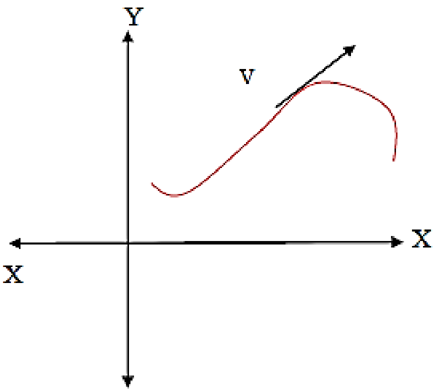
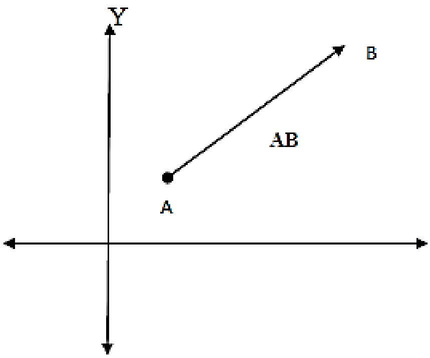
2.1 Vector

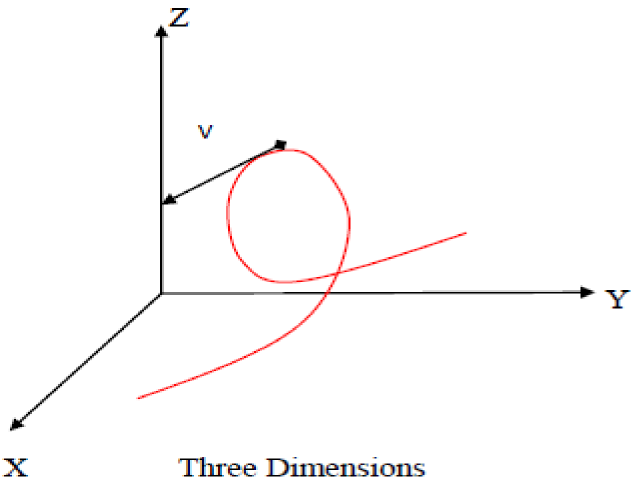
A quality such as force, displacement, or velocity is called a vector and represented by a direct line segment.



Two Dimensions



initial and terminal points.



Three Dimensions

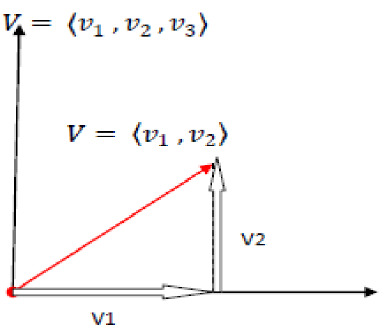
Definition of Vector:

A vector is a directed line segment $\overrightarrow{AB}$ has initial point (A) and terminal point (B), it's length denoted by $|\overrightarrow{AB}|$.

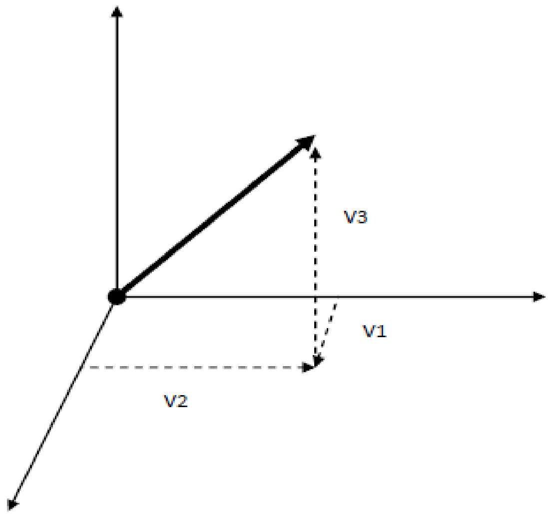
If $\vec{V}$ is a two dimensional vector in plane equal to the vector with initial points at the origin and terminal point (v_1, v_2), then the component of (V):

$$V = \langle v_1 , v_2 \rangle$$

If $\vec{V}$ is a three-dimensional vector with initial point at the origin at the terminal point(v_1, v_2, v_3), then the component of (V):



Component of two dimensional Vector



Component of three dimensional vectors

The magnitude of length of the vector $v = \overrightarrow{PQ}$ is the non-negative number.

$$|V| = \sqrt{{v_1}^2 + {v_2}^2 + {v_3}^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

Vectors Operation:

