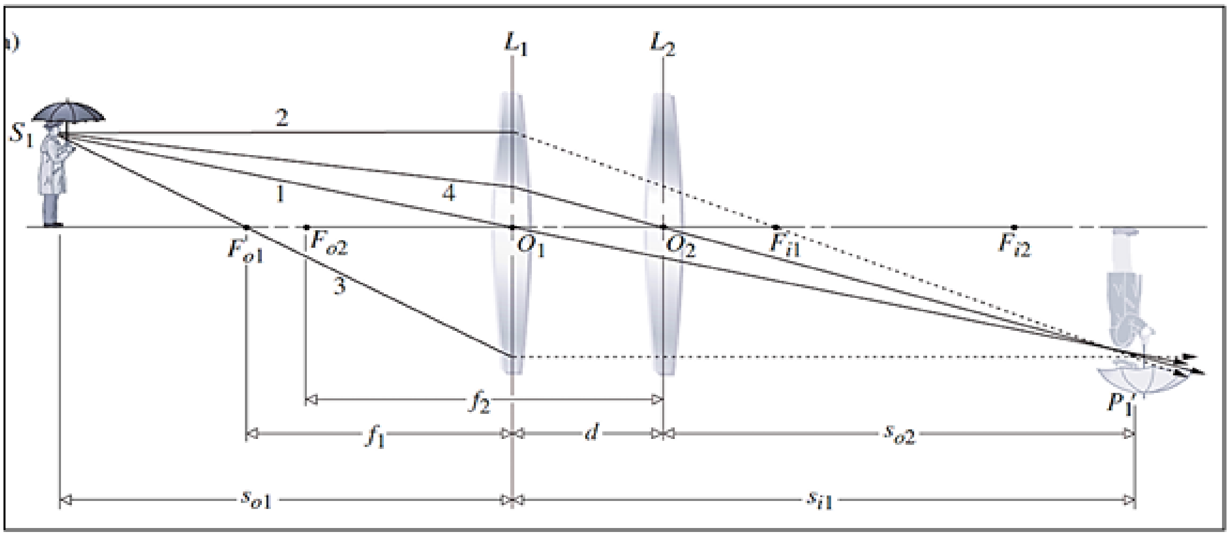
Thin-Lens Combinations



Consider two thin positive lenses L1 and L2 separated by a distance d, which is smaller than either focal length. The resulting image can be located graphically as follows. Overlooking L2 for a moment, construct the image formed exclusively by L1 using rays-2 and -3. As usual, these pass through the lens object and image foci, Fo1 and Fi1, respectively. The object is in a normal plane, so that two rays determine the top of the image, and a perpendicular to the optical axis finds its bottom. Ray-4 is then constructed running backward from P'1 through O2. Insertion of L2 has no effect on ray-4, whereas ray-3 is refracted through the image focus Fi2 of L2. The intersection of rays-4 and -3 fixes the image, which in this particular case is real, minified, and inverted. Notice that if the focal length of L2 is increased with all else constant, the size of the image increases as well. Analytically, looking for

$$\frac{1}{f1} = \frac{1}{so1} + \frac{1}{si1} \qquad (1)$$

This is positive, and the intermediate image (at P'1) is to the right of L1, when so1> $f1$ and $f1 > 0$.

Now considering the second lens L_2 with its object at P'_1

$$s_{o2} = d - s_{i1} \qquad (2)$$

if $d > s_{i1}$, the object for L_2 is real, whereas if $d < s_{i1}$, it is virtual ($s_{o2} < 0$). In the former instance the rays approaching L_2 are diverging from P'_1, whereas in the latter they are converging toward it. The intermediate image formed by L_1 is the virtual object for L_2. Furthermore, for L_2

$$\frac{1}{f_2} = \frac{1}{s_{o2}} + \frac{1}{s_{i2}} \qquad (3)$$

By substituting (2) in (3), we obtain;

$$s_{i2} = \frac{f_2 d - \frac{f_2 s_{o1} f_1}{s_{o1} - f_1}}{d - f_2 - \frac{s_{o1} f_1}{s_{o1} - f_1}} \qquad (4)$$

Here s_{o1} and s_{i2} are the object and image distances, respectively, of the compound lens.

