Al-Mustaqbal University

Department of Biomedical Engineering

Third Stage / 1st Course

“Transport Phenomena for BME”

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Lecture 3
Fluid Statics or Hydrostatics

What is pressure?

- **Pressure** is a measure of how concentrated a force is

Pressure (P) = $Force/\ Area$

Components in the picture:

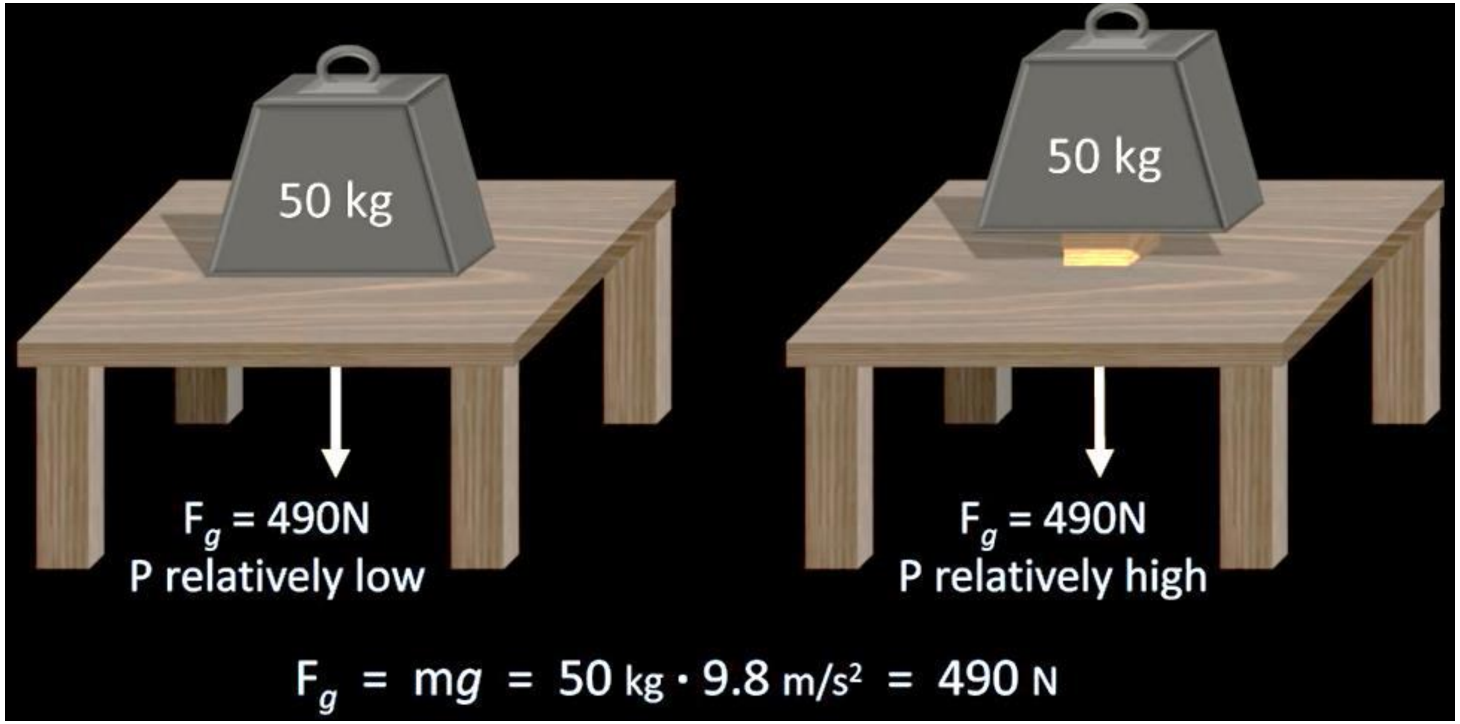
There are two weights (each 50 kg) placed on two tables.

The force due to the weight (F_g) is 490 Newtons (N) per mass, which is calculated using the acceleration due to gravity (9.8 m/s^2)

Pressure on the tables:

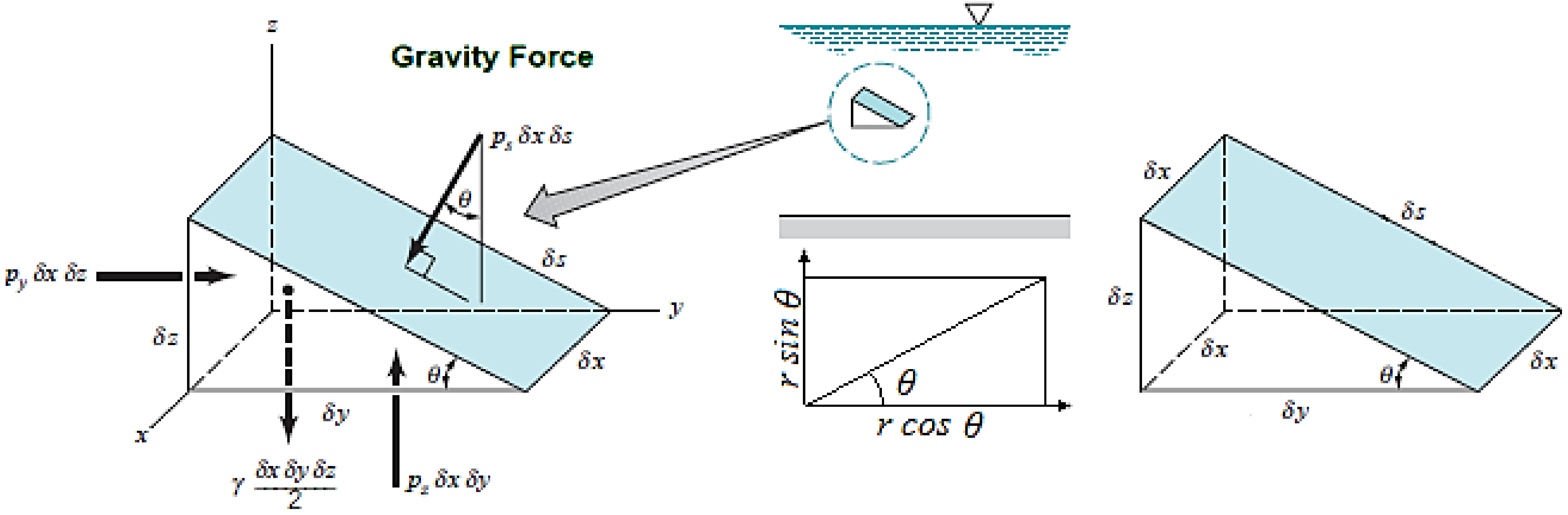
The pressure on the table depends on the area over which the force (weight) acts. **If the area is small, the pressure will be greater**, and **if the area is large, the pressure will be less.**

Note: The picture shows that pressure can vary even with the same force if the area changes.



