1.1. Discrete Time System

A discrete-time system is a system where signals (both input and output) are defined only at specific, separate points in time, rather than continuously. The system processes these discrete signals to produce an output, typically denoted as y(n), for each discrete time step n.

y[n]=T[x(n)]

Where,

T is a transformation or operation performed by the system on the input signal x[n].

Example: Find out the response y(n) if x(n) is equal:

$$x[n] = \begin{cases} 1 & \text{for } 0 \leq n \leq 3 \\ 0 & \text{elsewhere} \end{cases}$$

1. y[n]=x[−n]
2. y[n]=x[n-1]
3. y[n]=x[n-1] + x[n+1]H.W
4. y[n]=2x[n] H.W

prof.dr.bayan.mahdi@uomus.edu.iq
zahraa.eisa@uomus.edu.iq

① $y(n) = x(-n)$:- Folding (mirroring) operation

Let us sketch the sequence $x(n)$.

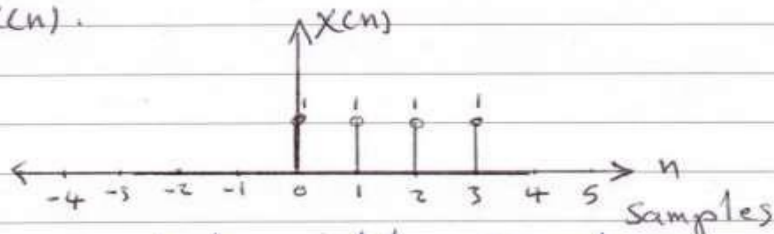


fig. graphical representation of input sequence $x(n)$

thus it is clear from above fig. that $x(n) = 1$ for $0 \le n \le 3$ and $x(n) = 0$ for $n = 4, 5, 6, \ldots \infty$. Similarly $x(n) = 0$ for $n = -1, -2, -3, -4, \ldots -\infty$. now we have $y(n) = x(-n)$

$n = 0 \rightarrow y(0) = x(0) = 1$

$n = 1 \rightarrow y(1) = x(-1) = 0$

$n = 2 \rightarrow y(2) = x(-2) = 0$

$n = 3 \rightarrow y(3) = x(-3) = 0$ ... and so on

Similarly

$n = -1 \rightarrow y(-1) = x(1) = 1$

$n = -2 \rightarrow y(-2) = x(2) = 1$

$n = -3 \rightarrow y(-3) = x(3) = 1$

$n = -4 \rightarrow y(-4) = x(4) = 0$ ... and so on.