The total transverse magnification of the compound lens is the product of the individual magnifications;

$$M_T = M_1 M_2 \qquad (5)$$

$$M_T = \frac{f_1 s_{i2}}{d(s_{o1} - f_1) - s_{o1} f_1} \qquad (6)$$

Example: A thin biconvex lens having a focal length of +40.0 cm is located 30.0 cm in front (i.e., to the left) of a thin biconcave lens of focal length -40.0 cm. If a small object is situated 120 cm to the left of the positive lens (a) determine the location of its image by calculating the effect of each lens. (b) Compute the magnification. (c) Describe the image.

Solution:

(a)

$$s_{i2} = \frac{f_2 d - \frac{f_2 s_{o1} f_1}{s_{o1} - f_1}}{d - f_2 - \frac{s_{o1} f_1}{s_{o1} - f_1}}$$

$$s_{i2} = 120\ cm$$

The image is formed 120 cm to the right of the negative lens.

(b)

$$\frac{1}{f_2} = \frac{1}{s_{o2}} + \frac{1}{s_{i2}} \rightarrow \frac{1}{-40} = \frac{1}{s_{o2}} + \frac{1}{120}$$

$$s_{o2} = -30\ cm$$

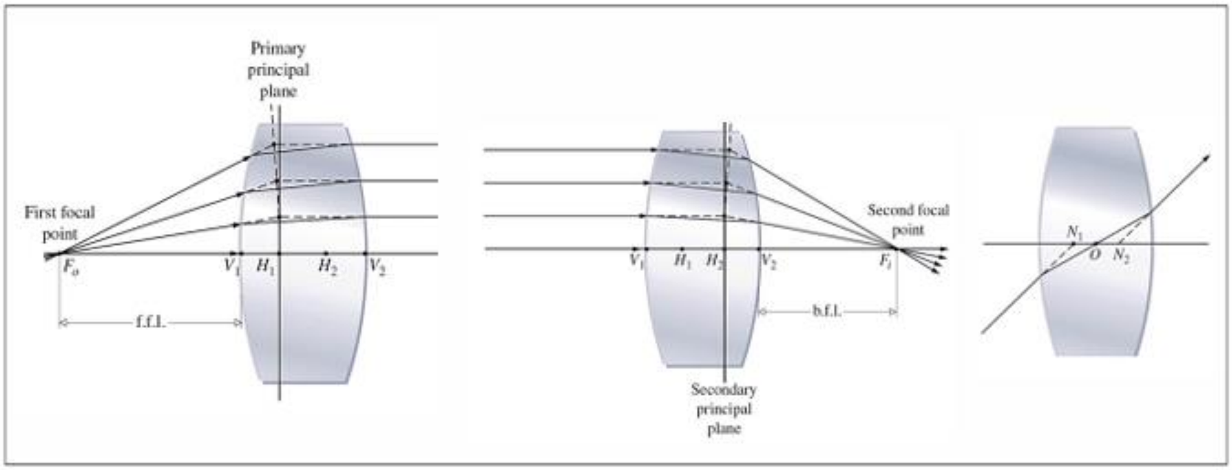
The magnification is

$$M_T = M_1 M_2 = \left(\frac{-s_{i1}}{s_{o1}}\right)\left(\frac{-s_{i2}}{-s_{o2}}\right) = -2$$

(c) The image is real, because $s_{i2} > 0$; inverted, because $M_T < 0$; and magnified.