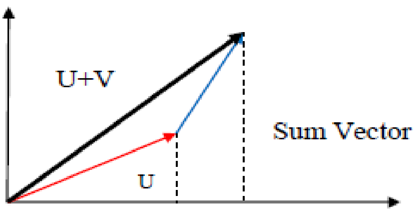
Vector Addition multiplication of a vector by scalar:

Let $U = (u_1, u_2, u_3), and\ V = (v_1, v_2, v_3)$

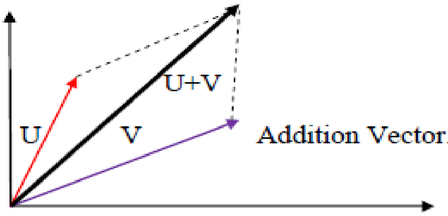
K is scalar.

Addition: $U + V = (u_1 + v_1, u_2 + v_2, u_3 + v_3)$

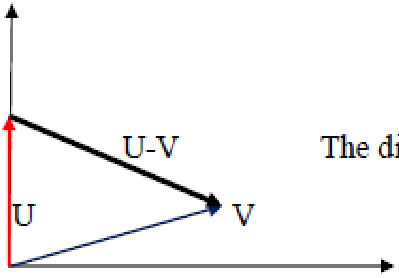
Scalar multiplication: $K.U = (K.u_1, K.u_2, K.u_3$



Sum Vector



Addition Vector.



The difference vector.

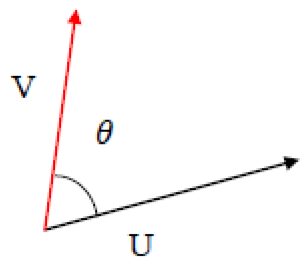
Properties of Vector

1. U±V=V±U

2. a(U+V)=aU+aV

3. U.V= V.U(Dot product)

4. angle between two vector

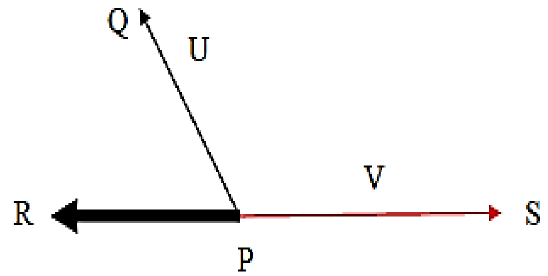


$$\theta = \cos^{-1}\left(\frac{U\,.V}{|u|.|v|}\right)$$

5.Vector Projection.

$U = \overrightarrow{PQ}$

$V = \overrightarrow{PS}$



The vector projection of $U = \overrightarrow{PQ}$ onto non-zero vector $V = \overrightarrow{PS}$ is the vector $\overrightarrow{PR}$.

$Proj_v\, U$ (*the vector Projection U onto V*)

$$Proj_v\, U = (\,|U|.\cos\theta)\,.\frac{V}{|V|}$$

$$\because\ |U|.\cos\theta = \frac{U.V}{|V|}$$

$$Proj_v\, U = \left(\frac{U.V}{|V|}\right).\frac{V}{|V|} = \left(\frac{U\,.V}{|V|^2}\right).V \qquad (vector\ value).$$

Example 1: Find the component form and length of vector with initial point P (-3, 4, 1) and terminal point Q (-5, 2, 2):

Solution:

a) The standard position vector V representing $\overrightarrow{PQ}$ has components:

$v_1 = x_2 - x_1 = -5 - 3 = -2$

$v_2 = y_2 - y_1 = 2 - 4 = -2$

$v_3 = z_2 - z_1 = 2 - 1 = 1$

The component form of $\overrightarrow{PQ}$ is:

$$V = (-2, -2, 1)$$

b) The magnitude of vector length $|V|$ or $|\overrightarrow{PQ}|$ is:

$$|V| = \sqrt{{v_1}^2 + {v_2}^2 + {v_3}^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

$$|V| = \sqrt{(-2)^2 + (-2)^2 + (1)^2} = \sqrt{9} = 3$$

Example 1: Let U = (-1, 3 , 1) , V = (4, 7, 0), Find:

1. 2U + 3V:

$$2U + 3V = 2(-1,3,1) + 3(4,7,0) = (-2,6,2) + (12,21,0) =$$

$$(10,27,2)$$

2. $U - V =$

$$U - V = (-1,3,1) - (4,7,0) = (-5,-4,1)$$

Example 6: Find the dot product of U.V, where $U = (1, -2. -1), and\ V = (-6, 2, -3)$.

