Mon. 21/10/2024

Partial Fraction Expansion

$$\mathcal{L}\left[\boxed{\frac{d^2y}{dt^2}+6\frac{dy}{dt}+8y(t)=2}\right]$$

Laplace	f(t)
$\frac{A}{S}$	A
$\frac{A}{s+b}$	Ae^{-bt}
$\frac{A}{S^2}$	At
$\frac{A\,S}{S^2+\omega^2}$	$A\cos\omega t$

$$\mathcal{L}\left[\frac{d^2y(t)}{dt^2}\right]=S^2Y(S)\,;\quad \mathcal{L}\left[6\frac{dy}{dt}\right]=6sY(s)$$

$$\mathcal{L}[8y(t)]=8Y(s)\,;\quad \mathcal{L}[2]=\frac{2}{s}$$

$$Y(s)\left[S^2+6S+8\right]=\frac{2}{S}$$

$$Y(s)=\frac{2}{S\left[S^2+6S+8\right]}=\frac{2}{s(s+2)(s+4)}$$

$$\mathcal{L}^{-1}Y(s)=\mathcal{L}^{-1}\frac{A}{s}+\mathcal{L}^{-1}\frac{B}{s+2}+\mathcal{L}^{-1}\frac{C}{s+4}$$

$$\boxed{y(t)=A+Be^{-2t}+Ce^{-4t}}$$

$$Y(s)=\frac{2}{s(s+2)(s+4)}=\frac{A}{s}+\frac{B}{s+2}+\frac{C}{s+4}$$

$$=\frac{A\left[s^2+6s+8\right]+B\left[s^2+4s\right]+C\left[s^2+2s\right]}{s(s+2)(s+4)}$$

$$\frac{0s^2+0s+2}{s(s+2)(s+4)}=\frac{s^2[A+B+C]+s[6A+4B+2C]+8A}{s(s+2)(s+4)}$$

$8A = 2$ —— (1)

$6A + 4B + 2C = 0$ —— (2)

$A + B + C = 0$ —— (3)

$-2: \; -2A - 2B - 2C = 0$

$4A + 2B = 0$ ——

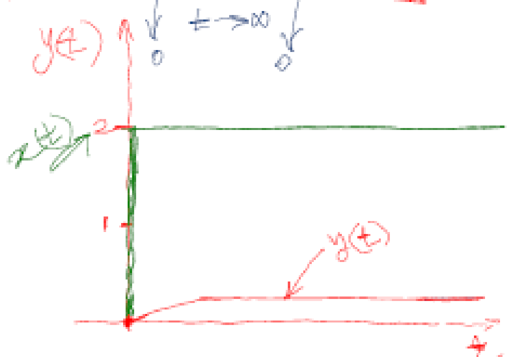
$1 + 2B = 0 \qquad 2B = -1 \Rightarrow B = -1/2$

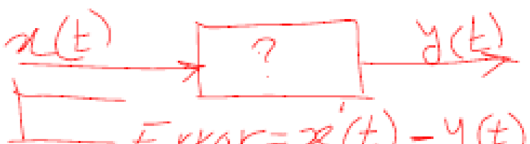
$1/4 - 1/2 + C = 0 \qquad C = 1/4$

$$\boxed{y(t) = 1/4 - 1/2\, e^{-2t} + 1/4\, e^{-4t}}$$

$t \to \infty$: $e^{-2t} \to 0$, $e^{-4t} \to 0$

$e^{-\infty} = \frac{1}{e^{\infty}} = \frac{1}{\infty} = 0$





$\text{Error} = x(t) - y(t)\Big|_{t=\infty}$

Final Value Theorem

$$Y(s) = \frac{2}{s(s^2+6s+8)}$$

$y(t=\infty) = \lim_{s \to 0} sY(s)$ ← Final Value Theorem

$y(t=0) = \lim_{s \to \infty} sY(s)$ ← initial Value Theorem

$$\lim_{s \to 0} \frac{\cancel{s}\, 2}{\cancel{s}(\cancel{s^2} + \cancel{6s} + 8)}$$