Pressure at a Point

- **Fluid statics** (also called hydrostatics is the branch of fluid mechanics that studies **fluids at rest and under pressure** in a fluid.



Pressure at a Point

- The equations of motion (Newton's second law, F m a in the y and z directions are, respectively, Force balance

$$\sum \bar{F} = m\bar{a}$$

m= density*volume

y - Direction

$$\sum F_y = p_y \delta x \delta z - p_s \delta x \delta s \sin\theta = \rho \delta \forall \, a_y$$

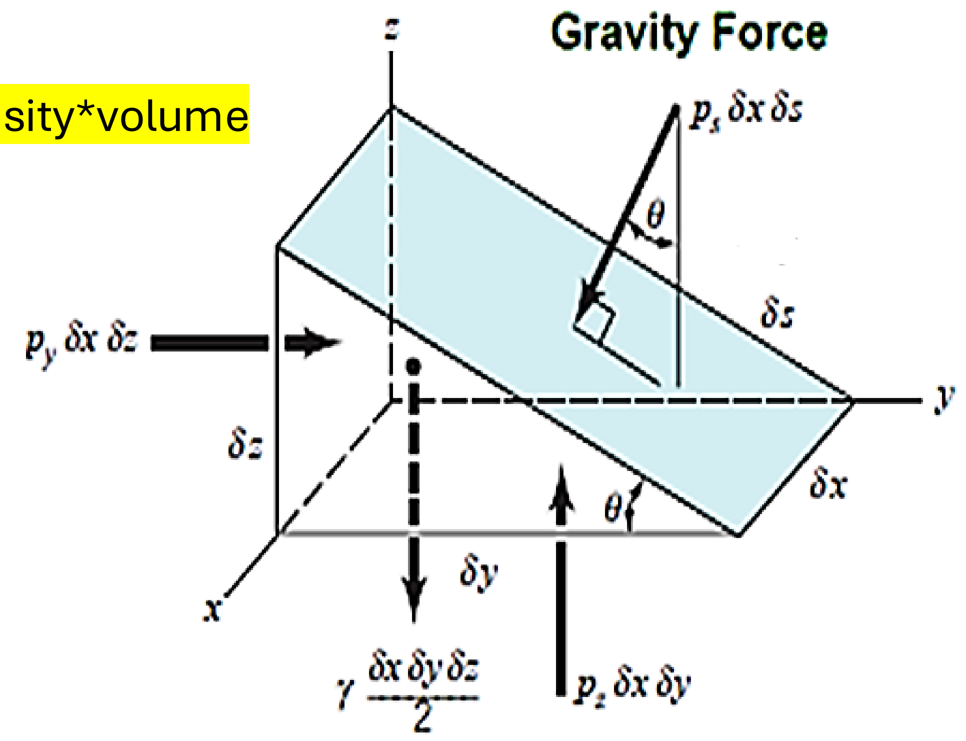
$$\delta \forall = \frac{\delta x \delta y \delta z}{2}$$

$$\sum F_y = p_y \delta x \delta z - p_s \delta x \delta s \sin\theta = \rho \frac{\delta x \delta y \delta z}{2} a_y$$

From the geometry that

$$\sin\theta = \frac{\delta z}{\delta s} \quad \rightarrow \quad \delta z = \delta s \sin\theta$$

$$p_y - p_s = \rho a_y \frac{\delta y}{2}$$



Pressure at a Point

- The equations of motion (Newton's second law, F m a in the y and z directions are, respectively, Force balance

W= density*volume *g

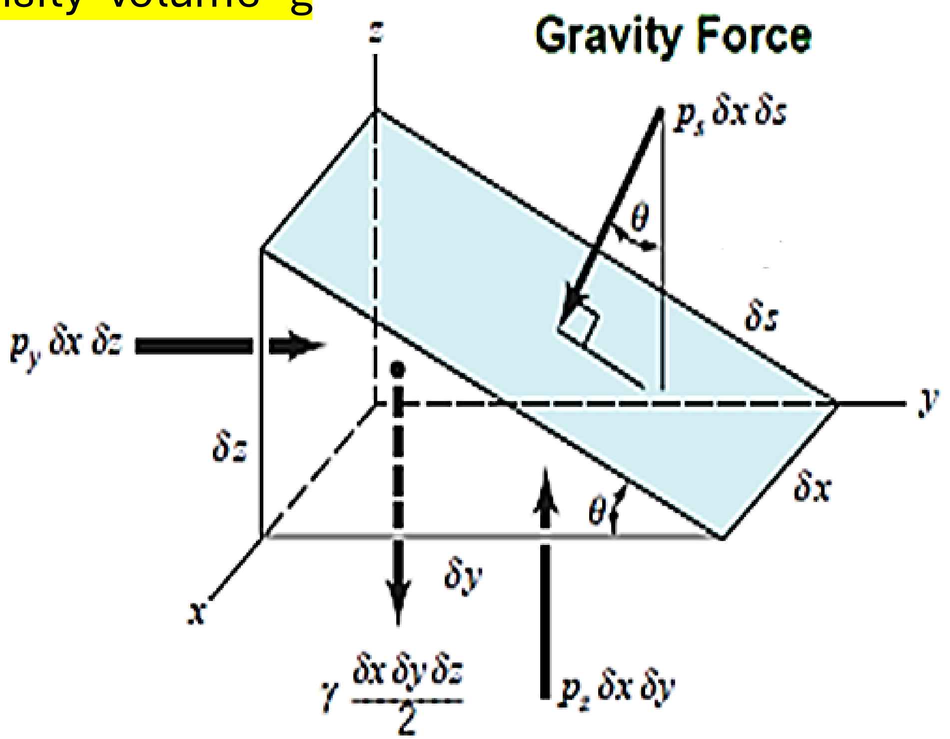
Apply Newton's second law, F = ma in the z direction

$$p_z\,\delta x\,\delta y - p_s\,\delta x\,\delta s\cos\theta - \gamma\frac{\delta x\,\delta y\,\delta z}{2} = \rho\frac{\delta x\,\delta y\,\delta z}{2}a_z$$

$$p_z - p_s = (\rho a_z + \gamma)\frac{\delta z}{2}$$

take the limit as δx, δy, and δz approach zero (while maintaining the angle θ),

it follows that: $p_y = p_s$; $p_z = p_s$ or:



