

Solution:

$$\text{Discharge (} Q_1 \text{) in pipe 1} = A_1 V_1 = \frac{\pi (0.3)^2}{4} \times 2.5 = 0.1767 \text{ m}^3 / \text{s}$$

$$Q_2 = A_2 V_2 = \frac{\pi (0.2)^2}{4} \times 2 = 0.0628 \text{ m}^3 / \text{s}$$

$$Q_1 = Q_2 + Q_3$$

$$Q_3 = Q_1 - Q_2 = 0.1767 - 0.0628 = 0.1139 \text{ m}^3 / \text{s}$$

$$Q_3 = A_3 V_3$$

$$V_3 = \frac{Q_3}{A_3} = \frac{0.1139}{\frac{\pi (0.15)^2}{4}} = 6.44 \text{ m / s}$$

Problem 4.3 /

A 25 cm diameter pipe carries oil of sp.gr. 0.9 at a velocity of 3 m/s . At another section the diameter is 20 cm. Find the velocity at this section and also mass rate of flow of oil.

Solution:

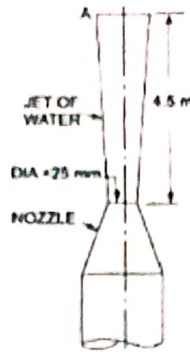
$$Q = A_1 V_1 = A_2 V_2$$

$$V_2 = \frac{A_1 V_1}{A_2} = \frac{\frac{\pi}{4} \times (0.25)^2 \times 3}{\frac{\pi}{4} (0.2)^2} = 4.68 \text{ m / s}$$

$$\begin{aligned} \text{Mass flow rate of oil (} \dot{m} \text{)} &= \rho A_1 V_1 = 0.9 \times 1000 \times \frac{\pi}{4} \times (0.25)^2 \times 3 \\ &= 132.23 \text{ kg / s} \end{aligned}$$

Problem 4.4/

A jet of water from a 25 mm diameter nozzle is directed vertically upwards. Assuming that the jet remains circular and neglecting any loss of energy, that will be the diameter at a point 4.5 m above the nozzle, if the velocity with which the jet leaves the nozzle is 12 m/s.



Solution:

Initial velocity(V_1) = 12 m /s

Final velocity (V_2)

But, $(\Delta V)^2 = 2 g h$ (opposite free fall equation)

$$V_2^2 - V_1^2 = 2 g h$$

$$V_2^2 - 12^2 = 2 (- 9.81) \times 4.5$$

$$V_2 = 7.46 \text{ m /s}$$

$$Q = A_1 V_1 = A_2 V_2$$

$$A_2 = \frac{A_1 V_1}{V_2} = \frac{\frac{\pi}{4} \times 0.025^2 \times 12}{7.46} = 0.0007896 \text{ m}^2$$

$$A_2 = \frac{\pi D_2^2}{4}, \quad D_2 = \sqrt{\frac{4 A_2}{\pi}} = \sqrt{\frac{4 \times 0.0007896}{\pi}} = 31.7 \text{ mm}$$

Problem 4.5 /

Which of the following velocity fields satisfies continuity equation ?

$$(A) \quad u = 4xy + y^2, \quad v = 6xy + 3x$$

To satisfy the continuity equation : $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

$$\frac{\partial u}{\partial x} = 4y \quad , \quad \frac{\partial v}{\partial y} = 6x$$

$4y + 6x = 0$, Therefore, it does not satisfy continuity equation.

$$(B) \quad u = 2x^2 + y^2 \quad , \quad v = -4xy$$

Solution:

$$\frac{\partial u}{\partial x} = 4x \quad , \quad \frac{\partial v}{\partial y} = -4x$$

$4x + (-4x) = 0$, Therefore, it does satisfy continuity equation.

$$(C) \quad u = 2x^2 - xy + z^2 \quad , \quad v = x^2 - 4xy + y^2 \quad , \quad w = -2xy - yz + y^2$$

Solution:

$$\frac{\partial u}{\partial x} = 4x - y \quad , \quad \frac{\partial v}{\partial y} = -4x + 2y \quad , \quad \frac{\partial w}{\partial z} = -y$$

$$4x - y + (-4x + 2y) + (-y) = 0$$

$$4x - y - 4x + 2y - y = 0$$

$4x - 2y - 4x + 2y = 0$, Therefore, it does satisfy continuity.

$$0 = 0$$

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