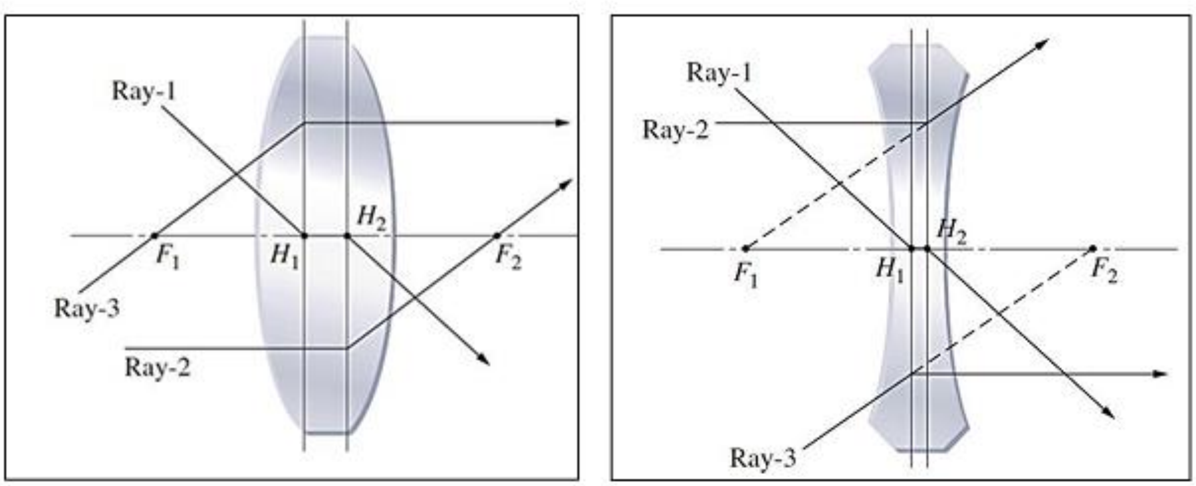
Thick lenses

The figure below shows a thick lens. The first and second focal points, or the object and image foci, fo and fi, can conveniently be measured from the two (outermost) vertices. In that case we have the familiar front and back focal lengths denoted by f.f.l. And b.f.l. Points where the primary and secondary principal planes intersect the optical axis are known as the first and second principal points, H1 and H2, respectively. Extending both the incoming and outgoing rays until they cross the optical axis locates what are called the nodal points, N1 and N2. When the lens is surrounded on both sides by the same medium, generally air, the nodal and principal points will be coincident. The six points, two focal, two principal, and two nodal, constitute the cardinal points of the system.



Tracing rays through a thick lens

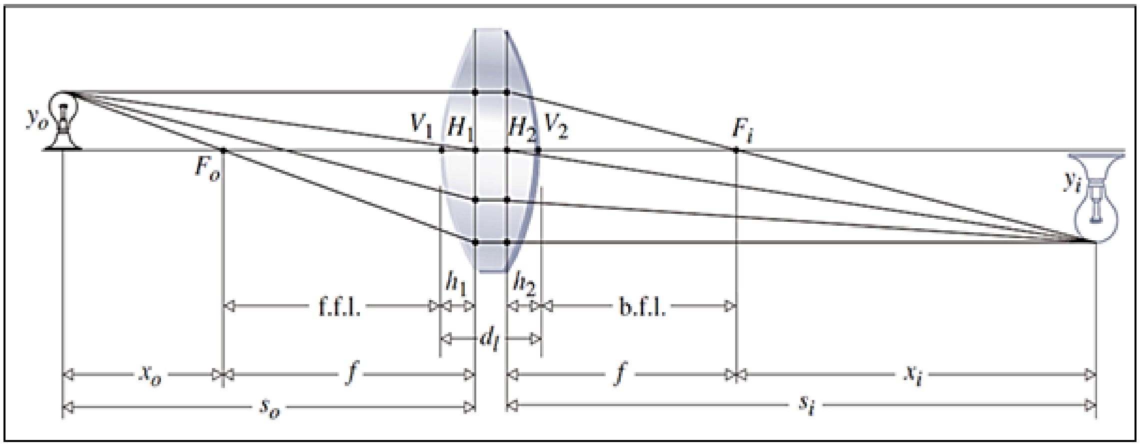
Any ray impinging on the first lens face must be extended until it intersects the first principal plane, the one at H1. This "ghost" ray traverses the gap between H1 and H2 parallel to the optical axis. It strikes the second principal plane, the one at H2, refracts, and passes straight out of the lens in a direction yet to be determined. Just as with the thin lens, there are three special rays whose passage into, across, and out of the thick lens.



- Ray-1 heading toward point-H1, after striking H1 it moves on to H2, traveling parallel to the central axis. At H2 it refracts and emerges parallel to the incoming ray.
- Ray-2, is traveling parallel to the central axis. It strikes the first principal plane and passes on, undeflected, to the second principal plane, where it refracts. If the lens is positive, ray-2 converges to back focal point-F2. If the lens is negative, ray-2 diverges as if from front focal point-F1
- For a positive lens, ray-3 is the one that passes through front focal point-F1, strikes the first principal plane, refracts parallel to the central axis, and, undeflected, continues on.
- For a negative lens, ray-3, heading toward back focal point-F2, strikes the first principal plane, refracts parallel to the central axis, and, undeflected.
- Any parallel bundle of rays entering a positive thick lens must emerge as a converging cone heading toward a point on its focal plane. And any parallel bundle of rays entering a negative thick lens must emerge as a cone diverging from a point on its focal plane.