Solution:

$$U.V = (1 \times\ -6) + \ (-2 \times 2) + \ (-1\ \times\ -3)$$

$$= \ -6 - 4 + 3 = \ -7$$

Example ☐: Find the angle θ between $U = i - 2j - 2k, and\ V = 6i + 3j + 2k$.

Solution:

$$U\,.V = (1 \times 6) + \ (-2 \times 3\,) + \ (-2 \times 2)$$

$$= 6 - 6 - 4 = \ -4$$

$$|U| = \ \sqrt{1^2 + \ (-2)^2 + (-2)^2} = \ \sqrt{9} = 3$$

$$|V| = \ \sqrt{6^2 + \ 3^2 + \ 2^2} = \ \sqrt{49} = 7$$

$$\theta = \ \cos^{-1}\frac{U\,.V}{|U|\,.|V|}$$

$$\theta = \ \cos^{-1}\frac{(-4)}{3 \times 7} \cong 1.76\,rad.$$

Example☐: Find the vector projection of $U = 6i + 3j + 2k$ $\ onto\ V = i - 2j - 2k$. and the scalar component of U in the direction of V.

Solution:

1) Find Proj_v u:

$$Proj_V u = \frac{U.V}{|V|.|V|} .V$$

$$Proj_V u = \frac{(6\times1)+(3\times-2)+(2\times-2)}{(1^2+(-2)^2+(-2)^2)} .(i - 2j - 2k)$$

$$Proj_V u = \frac{-4}{9} .(i - 2j - 2k) = \frac{-4}{9} i + \frac{8}{9} j + \frac{8}{9} k$$

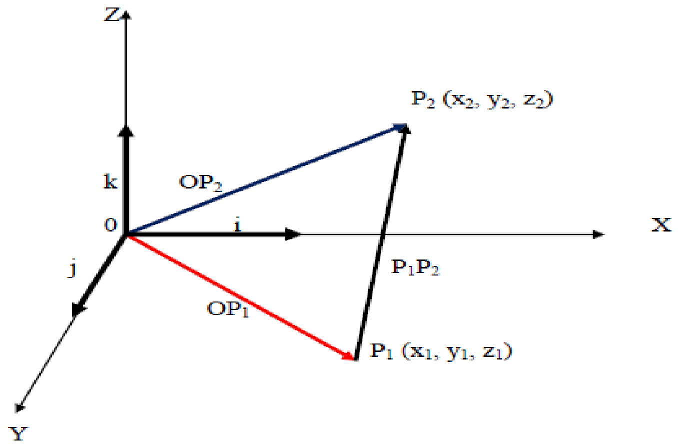
2) The scalar component of U in the direction of V.

$$|U|.\cos\theta = \frac{U.V}{|V|} = \frac{(6\times6)+(3\times-2)+\ (2\times-2)}{\sqrt{(1)^2+(-2)^2+(-2)^2}} = \frac{6-6+4}{\sqrt{9}} = \frac{4}{3}$$

Unit Vector

The length of vector is called unit vector. The standard of units vectors are:

$$i = (1, 0, 0), \qquad j = (0, 1, 0), \qquad k = (0, 0, 1)$$



$$V = v_1\, i + v_2\, j + v_3\, k$$

$i, j, and\ k\ \ as\ scalar\ vector.$

$i = component\ of\ vector\ v_1$, $\quad j = component\ of\ vector\ v_2$

$k = component\ of\ vector\ v_3$

$V_1=(X_2-X_1)$, $V_2=(Y_2-Y_1)$, $V_3=(Z_2-Z_1)$

$$\textbf{Unit vector}=\frac{v}{|v|} = \frac{\overline{P1P2}}{|\overline{P1P2}|}$$

Example 2: Find a unit vector in the direction of the vector $P_1(1, 0, 1)$ to $P_2(3, 2, 0)$.

Solution:

$$1.\ \overrightarrow{P1P2} = V1\ i + v2\ j + v3\ k$$
$$= (x2 - x1)i + (y2 - y1)j + (z2 - z1)k$$

$$=(3-1)i+(2-0)j+(0-1)k=2i+2j-k$$

$$2.\ |\overrightarrow{P_1P_2}| = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2 + (z_2-z_1)^2}$$
$$= \sqrt{(3-1)^2 + (2-0)^2 + (0-1)^2}$$
$$= \sqrt{2^2 + 2^2 + 1^2} = \sqrt{9} = 3$$

3. Unit vector:

$$U = \frac{\overrightarrow{P_1P_2}}{|\overrightarrow{P_1P_2}|} = \frac{2\,i + 2j - k}{3}$$

$$U = \frac{2}{3}\,i + \frac{2}{3}\,j - \frac{1}{3}\,k$$