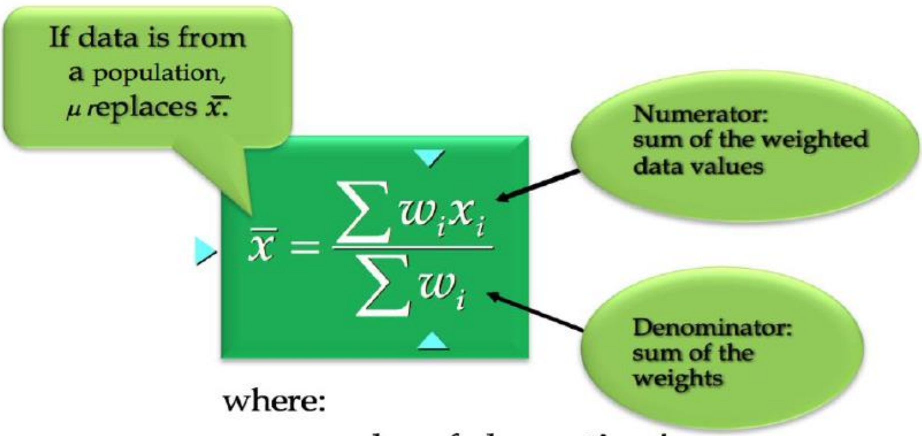
3.1 Weighted Mean

The weighted mean is a type of mean that is calculated by multiplying the weight (or probability) associated with a particular event or outcome with its associated quantitative outcome and then summing all the products together. It is very useful when calculating a theoretically expected outcome where each outcome has a different probability of occurring, which is the key feature that distinguishes the weighted mean from the arithmetic mean.



where:

x_i = value of observation i

w_i = weight for observation i

Example

Find the mean of the following data set.

1	1	1	1		
10	10	10	10	10	
5	5	5	5	5	5

Solution.

Use the Weighted Mean formula.

The w terms are the weights.

$$\text{Weighted Average} = \frac{\text{Sum of weighted terms}}{\text{total number of terms}} = \frac{w_1 \cdot x_1 + w_2 \cdot x_2 + \cdots + w_n \cdot x_n}{w_1 + w_2 \cdots + w_n}$$

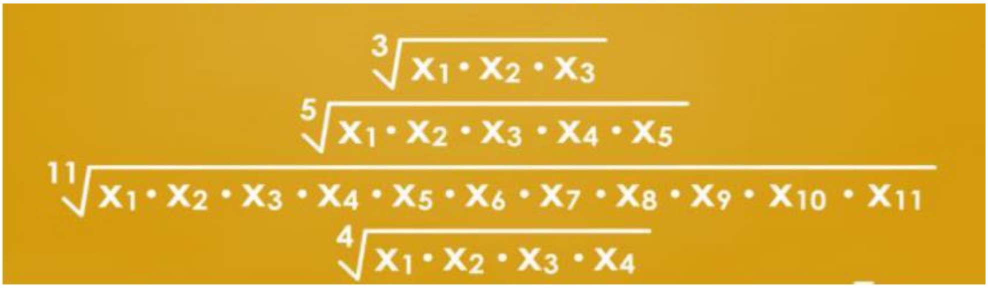
$$\text{Weighted Mean} = \frac{4 \cdot 1 + 5 \cdot 10 + 6 \cdot 5}{15} = \frac{4 + 50 + 30}{15} = \frac{84}{15} = 5.6$$

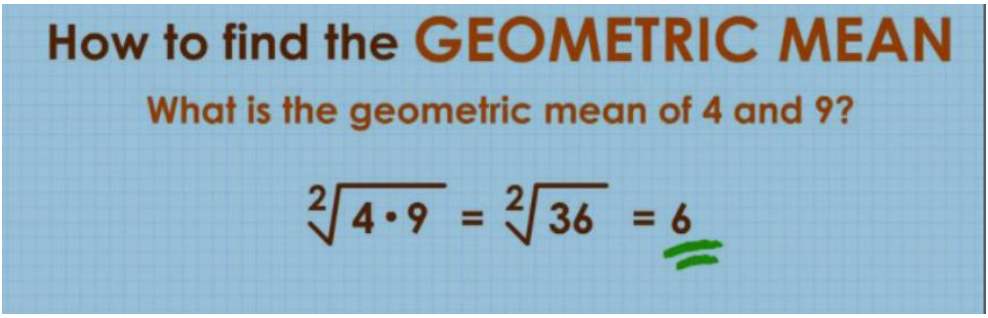
The numbers in red are the weights.

Geometric mean

In mathematics, the geometric mean is a mean or average, which indicates the central tendency or typical value of a set of numbers by using the product of their values (as opposed to the arithmetic mean which uses their sum). The geometric mean is defined as the nth root of the product of n numbers, i.e., for a set of numbers $x1, x2, \ldots, xn$, the geometric mean is defined as:

$$Geometric\ mean = \sqrt[n]{x_1 \cdot x_2 \cdot \ldots \ldots \cdot x_n}$$





Harmonic mean

The harmonic mean can be expressed as the reciprocal of the arithmetic mean of the reciprocals of the given set of observations.

$$Harmonic\ mean = \frac{n}{\sum \frac{1}{x_i}}$$

Where:

n : the number of the values in a dataset

xi : the point in a dataset

Example/ consider 2, 3, 5, 7, and 60 with a number of observations as 5 find Harmonic mean?

$$\text{Harmonic Mean} = \frac{n}{\left(\frac{1}{x_1} + \frac{1}{x_2} + \ldots + \frac{1}{x_n}\right)}$$

$$= \frac{5}{\left(\frac{1}{2} + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \frac{1}{60}\right)}$$

$$= \frac{5}{(0.5 + 0.33 + 0.2 + 0.14 + 0.017)}$$

$$= \frac{5}{1.187}$$

$$= 4.21$$