Thick-lenses Equations

The thick lens can be treated as consisting of two spherical refracting surfaces separated by a distance dl between their vertices.



$$\frac{1}{f} = \frac{1}{s_o} + \frac{1}{s_i} \qquad (6)$$

$$\frac{1}{f} = (n_l - 1)\left(\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n_l - 1)d_l}{n_l R_1 R_2}\right) \qquad (7)$$

f is the focal length

n_l is the refractive index of the lens

R_1 and R_2 are the radii of carvature

d_l is the thickness of the lens

The principal planes are located at distances of $\overline{V_1H_1} = h_1$. And $\overline{V_2H_2} = h_2$, which are positive when the planes lie to the right of their respective vertices.

$$h_1 = -\frac{f(n_l - 1)d_l}{n_l R_2} \qquad (8)$$

$$h_2 = -\frac{f(n_l - 1)d_l}{n_l R_1} \qquad (9)$$

The magnification is;

$$M = \frac{y_i}{y_o} = \frac{-x_i}{f} = \frac{-f}{x_o} \qquad (10)$$

Example: Find the image distance for an object positioned 30 cm from the vertex of a double convex lens having radii of 20 cm and 40 cm, a thickness of 1.0 cm, and an index of 1.5

Solution:

$$\frac{1}{f} = (n_l - 1)(\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n_l - 1)d_l}{n_l R_1 R_2})$$

$$f = 26.8\ cm$$

$$h_1 = -\frac{f(n_l - 1)d_l}{n_l R_2} = +0.22\ cm$$

$$h_2 = -\frac{f(n_l - 1)d_l}{n_l R_1} = -0.44\ cm$$

which means that H_1 is to the right of V_1, and H_2 is to the left of V_2.

Finally, $s_o = 30 + 0.22$

$$\frac{1}{f} = \frac{1}{s_o} + \frac{1}{s_i}$$

$s_i = 238\ cm$ measured from H_2

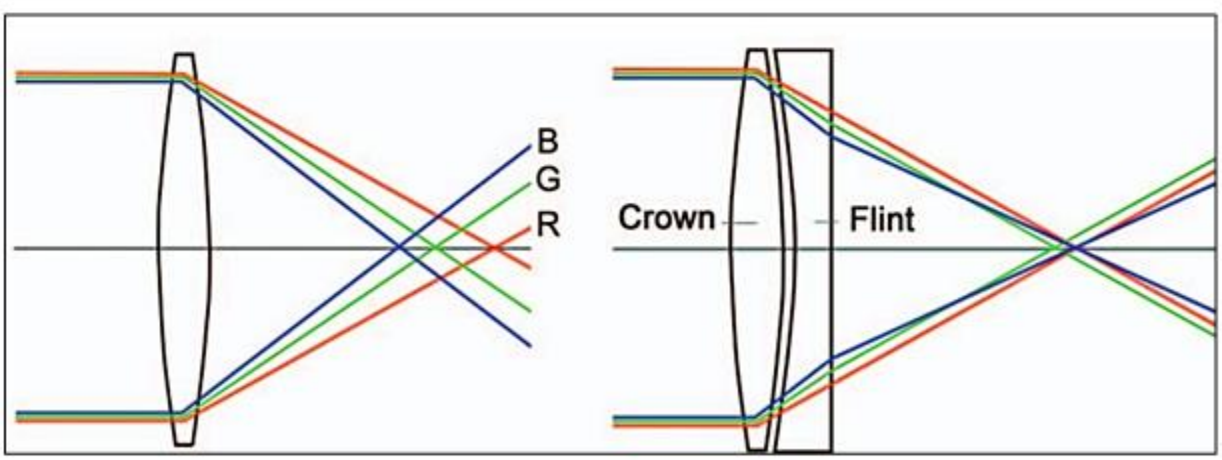
Abbreviation

Abbreviation occurs when the rays pass through a lens with spherical surfaces, some of them will converge to a point, and some of them will converge at other points. There is no single point at which they all come to a sharp focus. It can be classified into two categories; chromatic and monochromatic abbreviations.

1. **Chromatic Abbreviation** For any given lens, the shape and thickness are fixed, and cannot vary. The index of refraction of any piece of glass depends on the wavelength of light. The index of refraction is higher for short wavelengths as compared to long

wavelengths. If two parallel rays of white light striking a convergent lens, the index of refraction for violet light is relatively high. The focal length for violet light is short, and the violet image is close to the lens. For red light, the index of refraction is lower. So, the focal length is longer, and the red image is far from the lens. Different image for each wavelength is produced and the images will be strung out along the axis. There is no point where they all come to a single focus.

