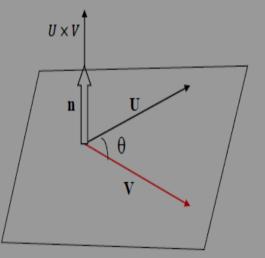


(Lecturer (Dr.alaa mohammed Hussein wais) 1st term – Lect. (Vector)

3. Vector/cross product

If U an V are not parallel, they determine a plane. A unit vector (n) is perpendicular to the plane by (right hand rule).

This mean that we choose (n) to be unit vector. The cross product ($U \times V$) means that (U cross V).



Properties of cross product

1.
$$U \times V = -V \times U$$

2. U×
$$(V + W) = U \times V + U \times W$$

3.
$$0 \times U = 0$$



(Lecturer (Dr.alaa mohammed Hussein wais) 1st term – Lect. (Vector)

Calculate cross products, by using Determinates.

If
$$U = u_1 i + u_2 j + u_3 k$$

$$V = v_1 i + v_2 j + v_3 k$$

$$U \times V = \begin{bmatrix} i & j & k \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_2 \end{bmatrix}$$

Area of plane= $|U \times V|$

Example 1: Find U× V. IF U = i + j + k . V = -2i + 2j + k and find area of plane.

Solution //

$$\mathbf{U} \times \mathbf{V} =$$

$$\begin{bmatrix} i & j & k \\ 1 & 1 & 1 \\ -2 & 2 & 1 \end{bmatrix}$$

$$=i\begin{bmatrix}1&1\\2&1\end{bmatrix}-j\begin{bmatrix}1&1\\-2&1\end{bmatrix}+k\begin{bmatrix}1&1\\-2&2\end{bmatrix}=-i-3j+4k$$

Area of plane= $|U \times V|$

$$=\sqrt{(-1)^2+(-3)^2+(4)^2}=\sqrt{26}$$



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Example 2: Find unit vector perpendicular to the plane P(1, -1, 2), Q(2, 0, -1), R(0, 2, 1) and find area of plane.

Solution //

$$PQ = (2-1)i + (0-(-1)j + (-1-2)k = i + j - 3k$$

$$PR = (0-1)i+(2-(-1)j+(1-2)k=-i+3j-k$$

$$PQ \times PR = \begin{bmatrix} i & j & k \\ 1 & 1 & -3 \\ -1 & 3 & -1 \end{bmatrix}$$

$$PQ \times PR = i \begin{bmatrix} 1 & -3 \\ 3 & -1 \end{bmatrix} - j \begin{bmatrix} 1 & -3 \\ -1 & -1 \end{bmatrix} + k \begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix}$$
$$= 8i + 4j + 4k$$

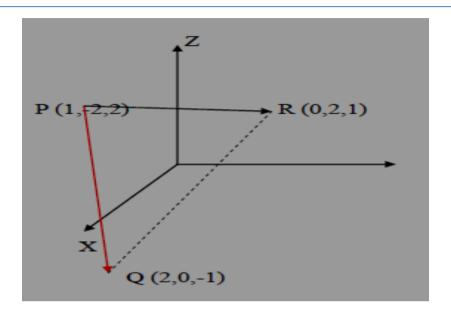
$$|PQ \times PR| = \sqrt{(8)^2 + (4)^2 + 4)^2} = \sqrt{96}$$

Unit vector=
$$\frac{PQ \times PR}{|PQ \times PR|} = \frac{8i + 4j + 4k}{\sqrt{96}}$$

Area of plane=
$$|PQ \times PR| = \sqrt{96}$$



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Exercise: Find unit vector perpendicular to the plane .P(1.1.1).Q(2.1.3).R(0.2.1) and find area of plane