Pressure at a Point

- The equations of motion (Newton's second law, F m a in the y and z directions are, respectively, Force balance

Y-direction $$p_y - p_s = \rho a_y \frac{\delta y}{2}$$

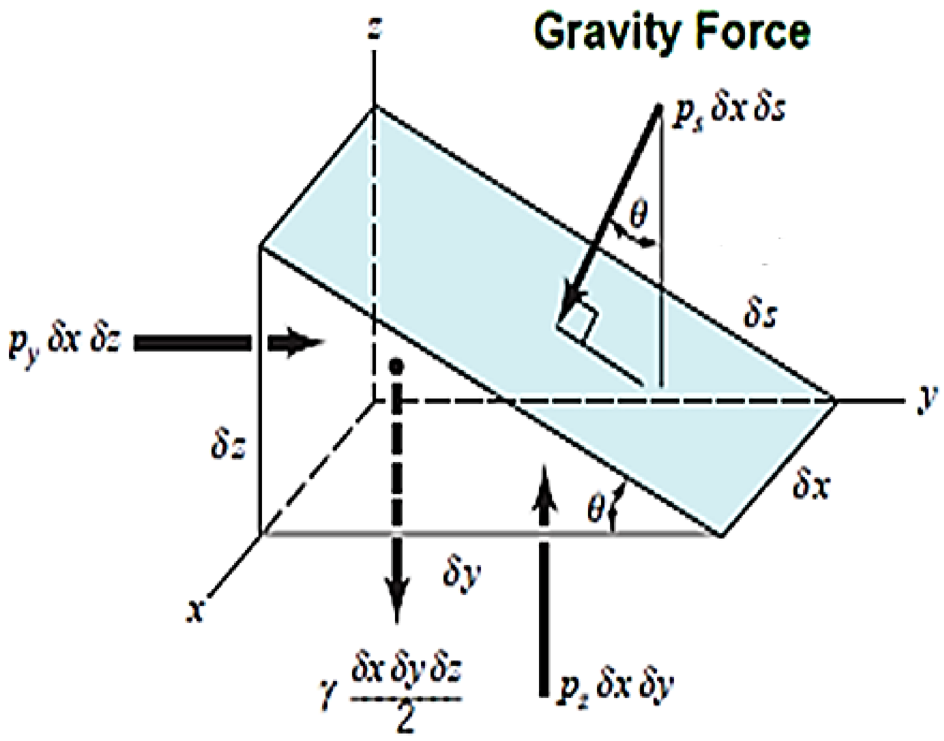
Z-direction $$p_z - p_s = (\rho a_z + \gamma)\frac{\delta z}{2}$$

take the limit as δx, δy, and δz approach zero (while maintaining the angle θ),

it follows that: $p_y = p_s$; $p_z = p_s$ or:

$$\therefore \quad p_y - p_s = 0$$
$$\therefore \quad p_z - p_s = 0 \rightarrow$$
$$\therefore \quad p_y = p_s = p_z$$

This important result is known as **Pascal's law**.

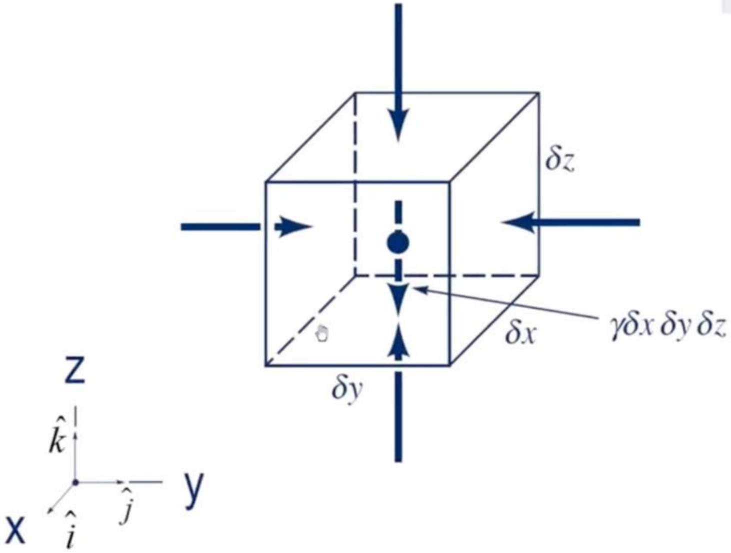


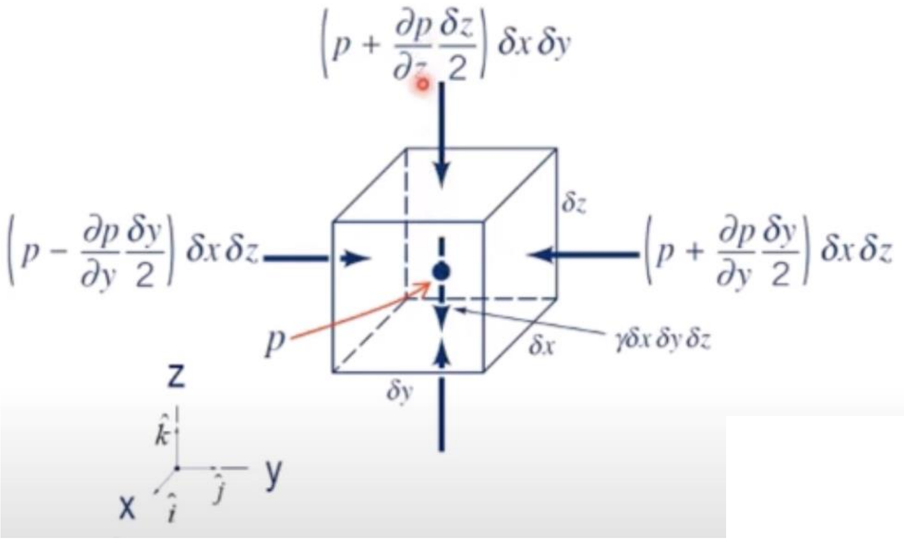
Pascal's Law the pressure at a point in a fluid at rest, or in motion, is **independent** of the direction as long as there are no shearing stresses present

Basic Equation for Pressure Field

- Although we have answered the question of how the **pressure at a point varies with direction**, we are now faced with an equally important question **how does the pressure in a fluid in which there are no shearing stresses vary from point to point**?