The size of the image depends on the focal length of the lens. Since the lens has a different focal length for each color (wavelength), each of the images will be a different size. If you bring any one of the point images to a sharp focus on a ground glass screen, it will be surrounded by colored halos. Chromatic abbreviation can be corrected by combining two lenses, so the abbreviation of one lens cancels the aberration of the other lens. For example, if you use two thin convergent lenses made from the same kind of glass, separated by half the sum of their focal lengths. Other eyepieces, and most objectives, are corrected by using two lenses of different kinds of glass, with little or no separation between them.



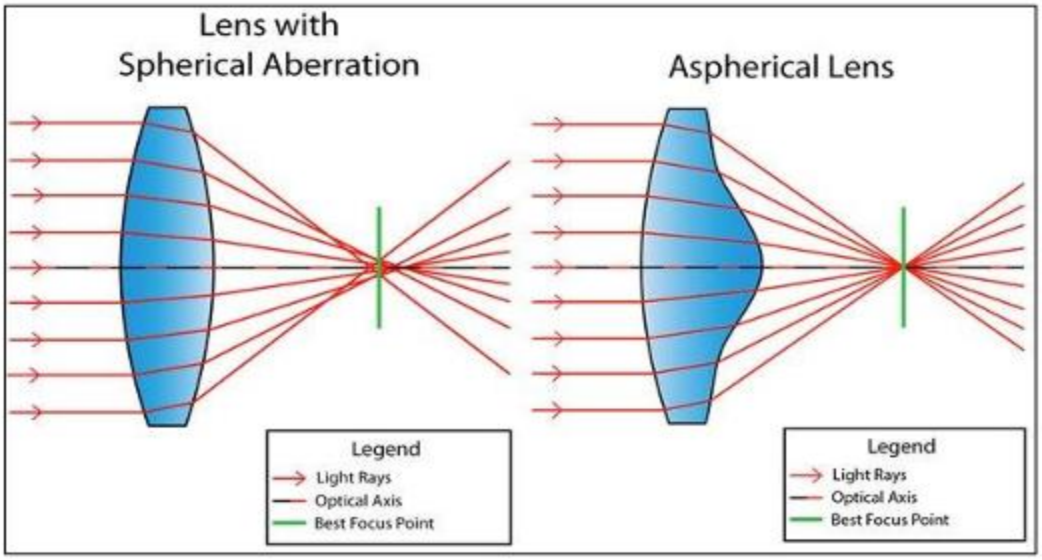
2. Spherical Abbreviation

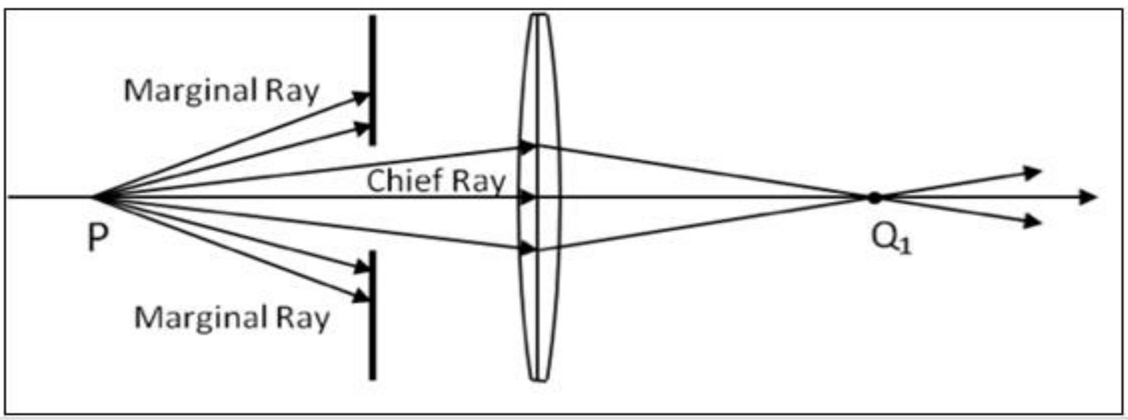
A convergent lens will bend parallel (monochromatic (single wavelength)) rays to a focus only if its surface is a rather small part of a sphere. The rays near the edges of the lens are bent more sharply than those near the axis. There is no single point at which they all come to a focus. This is not a defect of the lens. It is just the way light acts when it strikes a spherical surface. The amount of spherical aberration in either a convergent or divergent lens is influenced by

- the thickness of the lens
- its focal length.

