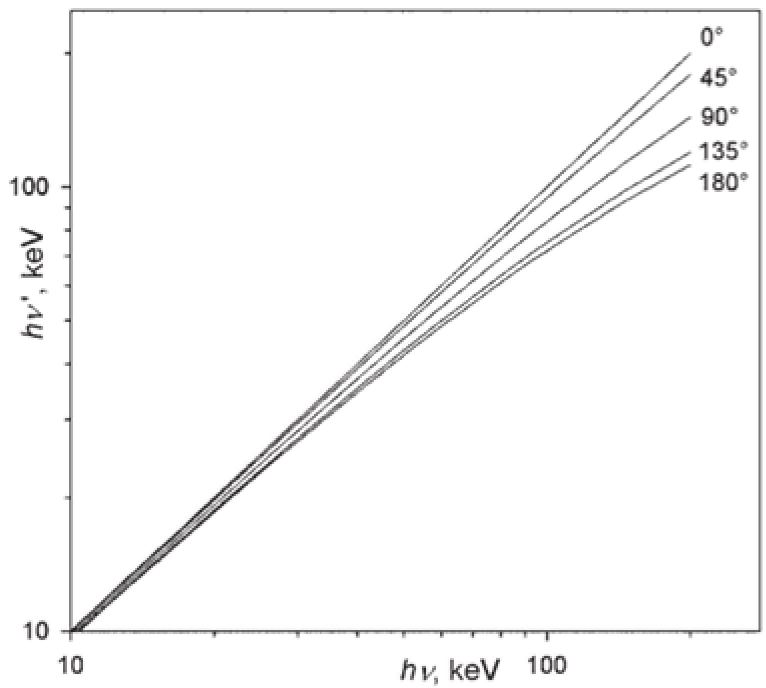

Figure (2.5): Secondary photon energy $h\upsilon'$ versus primary photon energy $h\upsilon$ for Compton interactions and various scattering angles

the total compton cross-section (probability of interaction per electron) for a photon of energy hν, is obtained by integrating eq. (2.16) using eq. (2.9) and the angular range 0 to π for θ. the result is:

$$\sigma_{KN}(h\nu)=2\pi r_0^2\left\{\left(\frac{1+\alpha}{\alpha^2}\right)\left(\frac{2(1+\alpha)}{1+2\alpha}-\frac{\ln(1+2\alpha)}{\alpha}\right)+\frac{\ln(1+2\alpha)}{2\alpha}-\frac{1+3\alpha}{(1+2\alpha)^2}\right\} \quad \text{……………….(2-18)}$$

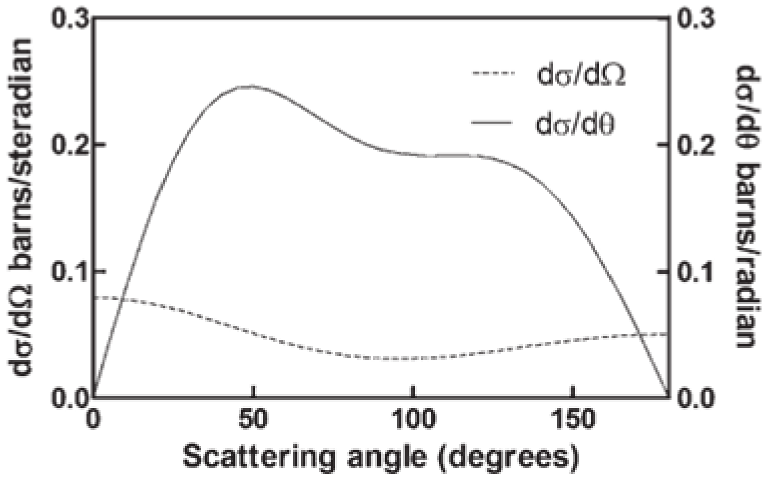

Figure (2.6). Compton differential cross-sections for scattering of 70 keV photons.