The pressures at the faces of the element can be expressed using
Taylor series expansion of the pressure





Basic Equation for Pressure Field

$$\sum \bar{F} = m\bar{a}$$

y - Direction

$$\left(p-\frac{\partial p}{\partial y}\frac{\delta y}{2}\right)\delta x\delta z-\left(p+\frac{\partial p}{\partial y}\frac{\delta y}{2}\right)\delta x\delta z=\delta x\delta y\delta z\rho a_y$$

$$\left(-\frac{\partial p}{\partial y}\frac{\delta y}{2}-\frac{\partial p}{\partial y}\frac{\delta y}{2}\right)\delta x\delta z=\delta x\delta y\delta z\rho a_y$$

$$-\frac{\partial p}{\partial y}=\rho a_y$$

Apply force balance in the x direction (not shown for simplicity),

$$\sum F_x = ma_x$$

$$\left(p-\frac{\partial p}{\partial x}\frac{\delta x}{2}\right)\delta y\delta z-\left(p+\frac{\partial p}{\partial x}\frac{\delta x}{2}\right)\delta y\delta z=\delta x\delta y\delta z\rho a_x$$

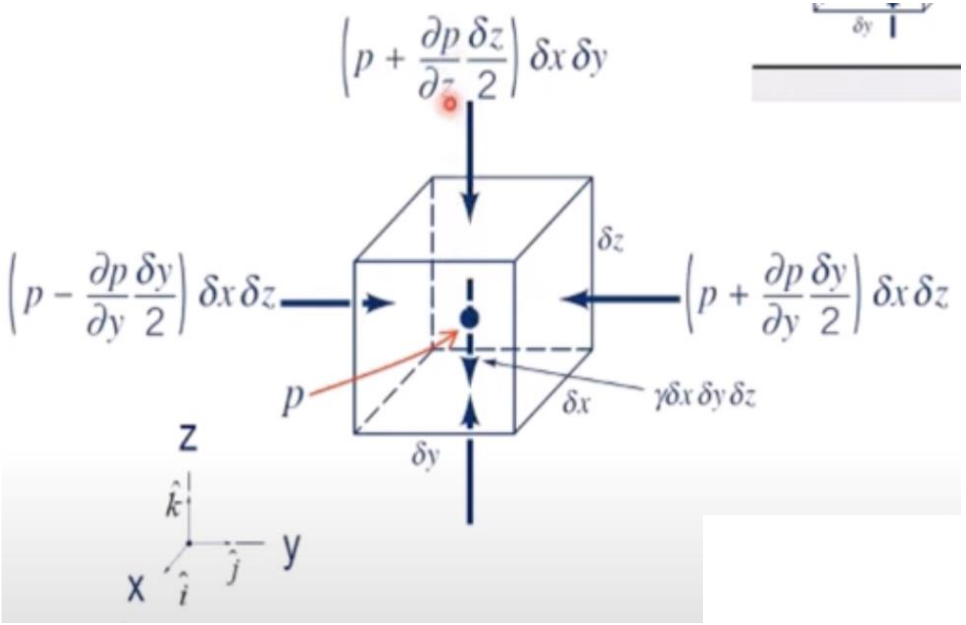
$$\left(-\frac{\partial p}{\partial x}\frac{\delta x}{2}-\frac{\partial p}{\partial x}\frac{\delta x}{2}\right)\delta y\delta z=\delta x\delta y\delta z\rho a_x$$

$$-\frac{\partial p}{\partial x}=\rho a_x$$

Apply force balance in the z-direction $\sum F_z = ma_z$

$$\left(p-\frac{\partial p}{\partial z}\frac{\delta z}{2}\right)\delta x\delta y-\left(p+\frac{\partial p}{\partial z}\frac{\delta z}{2}\right)\delta x\delta y-\underbrace{\gamma\delta x\delta y\delta z}_{\text{element weight}}=\delta x\delta y\delta z\rho a_z$$

$$-\frac{\partial p}{\partial z}=\rho a_z+\rho g$$



Pressure Variation in a Fluid at Rest

So, we ended up with 3 scaler equations.

If the acceleration $\vec{a}$ is zero, then

$$-\frac{\partial p}{\partial y} = \rho a_y \qquad if\ a_y = 0 \Rightarrow -\frac{\partial p}{\partial y} = 0$$

$$-\frac{\partial p}{\partial x} = \rho a_x \qquad if\ a_x = 0 \Rightarrow -\frac{\partial p}{\partial x} = 0$$

$$-\frac{\partial p}{\partial z} = \rho a_z + \rho g \qquad if\ a_z = 0 \Rightarrow -\frac{\partial p}{\partial z} = \rho g$$

Hydrostatic Condition(a = 0)

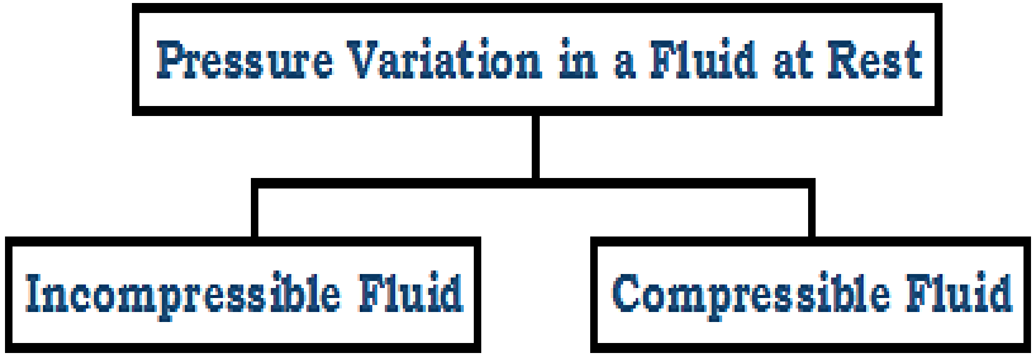
Hydrostatic Equation

Hydrostatic pressure is the pressure present within in s fluid when it is a rest

- It acts equally in all direction
- It acts at a right angle to any surface in contact with the fluid What

Hydrostatic Condition: Physical Implications

- Pressure changes with elevation.
- Pressure does not change in the horizontal x y plane.
- The pressure gradient in the vertical direction is negative.
- The pressure decreases as we move upward in a fluid at rest.
- Pressure in a liquid does not change due to the shape of the container.
- Specific Weight γ does not have to be constant in a fluid at rest.
- Air and other gases will likely have a varying γ.
- Thus, fluids could be incompressible or compressible statically.