$y(t=\infty) = \frac{2}{8} \to 1/4$ ← Final Value

$=$ steady state o/p

Example 2

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 6y(t)$$

$$s^2Y(s) + 5Y(s) + 6Y(s)$$

$$Y(s)\left[\underbrace{s^2+5s+6}\right]$$

$$Y(s) = \frac{s+1}{\underbrace{(s+2)(s+3)}_{s^2+5s+6}} = \ldots$$

$y(t)$

$$Y(s) = \frac{A}{s+2} + \frac{B}{s+3}$$

$$Y(s) = \mathcal{L}^{-1}\left[\frac{s+1}{(s+2)(s+3)}\right] = \frac{A(s+3)+B(s+2)}{(s+2)(s+3)} \longrightarrow \boxed{X}$$

$$\mathcal{L}^{-1}\frac{A}{s+2} + \mathcal{L}^{-1}\frac{B}{s+3}$$

$$\mathcal{L}\left[e^{-at}\right] = \frac{1}{s+a}$$

$$y(t) = Ae^{-2t} + Be^{-3t}$$

من البسط لمعادلة $Y(s)$ للطرفين معادلة $\boxed{X}$
نستنتج التالي

البسط الايمن $A(s+3)+B(s+2) = (A+B)s + (3A+2B)$

البسط الايسر $= s+1$

المفروض حدود البسط الحقيقي = حدود البسط لابلاس

اي ان

$$\cancel{s} = (A+B)\cancel{s}$$

$$1 = A+B \quad \text{——— } (1)$$

و

$$1 = 3A+2B \quad \text{——— } (2)$$

from (1) $A = 1-B$ ——— (3)

عوض (3) في (2)

$$3(1-B)+2B = 1$$

$$3-3B+2B = 1$$

$$-B = -2 \quad \therefore B = 2$$

بما أن $A = 1-B$

$$\therefore A = 1-2 = -1$$

and

$$y(t) = Ae^{-2t} + Be^{-3t} = -e^{-2t} + 2e^{-3t}$$

طريقة اخرى لايجاد B & A

$$A = \left.\frac{(s+1)\cancel{(s+2)}}{\cancel{(s+2)}(s+3)}\right|_{s=-2} = \frac{-2+1}{-2+3} = \frac{-1}{1} = -1$$

$$B = \left.\frac{(s+1)\cancel{(s+3)}}{(s+2)\cancel{(s+3)}}\right|_{s=-3} = \frac{-3+1}{-3+2} = \frac{-2}{-1} = 2$$

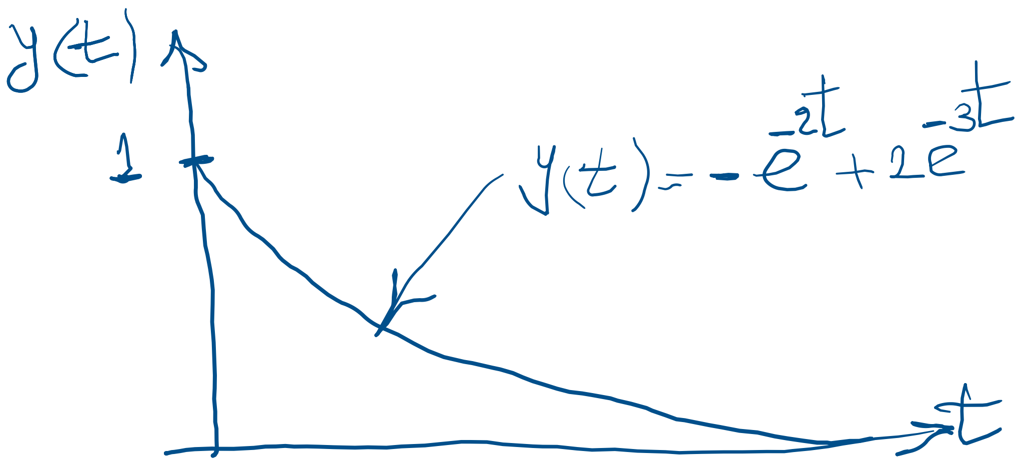
$$\boxed{y = -e^{-2t} + 2e^{-3t}}$$

ونبدل

Find $y(t=0)$ and $y(t=\infty)$?

$$y(t=0) = -e^{-0} + 2e^{-0} = -1+2 = 1$$

$$y(t=\infty) = -e^{-\infty} + 2e^{-\infty} = -\frac{1}{e^{\infty}} + \frac{2}{e^{\infty}} = 0$$



Osama.ali.awad@uomus.edu.iq