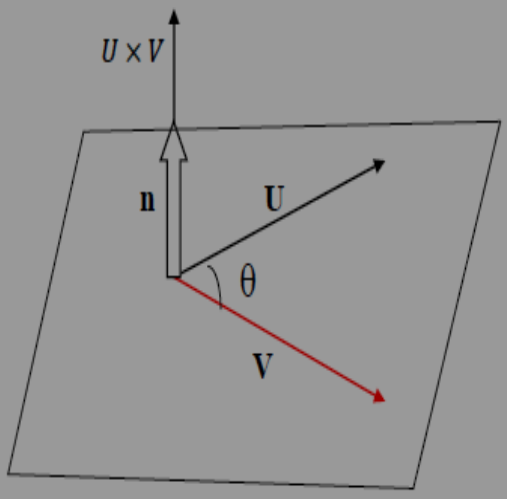
3. Vector/cross product

If U an V are not parallel, they determine a plane. A unit vector (n) is perpendicular to the plane by (right hand rule).

This mean that we choose (n) to be unit vector. The cross product ($U \times V$) means that (U cross V).

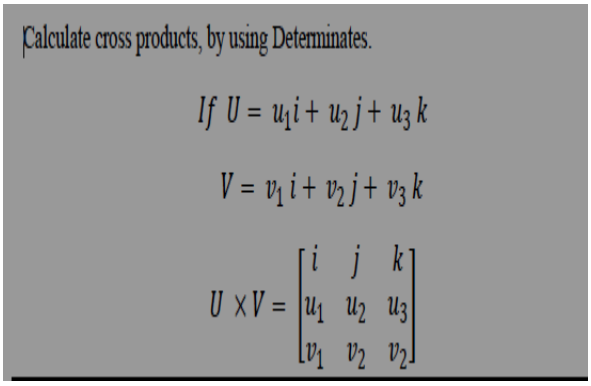


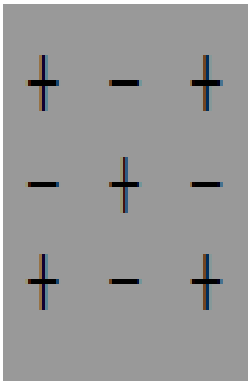
Properties of cross product

1. $\mathbf{U} \times \boldsymbol{V} = -\boldsymbol{V} \times \boldsymbol{U}$

2. $\mathbf{U} \times (\boldsymbol{V} + \boldsymbol{W}) = \boldsymbol{U} \times \boldsymbol{V} + \boldsymbol{U} \times \boldsymbol{W}$

3. $\mathbf{0} \times \boldsymbol{U} = \mathbf{0}$





Area of plane=$|U \times V|$

Example 1: Find U$\times \boldsymbol{V}$. $\boldsymbol{IF\ U = i + j + k}$ $\boldsymbol{.V = -2i + 2j + k}$ and find area of plane.

Solution //

$$\mathbf{U} \times \boldsymbol{V} = \begin{bmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ \mathbf{1} & \mathbf{1} & \mathbf{1} \\ \mathbf{-2} & \mathbf{2} & \mathbf{1} \end{bmatrix}$$

$$= \boldsymbol{i}\begin{bmatrix} \mathbf{1} & \mathbf{1} \\ \mathbf{2} & \mathbf{1} \end{bmatrix} - \mathbf{j}\begin{bmatrix} \mathbf{1} & \mathbf{1} \\ \mathbf{-2} & \mathbf{1} \end{bmatrix} + \boldsymbol{k}\begin{bmatrix} \mathbf{1} & \mathbf{1} \\ \mathbf{-2} & \mathbf{2} \end{bmatrix} = \boldsymbol{-i - 3j + 4k}$$

Area of plane=$|\boldsymbol{U \times V}|$

$$= \sqrt{(\mathbf{-1})^2 + (\mathbf{-3})^2 + (\mathbf{4})^2} = \sqrt{\mathbf{26}}$$

Example 2: Find unit vector perpendicular to the plane $P(1.-1.2).Q(2.0.-1).R(0.2.1)$ and find area of plane.

Solution //

PQ= (2-1)i+(0-(-1)j+(-1-2)k=i+j-3k

PR= (0-1)i+(2-(-1)j+(1-2)k=-i+3j-k

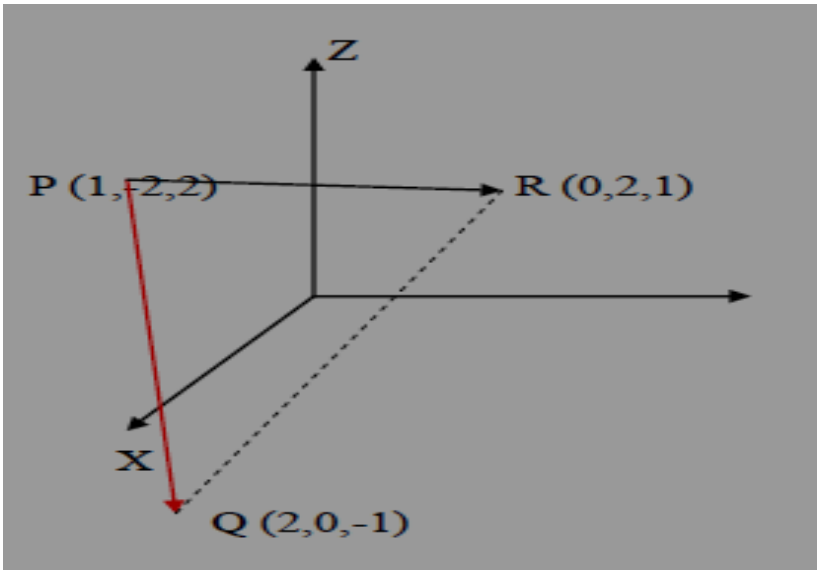
$$\text{PQ}\times PR = \begin{bmatrix} i & j & k \\ 1 & 1 & -3 \\ -1 & 3 & -1 \end{bmatrix}$$

$$PQ \times PR = i\begin{bmatrix} 1 & -3 \\ 3 & -1 \end{bmatrix} - \text{j}\begin{bmatrix} 1 & -3 \\ -1 & -1 \end{bmatrix} + k\begin{bmatrix} 1 & 1 \\ -1 & 3 \end{bmatrix}$$
$$= 8i + 4j + 4k$$

$$|PQ \times PR| = \sqrt{(8)^2 + (4)^2 + 4)^2} = \sqrt{96}$$

$$\text{Unit vector}=\frac{PQ\times PR}{|PQ\times PR|} = \frac{8i+4j+4k}{\sqrt{96}}$$

$$\text{Area of plane}=|PQ \times PR| = \sqrt{96}$$



Exercise: Find unit vector perpendicular to the plane $.P(1.1.1).Q(2.1.3).R(0.2.1) and\ find\ area\ of\ plane$