Compressible Fluid

$$\frac{dp}{dz} = -\rho g$$

$$\rho = \frac{p}{RT}$$

$$\frac{dp}{dz} = -\frac{gp}{RT}$$

$$\int_{p_1}^{p_2} \frac{dp}{p} = -\frac{g}{RT_o} \int_{z_1}^{z_2} dz$$

$$p_2 = p_1 \exp\left[-\frac{g(z_2 - z_1)}{RT_o}\right]$$

Incompressible Fluid

$$\frac{dp}{dz} = -\rho g$$

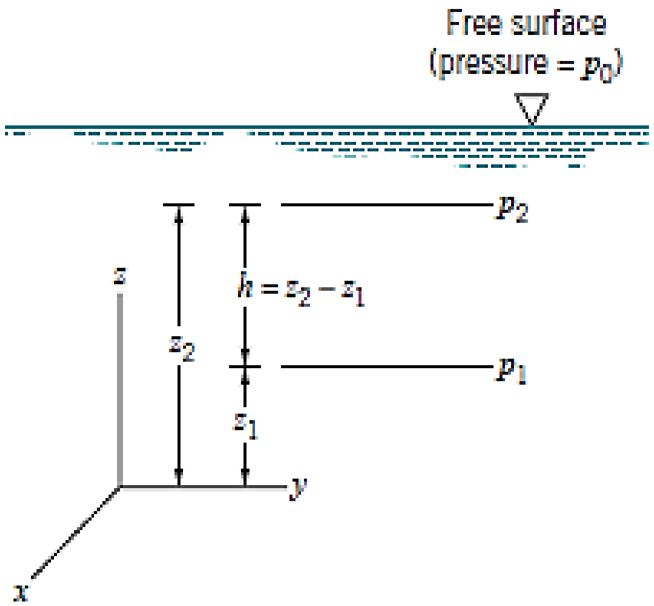
$$\int dp = -\rho g \int dz$$

$$p_2 - p_1 = -\rho g(z_2 - z_1)$$

$$p_1 - p_2 = \rho g(z_2 - z_1)$$

$$p_1 - p_2 = \rho g h$$

In this case h is called the pressure head



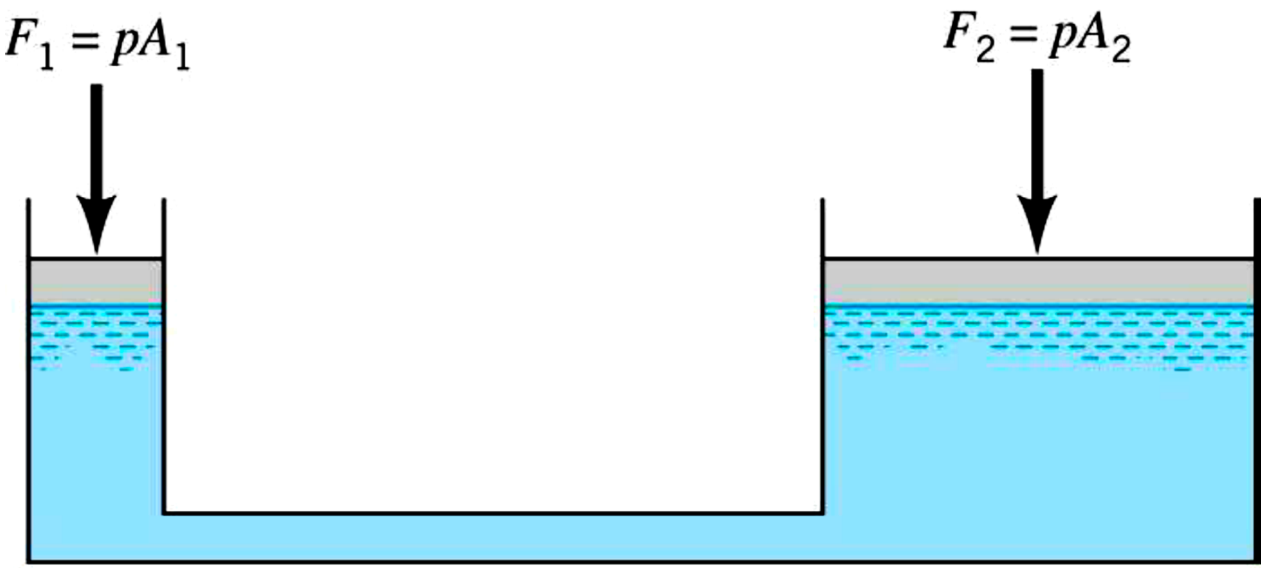
$$p_1 = \rho g h + p_2$$

This type of pressure distribution is commonly called a hydrostatic distribution

$$h = \frac{p_1 - p_2}{\rho g} = \frac{p_1 - p_2}{\gamma}$$

$$p = \rho g h + p_o$$

Hydrostatic Application: Transmission of Fluid Pressure



- Mechanical advantage can be gained with equality of pressures
- A small force applied at the small piston is used to develop a large force at the large piston.
- This is the principle between hydraulic jacks, lifts, presses, and hydraulic controls
- Mechanical force is applied through jacks action or compressed air for example

$$F_2 = \frac{A_2}{A_1} F_1$$

Measurement of Pressure

The **difference** in the levels of the **liquid** raised in the two tubes **will** denote the **pressure difference** between the **two points**

- **Mercury Barometer**

$$\sum F_y = p_{atm}A - W - p_{vapor}A = 0$$

$$\sum F_y = p_{atm}A - \rho g.Ah - p_{vapor}A = 0$$

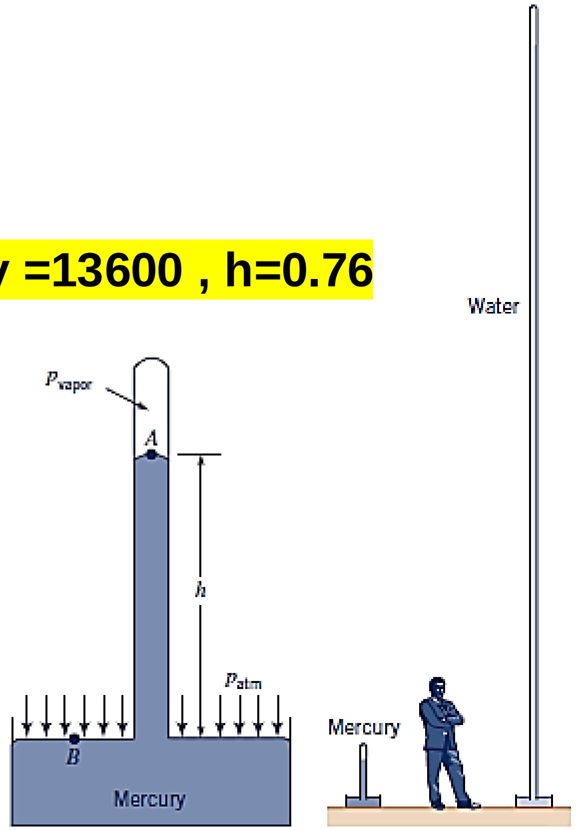
$$\therefore \; p_{atm} = \rho gh + p_{vapor} = \gamma h + p_{vapor}$$

$$P_{atmospher} = \rho_{mercery}.g.h_{mercery}$$

$$P_{atmospher} = 1.013 \times 10^5 \frac{N}{m^2} = 1.013 \times 10^5 \; pa = 101{,}325 \; pa$$

$$1\, atmosphere = 760\, mmHg = 29.92\, in\, of\, Hg = 101{,}325\, pa = 14.7\, psi$$