2.5. Scattering and energy transfer coefficients

in the incoherent free electron scattering process, the initial photon energy is divided between the scattered photon and the recoiling electron. a differential energy transfer coefficient can be obtained by using the equation

$$\frac{d\sigma_{tr}}{d\Omega}=\frac{r_0^2}{2}\left(1+\cos^2\theta\right)f_{KN}\left(\frac{\alpha(1-\cos\theta)}{1+\alpha(1-\cos\theta)}\right) \qquad \text{...............(2-19)}$$

this can be integrated over all angles to give $\boldsymbol{\sigma}_{tr}$, the energy transfer coefficient. the scattering coefficient is then, by definition, the difference between the total cross-section for compton scattering and the energy transfer coefficient:

$$\sigma_s=\sigma_{KN}-\sigma_{tr} \qquad \text{........................(2-20)}$$

2.6. Incoherent scattering

for the compton effect, as with thomson scattering, it is assumed that the electron is free and at rest. for incoherent scattering by bound atomic electrons, the contributions from individual electrons are added and the differential cross-section takes the form:

$$\frac{d\sigma_{incoh}}{d\Omega} = \frac{r_0^2}{2}\left(1+\cos^2\theta\right) f_{KN} S(x,Z) \qquad \text{……………………..(2-21)}$$

the function S is known as the incoherent scattering function and, as with the coherent form factor, is a universal function of the momentum transfer quantity x and the atomic number. the value for S is zero in the forward direction and increases with increasing momentum transfer, reaching the value of Z, the number of electrons per atom. this increase becomes slower as the atomic number increases. this is illustrated in fig. 2.7, which shows the normalized incoherent scatter function (*S/Z*) for three elements:

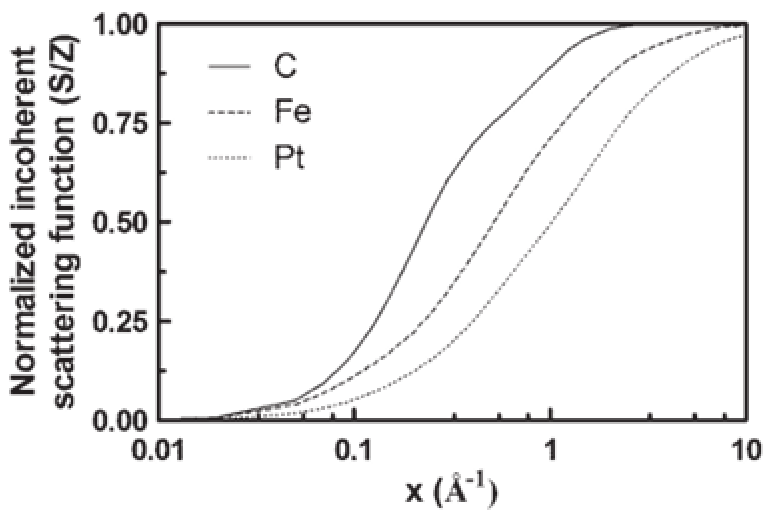

Figure (2.7): Variation of the normalized incoherent scattering function S/Z with the momentum transfer parameter x. Data are given for carbon (C), iron (Fe) and platinum (Pt).

the total cross-section for incoherent scattering is obtained by numerical integration of eq. (2.21). in many situations, it is very nearly equal to the single electron cross-section multiplied by the number of electrons in the atom

$$\sigma_{\text{incoh}} \approx Z\sigma_{\text{KN}} \qquad \text{……………………..(2-22)}$$

2.7. Pair and triplet production

When a high energy photon passes near to an atomic nucleus, the photon may interact with the nuclear coulomb field by a process called pair production. the photon is converted into an electron–positron pair, each with its own kinetic energy. the energy balance is:

$$h\nu = T_{+} + T_{-} + 2m_0c^2 \qquad \text{……………………………(2-23)}$$

On condition that the photon energy exceeds an energy threshold for the interaction of 2 m_0c^2 (1022 keV). Pair production cannot take place for photons with energies less than this. as pair production occurs in the field of the nucleus, the cross-section for this interaction varies approximately as Z^2, where Z is the nuclear charge. the process can also take place in the field of an electron. it is then called triplet production because the target electron is itself ejected with considerable energy. two electrons and one positron are thus set in motion. the energy threshold for triplet production is $4m_0$ c_2. thresholds for pair and triplet production are much higher than the photon energies relevant to diagnostic radiology.


Thank you