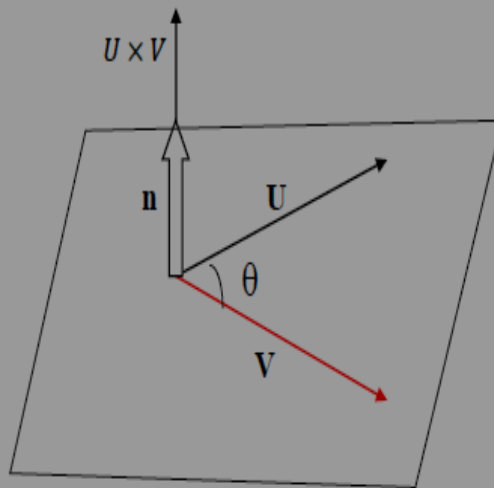


2.2 Vector/cross product

If U and V are not parallel, they determine a plane. A unit vector (n) is perpendicular to the plane by (right hand rule).

This means that we choose (n) to be unit vector. The cross product ($U \times V$) means that ($U \times V$).



Properties of cross product

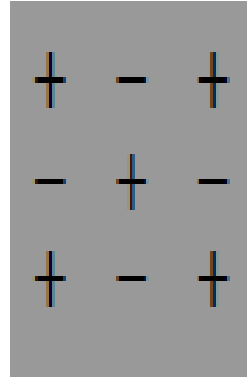
1. $U \times V = -V \times U$
2. $U \times (V + W) = U \times V + U \times W$
3. $0 \times U = 0$

Calculate cross products, by using Determinates.

$$\text{If } U = u_1 i + u_2 j + u_3 k$$

$$V = v_1 i + v_2 j + v_3 k$$

$$U \times V = \begin{bmatrix} i & j & k \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{bmatrix}$$



$$\text{Area of plane} = |U \times V|$$

Example 1: Find $U \times V$. IF $U = i + j + k$. $V = -2i + 2j + k$ and find area of plane.

Solution //

$$U \times V =$$

$$\begin{bmatrix} i & j & k \\ 1 & 1 & 1 \\ -2 & 2 & 1 \end{bmatrix}$$

$$= i \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix} - j \begin{bmatrix} 1 & 1 \\ -2 & 1 \end{bmatrix} + k \begin{bmatrix} 1 & 1 \\ -2 & 2 \end{bmatrix} = -i - 3j + 4k$$

$$\text{Area of plane} = |U \times V|$$

$$= \sqrt{(-1)^2 + (-3)^2 + (4)^2} = \sqrt{26}$$

Example 2: Find unit vector perpendicular to the plane
.P(1. -1. 2). Q(2. 0. -1). R(0. 2. 1)and find area of plane.

Solution //

$$\mathbf{PQ} = (2-1)\mathbf{i} + (0-(-1))\mathbf{j} + (-1-2)\mathbf{k} = \mathbf{i} + \mathbf{j} - 3\mathbf{k}$$

$$\mathbf{PR} = (0-1)\mathbf{i} + (2-(-1))\mathbf{j} + (1-2)\mathbf{k} = -\mathbf{i} + 3\mathbf{j} - \mathbf{k}$$

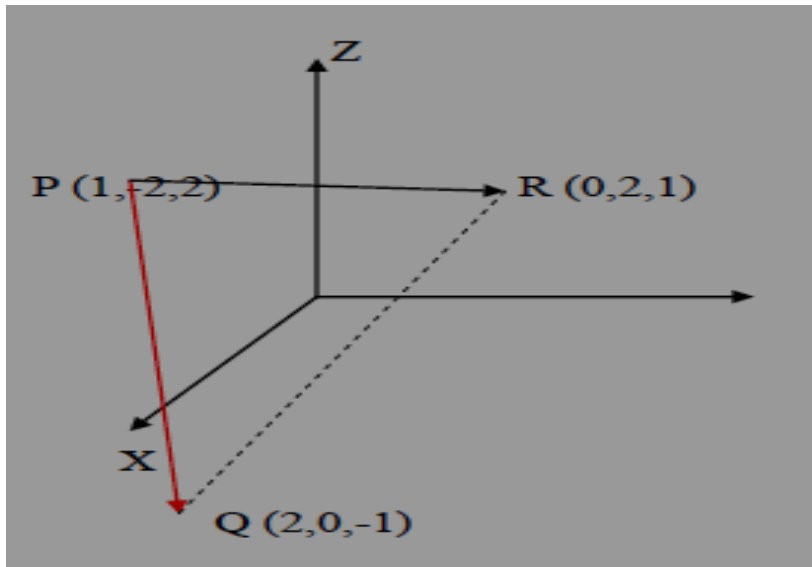
$$\mathbf{PQ} \times \mathbf{PR} = \begin{bmatrix} i & j & k \\ 1 & 1 & -3 \\ -1 & 3 & -1 \end{bmatrix}$$

$$\begin{aligned} \mathbf{PQ} \times \mathbf{PR} &= i \begin{bmatrix} 1 & -3 \\ 3 & -1 \end{bmatrix} - j \begin{bmatrix} 1 & -3 \\ -1 & -1 \end{bmatrix} + k \begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix} \\ &= 8\mathbf{i} + 4\mathbf{j} + 4\mathbf{k} \end{aligned}$$

$$|\mathbf{PQ} \times \mathbf{PR}| = \sqrt{(8)^2 + (4)^2 + (4)^2} = \sqrt{96}$$

$$\text{Unit vector} = \frac{\mathbf{PQ} \times \mathbf{PR}}{|\mathbf{PQ} \times \mathbf{PR}|} = \frac{8\mathbf{i} + 4\mathbf{j} + 4\mathbf{k}}{\sqrt{96}}$$

$$\text{Area of plane} = |\mathbf{PQ} \times \mathbf{PR}| = \sqrt{96}$$



Exercise: Find unit vector perpendicular to the plane $P(1,1,1), Q(2,1,3), R(0,2,1)$ and find area of plane