Density of mercury =13600 , h=0.76



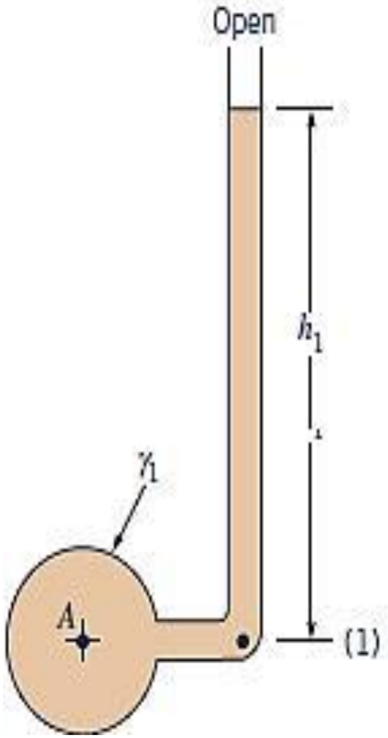
Measurement of Pressure

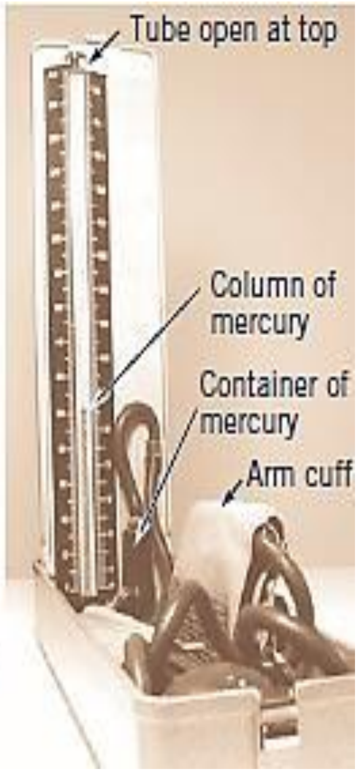
- **A. The Piezometer Tube Manometer**

$$p_A = \gamma h + p_o$$

Note that since the tube is open at the top, the pressure po can be set equal to zero (we are now using gage pressure)

$$p_A = \gamma_1 h_1$$





Measurement of Pressure

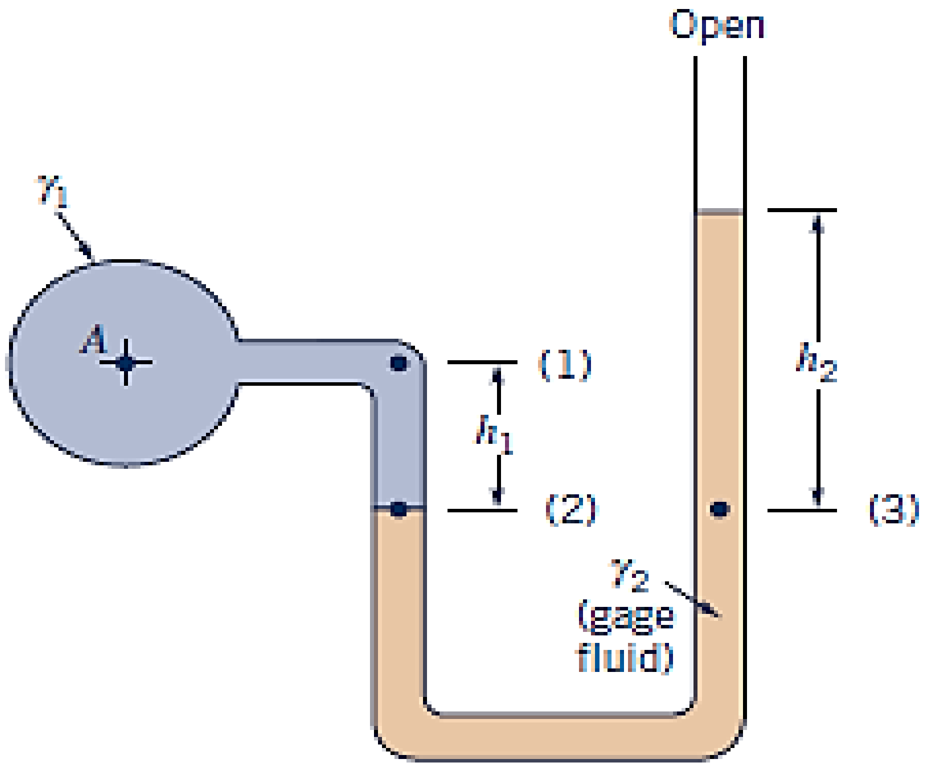
- **B. U.-Tube Manometer**

$$p_A + \gamma_1 h_1 - \gamma_2 h_2 = 0$$

$$p_A = \gamma_2 h_2 - \gamma_1 h_1$$

If A does contain a gas, the contribution of the gas column, $\gamma 1 h 1$ is almost always negligible so that $pA \approx p2$ and in this instance Eq above becomes