A thin lens with a long focal length has less aberration than a thick lens with a short focal length.

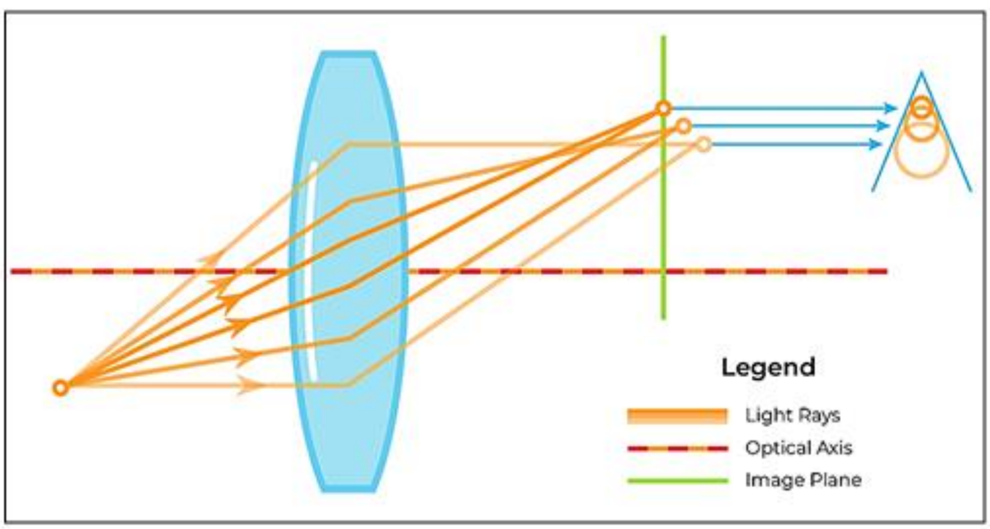
One method of reducing spherical aberration, is to test a lens to find out how much of the area around the optical axis (where the lens is most free of aberration) may be used to form a sharp image. Then use a field stop to mask out all rays that pass through the lens beyond this circle. Note the rays blocked by the field stops. The field stop is a flat ring or diaphragm made of metal (or other suitable opaque material) to mask the outer portion of the lens. The stop prevents rays from striking the lens and thus reduces the amount of light that passes through it. Spherical aberration in a lens can also be minimized by bending the lens. This can be accomplished by increasing the curvature of one surface and decreasing the curvature of the other surface. This process retains the same focal length, but reduces the amount of aberration.


Lens with
Spherical Aberration
Aspherical Lens
Legend
Light Rays
Optical Axis
Best Focus Point
Legend
Light Rays
Optical Axis
Best Focus Point