Note: $x(-n)$ is the mirror Image of $x(n)$

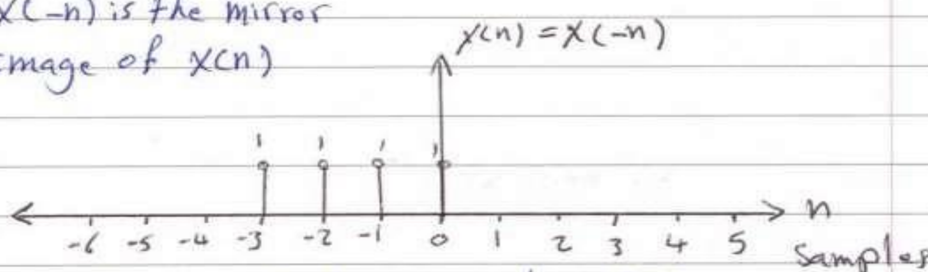


Fig. folding or reflection operation of the sequence

prof.dr.bayan.mahdi@uomus.edu.iq
zahraa.eisa@uomus.edu.iq

② $y(n) = x(n-1)$: Delay operation

$n=0 \rightarrow y(0) = x(0-1) = x(-1) = 0$

$n=1 \rightarrow y(1) = x(1-1) = x(0) = 1$

$n=2 \rightarrow y(2) = x(2-1) = x(1) = 1$

$n=3 \rightarrow y(3) = x(3-1) = x(2) = 1$

$n=4 \rightarrow y(4) = x(4-1) = x(3) = 1$

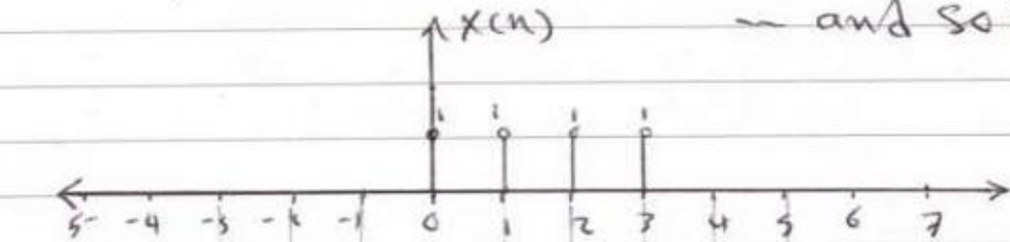
$n=5 \rightarrow y(5) = x(5-1) = x(4) = 0$

$n=6 \rightarrow y(6) = x(6-1) = x(5) = 0$ and so on

Similarly :-

$n=-1 \rightarrow y(-1) = x(-1-1) = x(-2) = 0$

$n=-2 \rightarrow y(-2) = x(-2-1) = x(-3) = 0$ — and so on



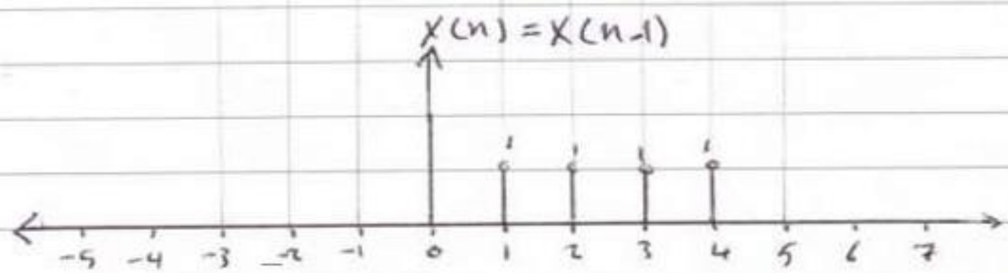


Fig. here y(n) is delayed by one sample

prof.dr.bayan.mahdi@uomus.edu.iq
zahraa.eisa@uomus.edu.iq

1.2. Properties of Discrete Time System

1. **Memoryless System** is a system in which the output at any given time depends solely on the input at that same instant, without regard to past or future input values.

$$y(n) \Longrightarrow x(n) \text{ at } n$$

Example: Check the following system is memoryless or not memoryless?

1. $y(n)=[x(n)]^2$
2. $y(n)=x(n-2)$

Solve:

$y(n)=[x(n)]^2$

$y(0)=[x(0)]^2$ the system is memoryless

And, $y(n)=x(n-2)$

$y(0)=x(-2)$ the system is not memoryless

2. **Linear System and nonlinear system (Linear property)**
   A system is said to be linear if it satisfies the superposition principle. Let $x_1(n)$ and $x_2(n)$ be two input sequence them the system is linear if only:

$$\mathbf{T[a_1x_1(n)+a_2x_2(n)] = a_1T[x_1(n)] + a_2T[x_2(n)]}$$

prof.dr.bayan.mahdi@uomus.edu.iq
zahraa.eisa@uomus.edu.iq

$y_1(n) \rightleftharpoons x_1(n)$

$y_2(n) \rightleftharpoons x_2(n)$

$T\{x_1(n)+x_2(n)\} = T(x_1(n)) + T(x_2(n)) = y_1(n)+y_2(n)$

$y(n) = a_1T(x_1(n)) + a_2T(x_2(n))$

Example: Determine the following system are linear or nonlinear

1. $y(n)=x(n^2)$
2. $y(n)=x^2(n)$

Solve:

1. $y(n)=x(n^2)$

$$y_1(n) = x_1(n^2) \quad \text{and} \quad y_2(n) = x_2(n^2)$$

Consider a linear combination of the inputs $a_1x_1(n)+a_2x_2(n)$

$$y_3(n) = T[a_1x_1(n) + a_2x_2(n)]$$

$$y_3(n) = a_1x_1(n^2) + a_2x_2(n^2)$$

prof.dr.bayan.mahdi@uomus.edu.iq
zahraa.eisa@uomus.edu.iq

$$y_3'(n) = a_1 y_1(n) + a_2 y_2(n)$$

Substituting the expressions for $y_1(n)$ and $y_2(n)$:

$$y_3'(n) = a_1 x_1(n^2) + a_2 x_2(n^2)$$

$$y_3(n) = y_3'(n)$$

The system is Linear

2. $y(n)=x^2(n)$

Solve,

$$y_1(n) = x_1^2(n) \quad \text{and} \quad y_2(n) = x_2^2(n)$$

Consider a linear combination of the inputs $a_1x_1(n)+a_2x_2(n)$

$$y_3(n) = T[a_1 x_1(n) + a_2 x_2(n)]$$

Substituting into the system equation:

$$y_3(n) = [a_1 x_1(n) + a_2 x_2(n)]^2$$

$$y_3(n)=a_1^2x_1^2(n)+2a_1a_2x_1(n)x_2(n)+a_2^2x_2^2(n)$$

The linear combination of two output will be:

$$y_3'(n)=a_1x_1^2(n)+a_2x_2^2(n)$$

$$y_3(n) \neq y_3'(n)$$

The system is nonlinear

prof.dr.bayan.mahdi@uomus.edu.iq
zahraa.eisa@uomus.edu.iq

3. **Time -invariant system** is one whose behavior and characteristics do not change over time. In other words, if you delay the input, the output will be delayed by the same amount, but the shape of the output will remain the same.

$$y(n-n_o) \rightleftharpoons x(n-n_o)$$

4. **Causality system**

The system described by the equation $\mathbf{y(n) = x(n) + x(n-1)}$ is **causal** because the value of the output at any time $n = no$ depends only on the input $x(n)$ at time no and at time $no - 1$. The system described by $\mathbf{y(n) = x(n) + x(n+1)}$, on the other hand, is noncausal because the output at time n = no depends on the value of the input at time $no + 1$is a system in which the output at any time depends only on the current and past values of the input.

Example,

1. y(0)=x(0)-x(-1), the system is causality
2. y(0)=x(0)+x(1), the system is noncausality

prof.dr.bayan.mahdi@uomus.edu.iq
zahraa.eisa@uomus.edu.iq

5. Stable and Unstable System (Stability Property)

When every bounded input produces a bounded output, the system is called Bounded Input Bounded Output (BIBO) stable.

- The input x(n) is **bounded** if there exists some finite number Bx such that:

$$|x(n)| \leq Bx < \infty$$

- Similarly, the output y(n) is **bounded** if there exists some finite number By such that:

$$|y(n)| \leq By < \infty$$

- If the output is unbounded for any bounded input, then the system is considered unstable.

prof.dr.bayan.mahdi@uomus.edu.iq
zahraa.eisa@uomus.edu.iq

Example: Characterization of the following system as either linear or not, time-invariant or not, and causal or not.

1. $y(n)=x^2(n+1)$

   Solve:

   1. Linearity

$$T[\alpha x(n)] = (\alpha x(n+1))^2 = \alpha^2 x^2(n+1)$$

This is not equal to $\alpha T[x(n)] = \alpha x^2(n+1)$, so the system is **nonlinear**.

   2. Time-invariant

$$T[x(n-n_0)]= x^2[(n-n_0)+1]=y(n-n_0)$$

$$T[x(n-n_0)]=y(n-n_0)$$

The system **is time-invariant**.

   3. Causality

$$y(n)=x^2(n+1)$$

the output $y(n)$ at time n depends on $x(n+1)$, which is the input value at **time n+1**, so the system is non-causal.

prof.dr.bayan.mahdi@uomus.edu.iq
zahraa.eisa@uomus.edu.iq

H.W/ Characterize the system below as linear/ nonlinear, causal/ noncausal, time invariant/time varying.

1. $y(n)= (n+a)^2 . x(n+4)$

2. $y(n)=\cos[x(n)]$

prof.dr.bayan.mahdi@uomus.edu.iq
zahraa.eisa@uomus.edu.iq