$$p_A = \gamma_2 h_2$$

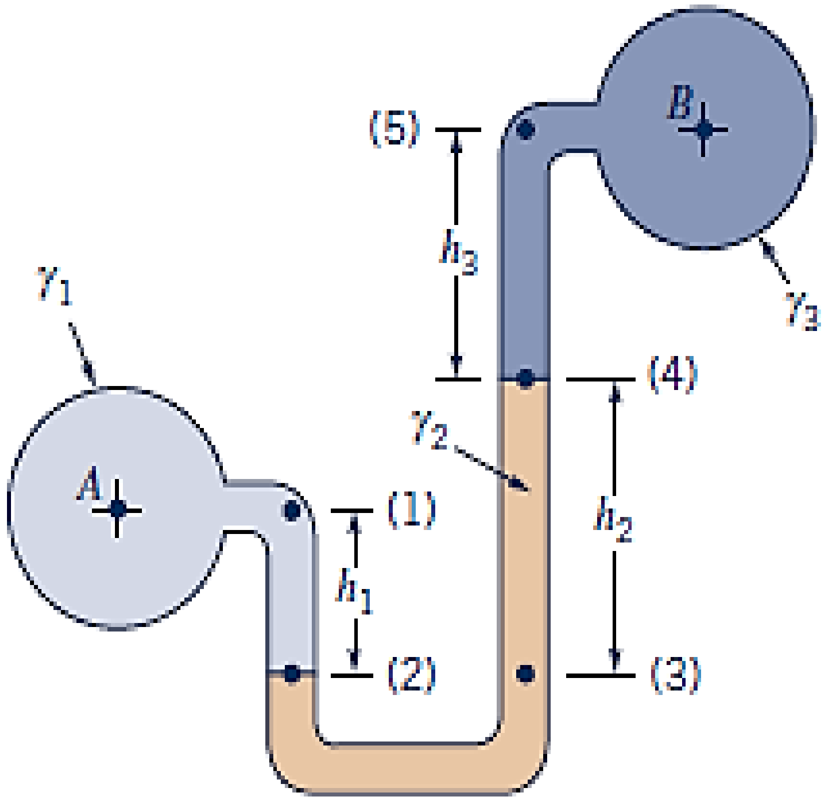


Measurement of Pressure

- **B. U.-Tube Manometer**

$$p_A + \gamma_1 h_1 - \gamma_2 h_2 - \gamma_3 h_3 = p_B$$

$$p_A - p_B = \gamma_2 h_2 + \gamma_3 h_3 - \gamma_1 h_1$$



Measurement of Pressure

- **C.Inclined-TubeManometer**

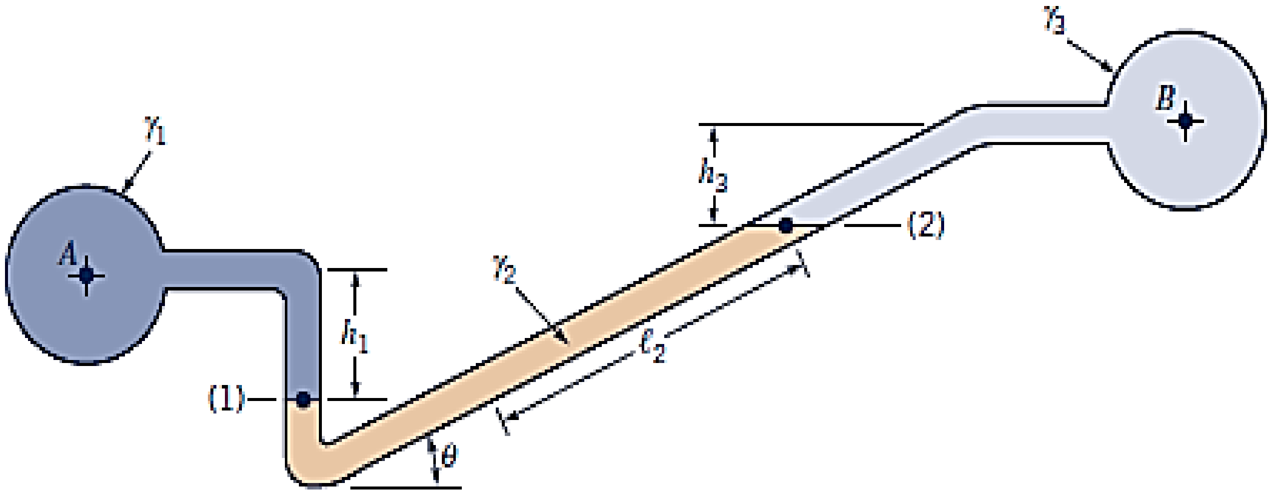
$$p_A + \gamma_1 h_1 - \gamma_2 \ell_2 \sin\theta - \gamma_3 h_3 = p_B$$

$$p_A - p_B = \gamma_2 \ell_2 \sin\theta + \gamma_3 h_3 - \gamma_1 h_1$$

If pipes **A** and **B** contain a gas, then

$$p_A - p_B = \gamma_2 \ell_2 \sin\theta$$

$$\ell_2 = \frac{p_A - p_B}{\gamma_2 \sin\theta}$$



Examples

- Freshwater and seawater flowing in parallel horizontal pipelines are connected to each other by a double U tube manometer, as shown in Figure Determine the pressure difference between the two pipelines ρw=1000 kg/m ^{3} ρHg=13600 kg/m ^{3} and ρsea=1035 kg/m ^{3} Can the air column be ignored in the analysis? hw=50cm,hHg=10cm,$hsea$=30cm

$$P_1 + \rho_w g h_w - \rho_{Hg} g h_{Hg} - \rho_{air} g h_{air} + \rho_{sea} g h_{sea} = P_2$$

Rearranging and neglecting the effect of air column on pressure,

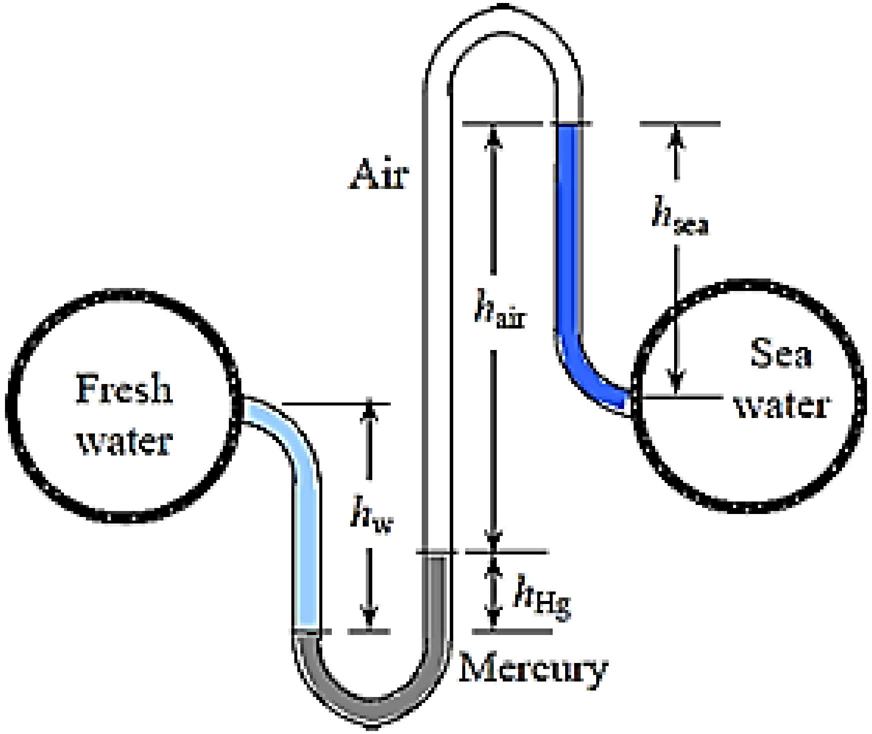
$$P_1 - P_2 = -\rho_w g h_w + \rho_{Hg} g h_{Hg} - \rho_{sea} g h_{sea}$$