Marginal Ray
Chief Ray
P
Q1
Marginal Ray

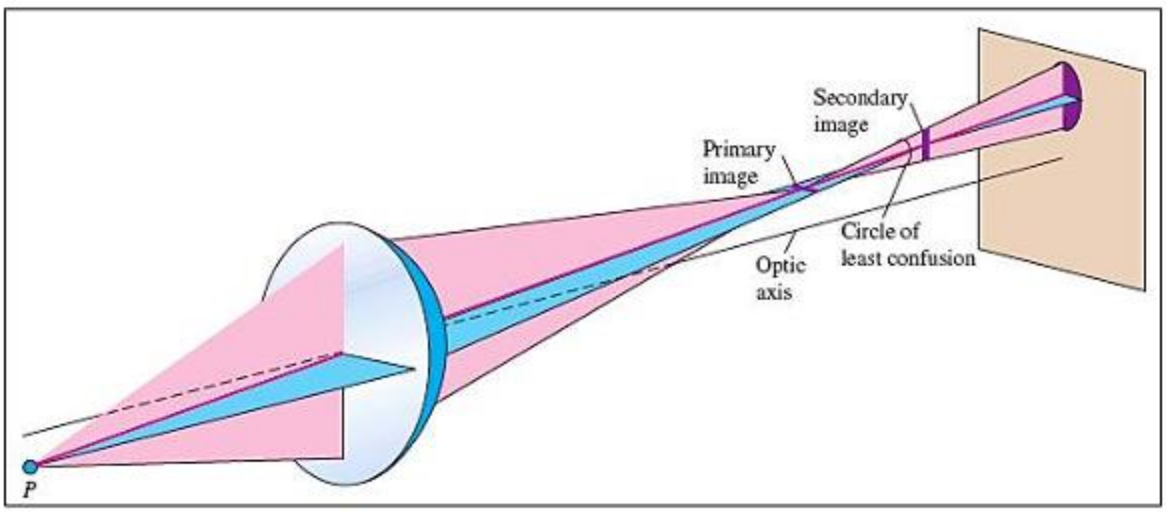
3. COMA

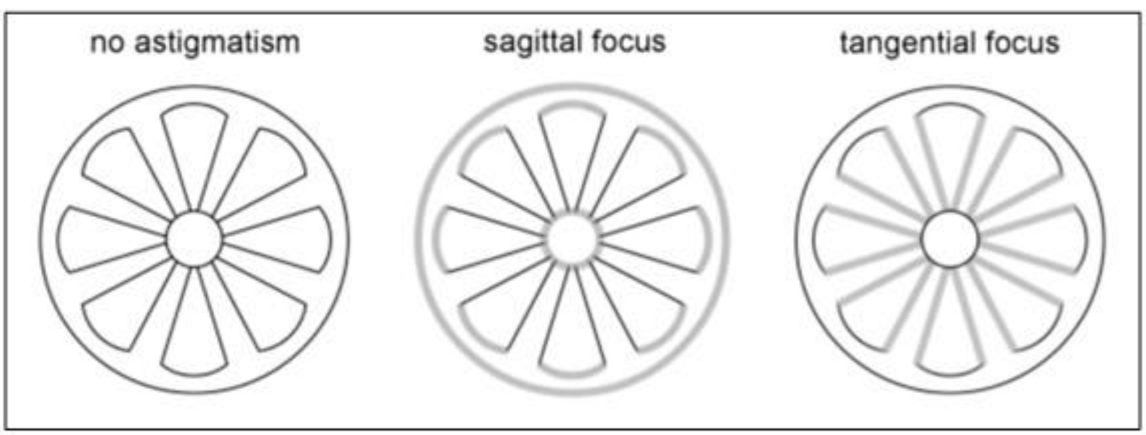
Coma has the same cause as spherical aberration; the edges of the lens have a higher refractive power than the center. Coma affects rays from a point OFF the lens axis. The image of that point, instead of being a disk, is shaped like a comet. The image is made up of a number of partly superimposed disks, each formed by a different part of the lens. Coma can be reduced by making a proper choice of curvatures for the refracting surfaces. Unlike spherical aberration, coma can be completely eliminated from a single lens, for one particular object distance. (It will still show coma for objects at other distances). Unfortunately, the curvatures that eliminate coma from a lens system are not the same as those that minimize spherical aberration. The people who design lens systems must choose between these two aberrations, or make compromises between them.



4. Astigmatism

ASTIGMATISM has two meanings. One refers to a true aberration, caused by the behavior of light at spherical surfaces. The other refers to a lens defect and happens when the surface is not truly spherical. Like coma, astigmatism affects the images of points some distance off the axis, but the effects are different. Coma spreads out the image in a plane perpendicular to the axis. Astigmatism spreads the image in a direction along the axis. What can we do about astigmatism? By properly selecting the curvatures of the lens surfaces, we can make the secondary image plane coincide with the first. The image will then be sharp all over. It will lie on a curved surface, and you will have curvature of field. The most common method of correcting this aberration is by use of a combination of lenses called field flatteners. They produce an opposite curvature of field and cancel aberration.





5. Distortion

All other aberrations affect the sharpness of an image. An image can be perfectly sharp all over, but it can still be distorted. The lens has a different magnification for objects off the axis than for objects on the axis. If magnification is less for objects off the axis, you will have BARREL DISTORTION. If it increases as you leave the axis, you will have PINCUSHION DISTORTION. A single thin lens will form an undistorted image. As soon as you put a diaphragm on the axis, you will get distortion. If you put the diaphragm in front of a lens, the image formed by that lens will show barrel distortion. If you put the diaphragm behind the lens, you will get pincushion distortion. To overcome the distortion abbreviation is to use a compound lens with a stop field between the two elements. Let the distortions cancel each other.