$$= g(\rho_{Hg} h_{Hg} - \rho_w h_w - \rho_{sea} h_{sea})$$

Substituting,

$$P_1 - P_2 = (9.81\,\text{m/s}^2)[(13{,}600\,\text{kg/m}^3)(0.1\,\text{m}) - (1000\,\text{kg/m}^3)(0.5\,\text{m}) - (1035\,\text{kg/m}^3)(0.3\,\text{m})]\left(\frac{1\,\text{kN}}{1000\,\text{kg}\cdot\text{m/s}^2}\right)$$

$$= 5.39\,\text{kN/m}^2 = \mathbf{5.39\,kPa}$$



Examples

- The water in a tank is pressurized by air, and the pressure is measured by a multifluid manometer as shown in Fig. 1-49. The tank is located on a mountain at an altitude of 1400 m where the atmospheric pressure is 85.6 kPa. Determine the air pressure in the tank if h1, = 0.1 m, h2, = 0.2 m, and h3= 0.35 m. Take the densities of water, oil, and mercury to be 1000 kg/m3, 850 kg/m3, and 13,600 kg/m3, respectively.

$$P_1 + \rho_{water} g h_1 + \rho_{oil} g h_2 - \rho_{mercury} g h_3 = P_{atm}$$

Solving for P_1 and substituting,

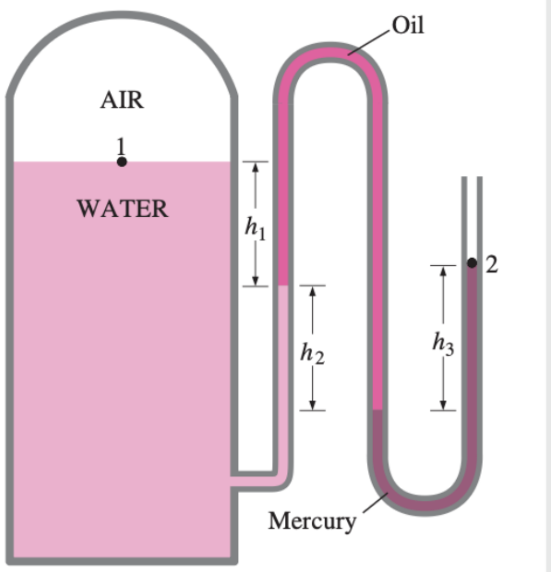
$$P_1 = P_{atm} - \rho_{water} g h_1 - \rho_{oil} g h_2 + \rho_{mercury} g h_3$$

$$= P_{atm} + g(\rho_{mercury} h_3 - \rho_{water} h_1 - \rho_{oil} h_2)$$

$$= 85.6\ \text{kPa} + (9.81\ \text{m/s}^2)[(13{,}600\ \text{kg/m}^3)(0.35\text{m}) - 1000\ \text{kg/m}^3)(0.1\ \text{m})$$

$$-(850\ \text{kg/m}^3)(0.2\ \text{m})]\left(\frac{1\ \text{N}}{1\ \text{kg} \cdot \text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1000\ \text{N/m}^2}\right)$$

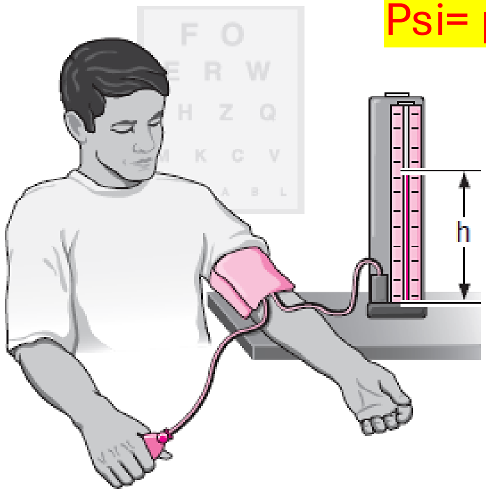
$$= 130\ \text{kPa}$$



Examples

Blood pressure is usually measured by wrapping a closed air filled jacket equipped with a pressure gage around the upper arm of a person at the level of the heart Using a mercury manometer and a stethoscope, the systolic pressure (the maximum pressure when the heart is pumping) and the diastolic pressure (the minimum pressure when the heart is resting) are measured in mmHg The systolic and diastolic pressures of a healthy person are about 120 mmHg and 80 mmHg, respectively, and are indicated as 120 /80 Express both of these gage pressures in kPa ,psi and meter water column $\rho water$=1000kgm^3,$\rho mercury$=13600kg/m^3)

Psi= pound per square



Solution

$$P_{high} = \rho g h_{high} = (13{,}600\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.12\ \text{m})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1000\ \text{N/m}^2}\right) = \mathbf{16.0\ kPa}$$

$$P_{low} = \rho g h_{low} = (13{,}600\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.08\ \text{m})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1000\ \text{N/m}^2}\right) = \mathbf{10.7\ kPa}$$

Noting that 1 psi = 6.895 kPa,

$$P_{high} = (16.0\ \text{kPa})\left(\frac{1\ \text{psi}}{6.895\text{kPa}}\right) = \mathbf{2.32\ psi} \quad \text{and} \quad P_{low} = (10.7\ \text{kPa})\left(\frac{1\ \text{psi}}{6.895\text{kPa}}\right) = \mathbf{1.55\ psi}$$

$$P = \rho_{water} g h_{water} = \rho_{mercury} g h_{mercury} \rightarrow h_{water} = \frac{\rho_{mercury}}{\rho_{water}} h_{mercury}$$

Therefore,

$$h_{water,\ high} = \frac{\rho_{mercury}}{\rho_{water}} h_{mercury,\ high} = \frac{13{,}600\ \text{kg/m}^3}{1000\ \text{kg/m}^3}(0.12\ \text{m}) = \mathbf{1.63\ m}$$

$$h_{water,\ low} = \frac{\rho_{mercury}}{\rho_{water}} h_{mercury,\ low} = \frac{13{,}600\ \text{kg/m}^3}{1000\ \text{kg/m}^3}(0.08\ \text{m}) = \mathbf{1.09\ m}$$